# ADDING AN INSURANCE MARKET TO OUR EXAMPLE

NOW, IN ADDITION TO OUR CREDIT MARKET, SUPPOSE WE ALSO HAVE A MARKET IN WHICH ONE CAN INSURE AGAINST STATE L. IN OTHER WORDS, WE HAVE TWO SECURITIES, WITH RETURNS VECTORS AS FOLLOWS:

$$d_1 = \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} \quad \text{AND} \quad d_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{array}{l} \leftarrow s=H \\ \leftarrow s=L \end{array}$$

$\uparrow$ A BOND OR SAVING ACCOUNT $\qquad$ $\uparrow$ AN INSURANCE CONTRACT

WHEN WE HAD ONLY A CREDIT MARKET, A CONSUMER COULD ACHIEVE ONLY $x_H - \mathring{x}_H = x_L - \mathring{x}_L$ — i.e., $x_H$ AND $x_L$ COULD NOT BE CHOSEN INDEPENDENTLY. BUT HERE $d_1$ AND $d_2$ SPAN $\mathbb{R}^2$, SO BY CHOOSING AMOUNTS $y_1$ AND $y_2$ OF THE TWO SECURITIES, A CONSUMER CAN ACHIEVE ANY $x_H$ AND $x_L$:

$$\begin{bmatrix} x_H \\ x_L \end{bmatrix} = \begin{bmatrix} \mathring{x}_H \\ \mathring{x}_L \end{bmatrix} + y_1 \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix},$$

i.e.,

$$\begin{bmatrix} x_H - \mathring{x}_H \\ x_L - \mathring{x}_L \end{bmatrix} = y_1 \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$z = Dy$

THIS SUGGESTS THAT PERHAPS AN EQUILIBRIUM WITH THESE TWO MARKETS MIGHT BE PARETO EFFICIENT. LET'S DETERMINE THE EQUILIBRIUM.

THE CONSUMER'S MAXIMIZATION PROBLEM:

$$\max_{y_1, y_2, x_0, x_H, x_L} u(x_0, x_H, x_L) \quad \text{s.t.} \quad \begin{aligned} x_0 + y_1 + q_2 y_2 &\leqq \mathring{x}_0 & &: \lambda_0 \\ x_H &\leqq \mathring{x}_H + (1+r)y_1 + 0 & &: \lambda_H \\ x_L &\leqq \mathring{x}_L + (1+r)y_1 + y_2 & &: \lambda_L \end{aligned}$$

FOMC:

$$\left.\begin{aligned} x_0 &: \quad u_0 = \lambda_0 \\ x_H &: \quad u_H = \lambda_H \\ x_L &: \quad u_L = \lambda_L \\ y_1 &: \quad 0 = \lambda_0 - (1+r)\lambda_H - (1+r)\lambda_L \\ y_2 &: \quad 0 = q_2 \lambda_0 - \lambda_L \end{aligned}\right\} \quad \begin{aligned} MRS_H + MRS_L &= \frac{u_H}{u_0} + \frac{u_L}{u_0} \\ &= \frac{\lambda_H + \lambda_L}{\lambda_0} = \frac{1}{1+r} \end{aligned}$$

AND

$$MRS_L = \frac{u_L}{u_0} = \frac{\lambda_L}{\lambda_0} = q_2 .$$

EQUILIBRIUM:

(1) $MRS_H^i + MRS_L^i = \frac{1}{1+r}, \quad i = A, B \qquad \therefore MRS_H^A + MRS_L^A = MRS_H^B + MRS_L^B$

(2) $MRS_L^i = q_2, \quad i = A, B \qquad \therefore MRS_L^A = MRS_L^B$

COMBINING (1) AND (2) ALSO YIELDS $MRS_H^A = MRS_H^B$.

$\therefore$ WE HAVE PARETO EFFICIENCY:

$$x_H^A = 18, \; x_L^A = 12, \quad x_H^B = 12, \; x_L^B = 18,$$

AND $MRS_H^A = MRS_H^B = \frac{1}{5}, \; MRS_L^A = MRS_L^B = \frac{2}{5}$.

$\therefore q_2 = \frac{2}{5}; \; \frac{1}{1+r} = \frac{1}{5} + \frac{2}{5} = \frac{3}{5}; \; 1+r = \frac{5}{3}$ AND $r = \frac{2}{3}$.

$z = DY$

WE CAN SOLVE FOR THE $y_s^i$ VALUES:

$$\begin{bmatrix} x_H - \mathring{x}_H \\ x_L - \mathring{x}_L \end{bmatrix} = y_1 \begin{bmatrix} 1+r \\ 1+r \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix} = y_1 \begin{bmatrix} \frac{5}{3} \\ \frac{5}{3} \end{bmatrix} + y_2 \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

$= \begin{bmatrix} 18-15 \\ 12-15 \end{bmatrix}$ $\qquad$ $z = DY$

WE HAVE

FOR A: $\begin{bmatrix} 3 \\ -3 \end{bmatrix} = y_1^A \begin{bmatrix} 5/3 \\ 5/3 \end{bmatrix} + y_2^A \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$\therefore\ y_1^A = \frac{9}{5}, \quad y_2^A = -3 - \left(\frac{5}{3}\right)\left(\frac{9}{5}\right) = -3-3 = -6.$$

CHECK THIS:

$$x_H^A = 15 + \frac{5}{3}y_1 + 0y_2 = 15 + \left(\frac{5}{3}\right)\left(\frac{9}{5}\right) = 15+3 = 18$$

$$x_L^A = 15 + \frac{5}{3}y_1 + 1y_2 = 15 + 3 - 6 = 15 - 3 = 12.$$

SIMILARLY FOR B: $y_1^B = -\frac{9}{5}, \quad y_2^B = 6.$

AND WE HAVE

$$x_0^A = 15 - q_1 y_1 - q_2 y_2$$
$$= 15 - 1y_1 - \frac{2}{5}y_2 = 15 - \frac{9}{5} - \left(\frac{2}{5}\right)(-6) = 15\tfrac{3}{5}$$

$$x_0^B = 14\tfrac{2}{5}.$$

WE'VE SHOWN THAT THE EQUILIBRIUM WITH THESE TWO MARKETS IS EXACTLY THE ARROW-DEBREU ALLOCATION

NOTE THAT $q_1$ AND $q_2$ SATISFY $q_k = d_{Hk}p_H + d_{Lk}p_L$:

$$q_1 = d_{H1}p_H + d_{L1}p_L = \left(\frac{5}{3}\right)\left(\frac{1}{5}\right) + \left(\frac{5}{3}\right)\left(\frac{2}{5}\right) = \frac{1}{3} + \frac{2}{3} = 1$$

$$q_2 = d_{H2}p_H + d_{L2}p_L = (0)\left(\frac{1}{5}\right) + (1)\left(\frac{2}{5}\right) = \frac{2}{5}.$$

A SECURITY'S PRICE, IN EQUILIBRIUM, IS THE SUM OF THE STATE-CONTINGENT A-D PRICES, WEIGHTED BY THE SECURITY'S STATE-CONTINGENT RETURNS.