

# ADDING PRODUCTION TO THE THEORY

WE FIRST CONSIDER THE SIMPLEST SITUATION THAT INCLUDES PRODUCTION: TWO GOODS, ONE OF WHICH CAN BE TRANSFORMED INTO THE OTHER. WE ALLOW FOR MANY PRODUCERS ("FIRMS") WITH DIFFERING TECHNOLOGICAL CAPABILITIES. WE FIRST DETERMINE THE PARETO EFFICIENT PROGRAMS, THEN INVESTIGATE THE WALRASIAN EQUILIBRIA.

CONSUMERS,  $i \in \{1, \dots, n\} = N$ : UTILITY FUNCTIONS  $u^i(x_i, y_i)$  AND ENDOWMENTS  $(\bar{x}_i, \bar{y}_i)$ .

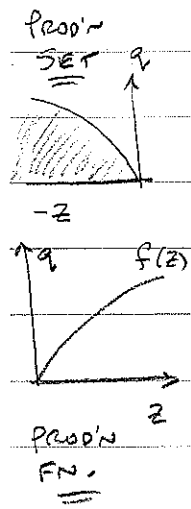
PRODUCERS,  $j \in \{1, \dots, m\} = M$ : PRODUCTION FUNCTIONS

$q_j = f_j(z_j)$  — i.e.,  $z_j$  IS INPUT LEVEL,  $q_j$  IS OUTPUT LEVEL.

INPUT IS X-GOOD, OUTPUT IS Y-GOOD: INPUT FROM  $i$  IS

$\bar{x}_i - x_i$  (X-ENDOWMENT NOT CONSUMED); OUTPUT

AUGMENTS Y-GOOD:  $\sum_{i=1}^n y_i \leq \bar{y} + \sum_{j=1}^m q_j$ .



THE "DATA" THAT DESCRIBE THE ECONOMY ARE THUS

THE  $n$ -TUPLES  $(u^i)_1^n$  AND  $((\bar{x}_i, \bar{y}_i))_1^n$  AND THE  $m$ -TUPLE  $(f_j)_1^m$ .

WE INVESTIGATE PARETO EFFICIENCY BY LOOKING ONLY FOR PROGRAMS (PROFILES OF CONSUMPTION AND PRODUCTION PLANS), WITHOUT MAKING ANY BEHAVIORAL ASSUMPTIONS (SUCH AS UTILITY- OR PROFIT-MAXIMIZATION) OR ASSUMING THAT MARKETS AND PRICES ARE AVAILABLE.

## THE FEASIBLE ALLOCATIONS, OR PROGRAMS:

$$(1) \quad \sum_{i=1}^n x_i + \sum_{j=1}^m z_j \leq \bar{x}$$

$$(2) \quad \sum_{i=1}^n y_i \leq \bar{y} + \sum_{j=1}^m q_j$$

$$(3) \quad q_j \leq f_j(z_j), \quad \forall j.$$

IF WE ASSUME THAT  $q_j = f_j(z_j)$  FOR EACH  $j$  (AS WILL BE THE CASE AT AN EFFICIENT ALLOCATION IF EACH  $f_j$  IS INCREASING AND EACH  $z_j$  INCREASING), THEN ~~WE CAN ELIMINATE THE VARIABLES  $q_1, \dots, q_m$~~  AND THE FEASIBLE PROGRAMS ARE THE ONES THAT SATISFY (1) AND

$$(4) \quad \sum_{i=1}^n y_i \leq \bar{y} + \sum_{j=1}^m f_j(z_j).$$

## PARETO EFFICIENT ALLOCATIONS:

WE CONTINUE TO DEFINE PARETO EFFICIENCY ONLY IN TERMS OF CONSUMERS' PREFERENCES OVER THEIR CONSUMPTION BUNDLES.

$$\max \lambda_1 u^1(x_1, y_1) \quad \text{s.t.} \quad x_i, y_i, z_j \geq 0 \quad (\forall i, j)$$

AND TO

$$\sum_{i=1}^n x_i + \sum_{j=1}^m z_j \leq \bar{x} \quad : \quad \sigma_x$$

$$\sum_{i=1}^n y_i \leq \bar{y} + \sum_{j=1}^m f_j(z_j) \quad : \quad \sigma_y$$

$$u^i(x_i, y_i) \geq c_i \quad (i=2, \dots, n) \quad : \quad \lambda_2, \dots, \lambda_n$$

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## FIRST-ORDER CONDITIONS:

### MARGINALS:

- (1)  $x_i$ :  $\lambda_i u_x^i \leq \sigma_x$ , w/ EQUALITY IF  $x_i > 0$  ( $\forall i$ )  
 (2)  $y_i$ :  $\lambda_i u_y^i \leq \sigma_y$ , w/ EQUALITY IF  $y_i > 0$  ( $\forall i$ )  
 (3)  $z_j$ :  $0 \leq \sigma_x - \sigma_y f_j'(z_j)$ , w/ EQUALITY IF  $z_j > 0$  ( $\forall j$ )

### ADDING-UP:

- (4)  $\sigma_x$ :  $\sum x_i + \sum z_j \leq \bar{x}$ , w/ EQUALITY IF  $\sigma_x > 0$   
 (5)  $\sigma_y$ :  $\sum y_i \leq \bar{y} + \sum f_j(z_j)$ , w/ EQUALITY IF  $\sigma_y > 0$ .  
 (6)  $\lambda_i$ :  $u^i(x_i) \geq c_i$  ( $i=2, \dots, n$ ), w/ EQUALITY IF  $\lambda_i > 0$ .

## ECONOMIC INTERPRETATION OF FOC:

ASSUME  $x_i, y_i, z_j > 0$  ( $\forall i, j$ ) AND  $\lambda_i > 0$  ( $\forall i$ );  $\sigma_x, \sigma_y > 0$

$\therefore$  ALL THE FOC ARE EQUATIONS.

THE CONDITIONS (1) AND (2) ARE THE FAMILIAR  $MRS^i = \sigma_x / \sigma_y$  FOR EACH  $i$ , WHICH YIELD EQUALITY OF ALL CONSUMERS' MRS'S. COMBINING THIS WITH (3) YIELDS

$$(7) \quad f_j'(z_j) = MRS_{xy}^i \quad (\forall i, j),$$

$$\text{i.e., } \frac{dq_j}{dz_j} = - \frac{dy_i}{dx_i}, \quad \text{EQUALITY OF}$$

MARGINAL VALUES OF  $x$  IN TERMS OF  $y$ , IN PRODUCTION AND CONSUMPTION; i.e.,

$$(8) \quad MP_j^i = MRS_{xy}^i \quad (\forall i, j).$$

WE COULD JUST AS WELL WRITE THE EQUATIONS (7) USING THE RECIPROCAL OF EACH SIDE OF THE EQUATION:

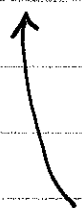
$$(9) \quad \frac{1}{f_j'(z_j)} = MRS_{yx}^i \quad (\forall i, j)$$

$$\text{i.e.,} \quad \frac{dz_j}{dq_j} = - \frac{dx_i}{dy_i}, \quad \text{EQUALITY OF}$$

MARGINAL VALUES OF  $y$  IN TERMS OF  $x$ ,  
IN PRODUCTION AND CONSUMPTION; i.e.,

$$(10) \quad MC^j = MV^i$$

↑  $i$ 'S MARGINAL VALUE (OR MARGINAL WILLINGNESS TO PAY) FOR THE  $y$ -GOOD, IN TERMS OF THE  $x$ -GOOD.



THE REAL MARGINAL COST TO  $j$  OF AN ADDITIONAL UNIT OF OUTPUT  $q_j$ , IN TERMS OF UNITS OF INPUT  $z_j$  REQUIRED.

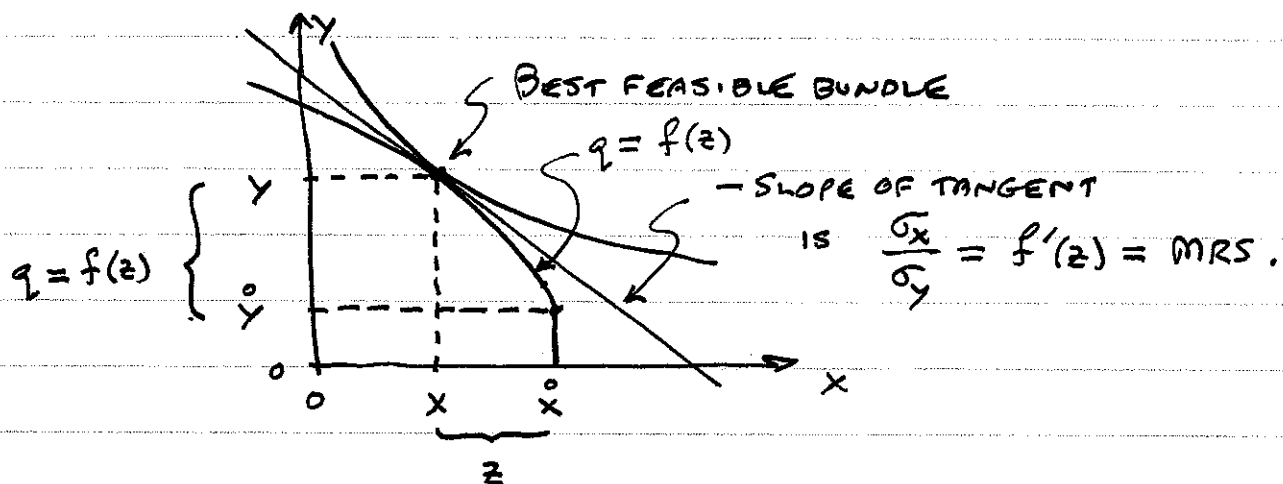
AND OF COURSE THE MARGINAL CONDITIONS (8) OR (10) MUST BE SUPPLEMENTED BY THE ADDING-UP CONDITIONS

$$\sum x_i + \sum z_j = \bar{x} \quad \text{AND} \quad \sum y_i = \bar{y} + \sum f_j(z_j)$$

(RECALL THAT WE HAVE ASSUMED THAT  $\sigma_x, \sigma_y > 0$ ).

## "FISHER'S SEPARATION THEOREM"

LET'S CONSIDER THE ROBINSON CRUSOE SITUATION, IN WHICH  $n=m=1$ . DIAGRAMMATICALLY:



THE PARETO EFFICIENCY PROBLEM IS ROBINSON'S OWN UTILITY-MAXIMIZATION PROBLEM (SUBJECT TO THE CONSTRAINTS  $x+z \leq \bar{x}$  AND  $y \leq \bar{y} + f(z)$ ; NOT TO A BUDGET CONSTRAINT). WE CAN WRITE IT AS A ONE-VARIABLE UNCONSTRAINED PROBLEM (BY RECOGNIZING THAT BOTH INEQUALITIES WILL BE EQUALITIES AT THE OPTIMUM):

$$\max_z u(\bar{x}-z, \bar{y}+f(z)).$$

THE FIRST-ORDER CONDITION:

$$\frac{d}{dz} u(\bar{x}-z, \bar{y}+f(z)) = 0;$$

$$\text{i.e., } -u_x + f'(z)u_y = 0;$$

$$\text{i.e., } f'(z) = \frac{u_x}{u_y}; \quad \text{i.e., } MP = MRS.$$

SUPPOSE THAT WE (OR ROBINSON) WANTED TO DECENTRALIZE THE ALLOCATION THAT MAXIMIZES  $u$ :

WE WANT TO FIND PRICES  $p_x$  AND  $p_y$  AT WHICH

(a) AS PRODUCTION MANAGER, ROBINSON WILL CHOOSE THE RIGHT  $z$  BY MAXIMIZING PROFIT, AND

(b) AS CONSUMPTION MANAGER ("BUYER"), ROBINSON WILL CHOOSE THE RIGHT  $(x, y)$  BY MAXIMIZING UTILITY SUBJECT TO THE  $(p_x, p_y)$ -BUDGET CONSTRAINT.

(TO SIMPLIFY THINGS, LET  $\bar{y} = 0$ .)

IT'S OBVIOUS THAT THE PRICES WILL HAVE TO BE IN THE RATIO

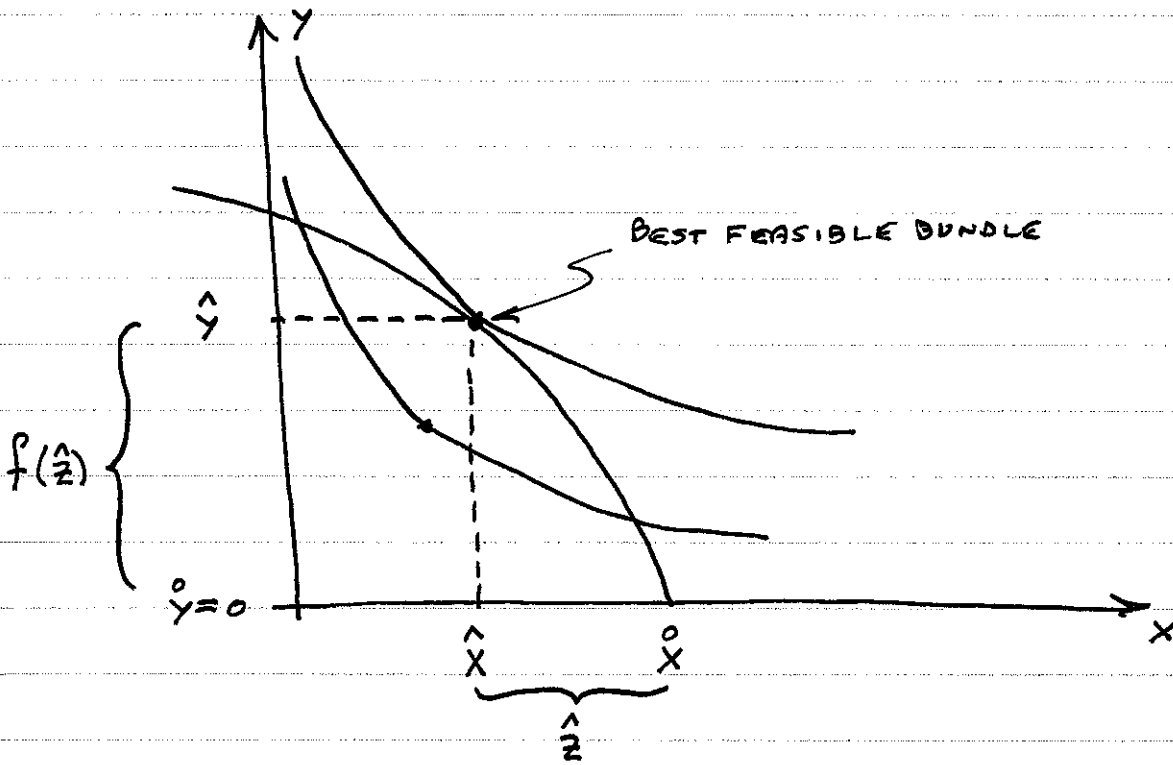
$$\frac{p_x}{p_y} = \frac{\sigma_x}{\sigma_y} \quad (= f'(z))$$

IN ORDER TO GENERATE THE RIGHT  $z$ . BUT THE BUDGET CONSTRAINT WITH THESE PRICES,

$$p_x x + p_y y \leq p_x \bar{x} + p_y \bar{y},$$

MAY NOT PERMIT THE CONSUMER—ROBINSON TO CHOOSE A LARGE ENOUGH BUNDLE  $(x, y)$  — AS IN FIGURE 1.

THE REASON FOR THIS: ALTHOUGH  $f'(z)$  IS THE MARGINAL RATE OF TRANSFORMATION, IT IS NOT THE AVERAGE RATE (IN GENERAL), WHICH IS TYPICALLY LARGER. IN OTHER WORDS, THERE IS TYPICALLY A "SURPLUS" — PROFIT IS POSITIVE.



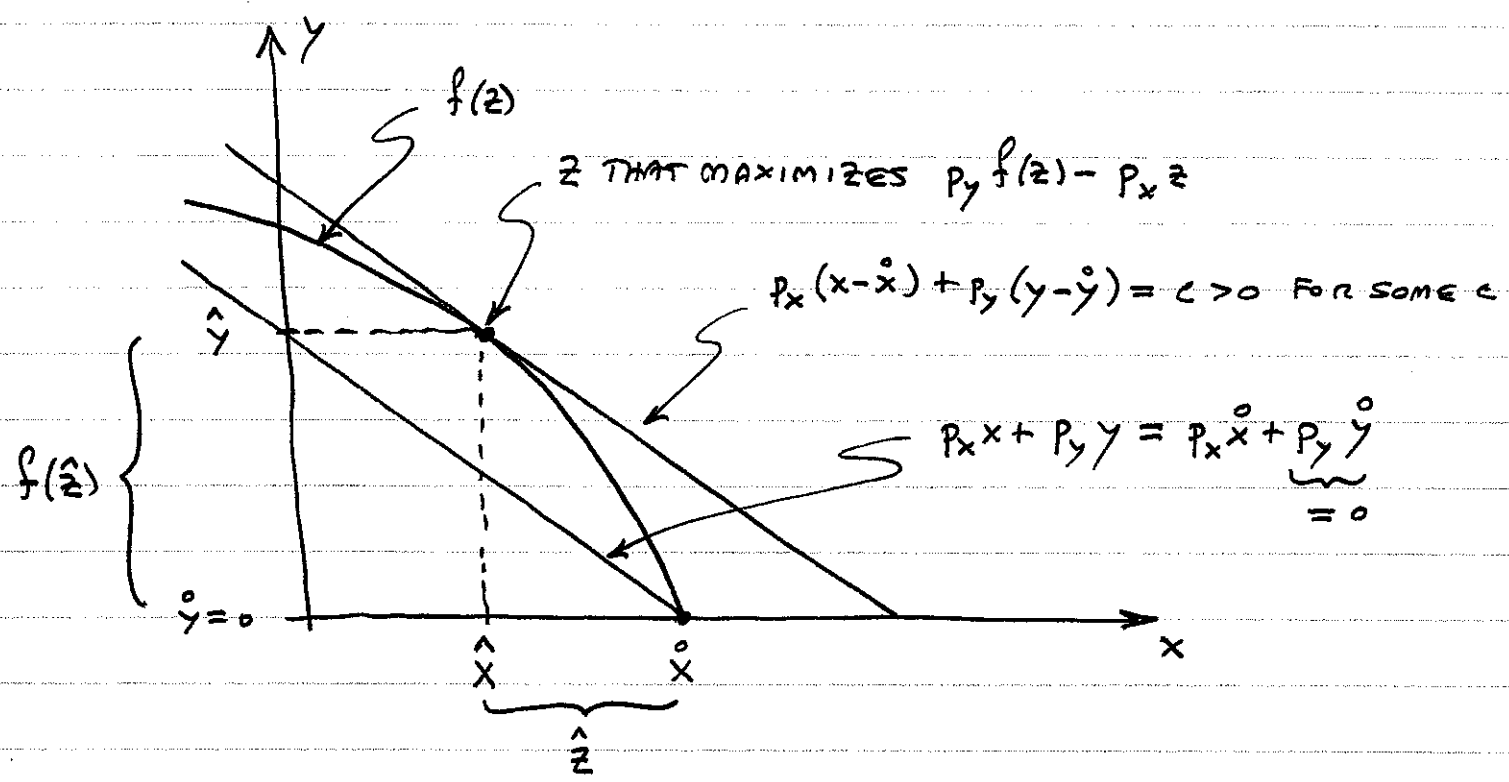


FIGURE 1

FORMALLY, AND RETURNING TO THE CASE WITH  $n$  CONSUMERS AND  $m$  FIRMS (BUT STILL ONLY TWO GOODS, ONE AN INPUT, ONE AN OUTPUT), SUPPOSE THAT THIS WERE OUR DEFINITION OF COMPETITIVE EQUILIBRIUM:

DEFN (??): A WALRASIAN EQUILIBRIUM IS A TRIPLE  $(\hat{p}; (\hat{z}_j)_{j=1}^m; (\hat{x}_i, \hat{y}_i)_{i=1}^n) \in \mathbb{R}_+^2 \times \mathbb{R}_+^m \times (\mathbb{R}_+^n \times \mathbb{R}_+^n)$  THAT SATISFIES

(U-M)  $\forall i \in N$ :  $(\hat{x}_i, \hat{y}_i)$  MAXIMIZES  $u_i$  SUBJECT TO

$$(BC) \quad \hat{p}_x x_i + \hat{p}_y y_i \leq \hat{p}_x \bar{x}_i + \hat{p}_y \bar{y}_i \quad ;$$

(P-M)  $\forall j \in M$ :  $\hat{z}_j$  MAXIMIZES  $\pi_j(z_j) := \hat{p}_y f_j(z_j) - \hat{p}_x z_j$  ;

$$(M-C) \quad \sum_{i=1}^n \hat{x}_i + \sum_{j=1}^m \hat{z}_j \leq \sum_{i=1}^n \bar{x}_i, \quad w/ \text{EQUALITY IF } \hat{p}_x > 0$$

$$\sum_{i=1}^n \hat{y}_i \leq \sum_{i=1}^n \bar{y}_i + \sum_{j=1}^m f_j(\hat{z}_j), \quad w/ \text{EQUALITY IF } \hat{p}_y > 0.$$

NOTICE THAT ANY ALLOCATION THAT SATISFIES THE BUDGET CONSTRAINTS (BC) FOR ALL  $i$  WILL SATISFY

... FOR ANY PRICES  $p_x, p_y$

$$p_x [\sum x_i - \sum \bar{x}_i] + p_y [\sum y_i - \sum \bar{y}_i] \leq 0.$$

IF TOTAL PROFITS HAPPEN TO BE POSITIVE — i.e., IF

$$\sum_{j=1}^m [p_y f_j(z_j) - p_x z_j] > 0$$

— THEN WE HAVE

$$p_x [\sum x_i - \sum \bar{x}_i] + p_y [\sum y_i - \sum \bar{y}_i] - [p_y \sum f_j(z_j) - p_x \sum z_j] < 0;$$

i.e.,

$$(*) \quad p_x [\sum x_i + \sum z_j - \sum \bar{x}_i] + p_y [\sum y_i - (\sum \bar{y}_i + \sum f_j(z_j))] < 0.$$

IF WE WRITE

$$E_x := \sum x_i + \sum z_j - \sum \bar{x}_i$$

AND

$$E_y := \sum y_i - (\sum \bar{y}_i + \sum f_j(z_j))$$

FOR THE AMOUNTS BY WHICH THE ALLOCATION  $((x_i, y_i)_N, (z_j)_M)$  EXCEEDS THE AVAILABLE RESOURCES, THEN (\*) IS WRITTEN

$$(*) \quad p_x E_x + p_y E_y < 0.$$

THE INEQUALITY (\*) HAS TWO IMPORTANT IMPLICATIONS:

(A) AN ALLOCATION THAT SATISFIES (\*) CANNOT BE PARETO EFFICIENT WITH EFFICIENCY PRICES PROPORTIONAL TO  $(p_x, p_y)$  — I.E., WITH  $MRS^i = \frac{p_x}{p_y}$  FOR EACH  $i$  — BECAUSE THAT WOULD REQUIRE

$$\sigma_x E_x + \sigma_y E_y = 0, \text{ WHERE } \sigma = \lambda p \text{ (cf. (4), (5)).}$$

(B) AN ALLOCATION THAT SATISFIES (\*) CANNOT BE A WALRASIAN EQUILIBRIUM (ACCORDING TO THE TENTATIVE DEFINITION WE HAVE GIVEN), BECAUSE IT CANNOT SIMULTANEOUSLY SATISFY BOTH OF THE MARKET-CLEARING CONDITIONS (M-C) IN THE DEFINITION — THEY IMPLY

$$p_x E_x + p_y E_y = 0.$$

WRITE  $x, y, \overset{\circ}{x}, \overset{\circ}{y}, z,$  AND  $q$  FOR THE AGGREGATE AMOUNTS (SUMMED OVER ALL CONSUMERS OR FIRMS).

IF  $x = \overset{\circ}{x} - z$  AND  $y = \overset{\circ}{y} + q$ , THEN

$$\begin{aligned}
 \underbrace{p_x x + p_y y}_{\substack{\uparrow \\ \text{VALUE OF} \\ \text{CONSUMPTION}}} &= p_x (\overset{\circ}{x} - z) + p_y (\overset{\circ}{y} + q) \\
 &= \underbrace{p_x \overset{\circ}{x} + p_y \overset{\circ}{y}}_{\text{VALUE OF ENDOWMENT}} + \underbrace{p_y q - p_x z}_{\text{NET VALUE OF PRODUCTION ACTIVITIES (i.e., PROFIT)}}
 \end{aligned}$$

IF PROFIT IS POSITIVE, THEN ~~CONSUMERS~~ THE VALUE OF CONSUMERS' ENDOWMENT IS NOT LARGE ENOUGH TO COVER THE ENTIRE VALUE OF THE GOODS AVAILABLE FOR CONSUMPTION AFTER PRODUCTION HAS TAKEN PLACE — UNLESS CONSUMERS' WEALTH (OR INCOME) IS AUGMENTED BY THE FIRMS' PROFITS. IN OTHER WORDS, WE HAVE TO ACCOUNT FOR THE DISTRIBUTION OF PROFITS ("SURPLUS VALUE" DUE TO PRODUCTION) IN OUR MODEL OF ALLOCATION VIA MARKETS AND PRICES.

WE ACCOUNT FOR PROFIT BY ASSUMING IT ALL GOES TO CONSUMERS. IN THE WALRASIAN MODEL, FOR EXAMPLE, WE ASSUME THAT CONSUMERS OWN (SHARES OF) THE FIRMS' PROFITS:  $\theta_{ij}$  DENOTES CONSUMER  $i$ 'S SHARE OF FIRM  $j$ 'S PROFIT [ $0 \leq \theta_{ij} \leq 1$  AND  $\sum_{i \in N} \theta_{ij} = 1$ ].

THE TENTATIVE DEFINITION OF WALRASIAN EQUILIBRIUM THAT WE GAVE EARLIER IS ALTERED BY CHANGING THE BUDGET-CONSTRAINT OF EACH IEN TO

$$(BC) \quad \hat{p}_x x_i + \hat{p}_y y_i \leq \hat{p}_x x_i^0 + \hat{p}_y y_i^0 + \sum_{j \in M} \theta_{ij} \pi_j(\hat{z}_j),$$

WHERE  $\pi_j(\hat{z}_j) := \hat{p}_y f_j(\hat{z}_j) - \hat{p}_x z_j$ .

WITH THIS NEW DEFINITION OF EQUILIBRIUM, IT IS STRAIGHTFORWARD TO USE THE SAME METHODS WE HAVE USED BEFORE IN ORDER TO ESTABLISH THE EXISTENCE OF AN EQUILIBRIUM AND THE VALIDITY OF THE TWO WELFARE THEOREMS (IF EVERY FIRM'S  $f_j$  IS CONTINUOUS AND CONCAVE) IN THIS TWO-GOOD (ONE INPUT AND ONE OUTPUT) CASE.

FOR THE MORE GENERAL CASE, IN WHICH A FIRM MAY HAVE MANY INPUTS AND MANY OUTPUTS, AND IN WHICH A PARTICULAR GOOD MIGHT BE AN INPUT FOR SOME FIRMS AND AN OUTPUT FOR OTHERS, WE REPRESENT A FIRM'S TECHNOLOGY BY A PRODUCTION SET (THE SET OF ITS FEASIBLE PRODUCTION PLANS — I.E., INPUT-OUTPUT LISTS),  $Y_j \subseteq \mathbb{R}^l$ , FROM WHICH IT MUST CHOOSE A PARTICULAR LIST, OR PRODUCTION PLAN  $y_j \in Y_j$ .

[SEE THE REMARK AT THE BOTTOM OF THE FOLLOWING PAGE FOR THE DIFFERING TREATMENT OF INPUTS AND OUTPUTS IN  $y_j$ .]

DEFN: A PRIVATE OWNERSHIP ECONOMY IS DESCRIBED BY THE DATA  $(u_i, \bar{x}_i)_{i \in N}$ ,  $(Y_j)_{j \in M}$ , AND  $(\theta_{ij})_{N \times M}$ , WHERE

$u_i: \mathbb{R}_+^l \rightarrow \mathbb{R}$  IS A UTILITY FUNCTION  
 $\bar{x}_i \in \mathbb{R}_+^l$  IS  $i$ 'S INITIAL ENDOWMENT  
 $Y_j \subseteq \mathbb{R}^l$  IS  $j$ 'S PRODUCTION SET  
 $\theta_{ij}$  IS  $i$ 'S SHARE OF FIRM  $j$   $[0 \leq \theta_{ij} \leq 1; \sum_{i \in N} \theta_{ij} = 1]$ .

DEFN: A WALRASIAN EQUILIBRIUM OF AN OWNERSHIP ECONOMY IS A TRIPLE  $(\hat{p}; (\hat{x}_i)_{i \in N}; (\hat{y}_j)_{j \in M}) \in \mathbb{R}_+^l \times \mathbb{R}_+^{nl} \times \mathbb{R}^{ml}$  THAT SATISFIES

(u.m.)  $\forall i \in N$ :  $\hat{x}_i$  MAXIMIZES  $u_i(x_i)$  S.T.  $x_i \geq 0$  AND

$$(bc) \hat{p} \cdot \hat{x}_i \leq \hat{p} \cdot \bar{x}_i + \sum_{j \in M} \theta_{ij} \hat{p} \cdot \hat{y}_j ;$$

(p.m.)  $\forall j \in M$ :  $\hat{y}_j$  MAXIMIZES  $\pi_j(y_j; \hat{p}) := \hat{p} \cdot y_j$  ON  $Y_j$  ;

(m.c.)  $\forall k = 1, \dots, l$ :  $\sum_N \hat{x}_{ik} - \sum_M \hat{y}_{jk} \leq \sum_N \bar{x}_{ik}$ , W/EQUALITY IF  $\hat{p}_k > 0$ .

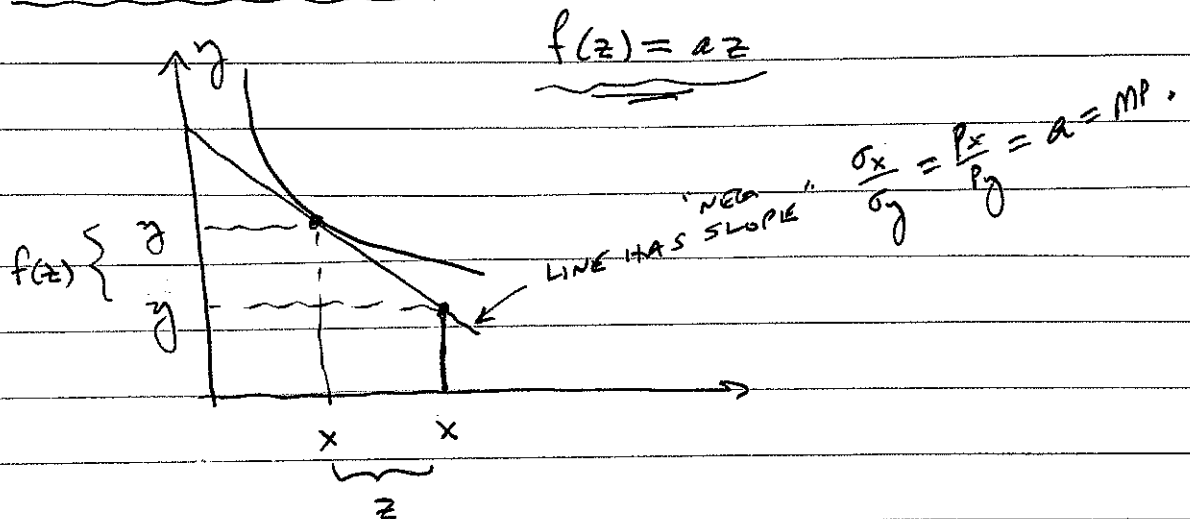
REMARK: NOTE THAT THE CONVENTION USED HERE IS THAT  $y_{jk} > 0$  MEANS THAT FIRM  $j$  IS (NET) PRODUCING GOOD  $k$  AS AN OUTPUT, AND  $y_{jk} < 0$  MEANS THAT FIRM  $j$  IS (NET) USING GOOD  $k$  AS AN INPUT.

INCIDENTALLY, IT IS WORTH NOTING THAT IF, AT THE EQUILIBRIUM, IT WERE TO HAPPEN THAT TOTAL PROFITS WERE ZERO, THEN OUR ORIGINAL DEFINITION, WITHOUT PROFIT DISTRIBUTION, WOULD HAVE BEEN OK.

← i.e., (BC) AND (BC) ARE EQUIVALENT

THIS IS IMPORTANT TO KNOW, BECAUSE THERE IS AN OFTEN-USED ASSUMPTION ABOUT THE ECONOMY'S STRUCTURE THAT ALWAYS YIELDS ~~ZERO~~ PROFITS <sup>FOR</sup> EVERY FIRM AT ITS PROFIT-MAXIMIZING DECISION: CONSTANT RETURNS TO SCALE. THUS, ONE NEEDN'T WORRY ABOUT PROFITS IN AN ECONOMY WITH CONSTANT RETURNS TO SCALE.

EXAMPLE (n=m=1):



! NOTE THAT THE PRICE RATIO IS COMPLETELY DETERMINED BY THE TECHNOLOGY:  $\frac{P_x}{P_y} = a.$  AND THE LEVEL OF PRODUCTION IS COMPLETELY DETERMINED (GIVEN  $a$ ) BY DEMAND.

EXAMPLE:

$$f(z) = \begin{cases} 40 \log z, & z \geq 1 \\ 0, & z \leq 1 \end{cases}$$

$$f'(z) = \frac{40}{z} \quad \text{IF } z > 1$$

$$u(x, y) = y + 80 \log x \quad (\bar{x}, \bar{y}) = (60, 40)$$

$$MRS = \frac{80}{x}$$

PARETO FOC:

$$f'(z) = MRS \text{ (INTERIOR)}$$

$$\text{i.e., } \frac{40}{z} = \frac{80}{x}$$

$$40x = 80z$$

$$40(\bar{x} - z) = 80z$$

$$120z = 40\bar{x}$$

$$z = \frac{1}{3}\bar{x} = 20$$

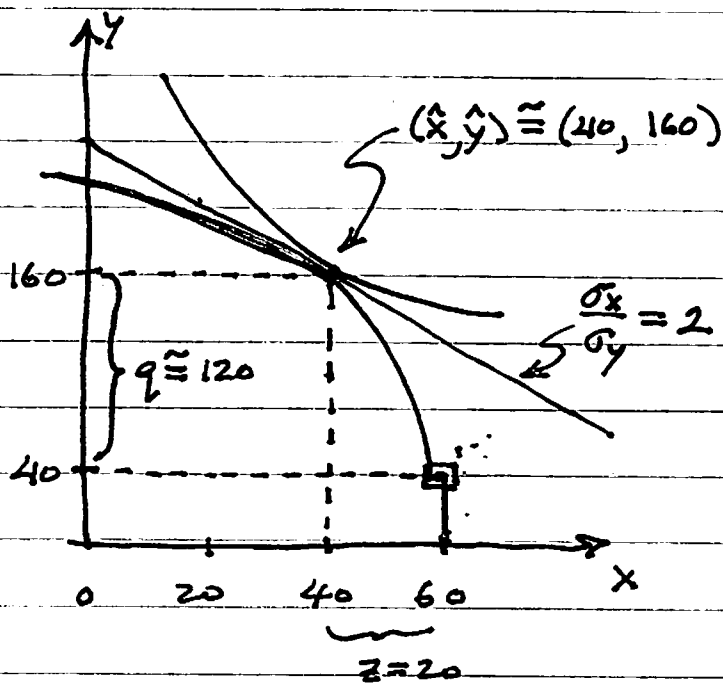
$$\therefore q = f(z) = 40 \log 20$$

$$\approx (40)(3.00)$$

$$= 120$$

$$\therefore x = \bar{x} - z = 40$$

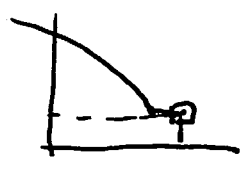
$$y = \bar{y} + q \approx 160$$



$$f'(20) = \frac{40}{20} = 2$$

$$MRS = \frac{80}{40} = 2$$

EFFICIENCY PRICES SATISFY  $\frac{P_x}{P_y} = 2$ .



$$f(z) = \begin{cases} \alpha \log z, & \text{if } z \geq 1 \\ 0, & \text{if } z \leq 1 \end{cases}$$

EXAMPLE:

$$f(z) = \alpha \log z$$

$$\therefore f'(z) = \frac{\alpha}{z}$$

LET  $\alpha = 40$ .

$$u(x,y) = x + 2y$$

$$\therefore \text{MRS} = \frac{1}{2}, \quad v(x,y)$$

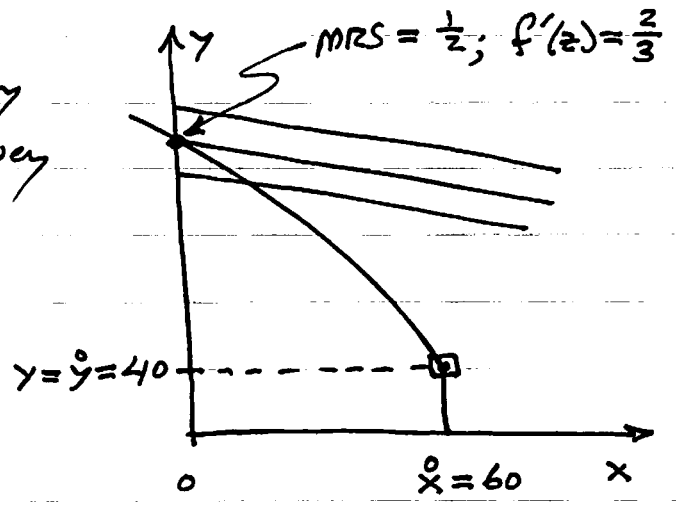
$$(x^0, y^0) = (60, 40)$$

NOTE THAT  $f'(60) = \frac{40}{60} = \frac{2}{3} > \text{MRS}$ .

$\therefore$  WE SHOULD SHIFT MORE OF THE X-GOOD INTO THE PRODUCTION PROCESS AS INPUT. BUT AT  $z = 60$  WE HAVE  $z = x^0$  — NO MORE OF THE X-GOOD IS AVAILABLE FOR INPUT.

$\therefore$  THE SOLUTION  $z = 60, x = 0, y = 40 + 40 \log 60 \approx 204$  IS THE UNIQUE EFFICIENT OUTCOME.

WHAT ABOUT EFFICIENCY PRICES? THE ONLY EFFICIENCY PRICES ARE THOSE THAT SATISFY  $\frac{P_x}{P_y} = \frac{2}{3}$ .



IT MIGHT SEEM FROM THE DIAGRAM THAT ANY  $(P_x, P_y)$  SATISFYING  $\frac{1}{2} \leq \frac{P_x}{P_y} \leq \frac{2}{3}$  WOULD DO. BUT IF  $\frac{P_x}{P_y} < \frac{2}{3}$  — i.e.,  $\frac{P_x}{P_y} < \text{MP}$  — THEN THE FIRM WOULD CHOOSE TO INCREASE ITS PRODUCTION LEVEL (i.e., INCREASE  $z$ , BEYOND  $z = 60$ ). OF COURSE, THE SUPPLY OF THE X-GOOD WOULD BE INSUFFICIENT TO CLEAR THE MARKET: IT WOULD NOT BE AN EQUILIBRIUM.