

## Intertemporal Exercise

Benjamin has just graduated from college. Now he's deciding how much further to invest in his own human capital. The result of his decision will be a consumption stream  $\mathbf{x} = (x_0, x_1)$ , where  $x_0$  denotes consumption "today" (say, during the next ten years) and  $x_1$  denotes consumption "tomorrow" (the remainder of his life). His preference is described by the utility function  $u(x_0, x_1) = x_0^3 x_1^2$ . If he undertakes no investment, and neither saves nor borrows, his consumption stream will be  $\hat{\mathbf{x}} = (\hat{x}_0, \hat{x}_1) = (38, 16)$ .

The investment possibilities available to Benjamin are described by the function

$$f(z) = \begin{cases} 4z - \frac{1}{8}z^2, & z \leq 16 \\ 32, & z \geq 16. \end{cases}$$

(a) Suppose Benjamin has no access to capital markets: he can neither borrow nor save. Verify that he will invest at level  $z = 8$ . What will be his resulting consumption stream, marginal rate of substitution, and marginal rate of transformation? Depict this decision in a diagram showing Benjamin's consumption-possibilities set and the best indifference curve he can attain.

Now suppose Benjamin has access to capital markets in which he can borrow and lend (i.e., save) at a common inter-period (not annual) interest rate,  $r$  – i.e., if he borrows  $B$  dollars today, he repays  $(1 + r)B$  dollars tomorrow; if he saves  $S$  today, he receives  $(1 + r)S$  tomorrow.

(b) If the interest rate is 100%, how much will Benjamin invest? How much will he borrow or lend? What will be his resulting consumption stream? Verify that the net present value of his consumption stream is equal to the net present value of his endowment stream plus the net present value of his investment plan.

(c) Now suppose the interest rate is 200%, and consider the investment and consumption plans in part (b). Verify that the present value of the consumption stream in (b) is still equal to the present value of Benjamin's endowment stream plus the present value of the investment plan in (b). In a diagram like the one in part (a), depict the present-value contour(s) on which the investment plan and consumption plan lie. Will Benjamin's investment and consumption plans be the same as in (b)? If not, will his investment, borrowing, lending, and consumption in each period be more or less than in (b)? Hint: If  $r = 200\%$ , the present value of the investment plan in part (b) is zero.

(d) If the interest rate is 200%, how much will Benjamin invest? How much will he borrow or lend? What will be his resulting consumption stream? Hint: Benjamin will choose a consumption plan that maximizes his utility among all plans with a net present value no greater than his wealth. His wealth is the present value of his endowment plus the net present value of his investment plan. You should find that his wealth is 44.