

General Equilibrium Exercise

In doing applied microeconomics you often have to compute equilibria of models that don't have closed-form solutions. The computation therefore must be done by iterative numerical methods. That's what you'll do in this exercise, for the Cobb-Douglas example in the first lecture. The iterative computation is pretty straightforward, because there are only two goods and the demand functions have simple closed-form solutions. Moreover, the equilibrium itself has a closed-form solution, so you can also have your program compute the equilibrium prices directly and then check whether your iterative program converges to the correct equilibrium prices.

Specifically, you are to use a spreadsheet program such as Excel, or a programming language such as C+ or Pascal, to compute the path taken by prices and excess demands in the two-person, two-good, pure exchange Cobb-Douglas example from the first lecture, assuming that prices adjust according to the transition function in the Arrow & Hahn proof of existence of equilibrium:

$$f(p) = \frac{1}{\sum_{k=1}^I [p_k + M_k(p)]} [p + M(p)]$$

where $M_k(p) = \max\{0, I z_k(p)\}$ for each good k .

Note that the proof did not actually require a I , *i.e.*, we could assume that $I = 1$. However, with $I = 1$ the iterative process defined by this transition function will not converge for the Cobb-Douglas example, as you can verify once you've created your computational program. You'll find that to achieve convergence you'll need to use a I equal to about .02 or smaller. Recall, too, that the proof does not actually apply to the Cobb-Douglas example, because demands are not defined for the price-lists (1,0) and (0,1). For the same reason, you can't start the iterative process off using either of these as the initial price-lists, because the "next p " defined by $f(p)$ won't be well-defined.

You will of course have to use specific parameter values for the two consumers' utility functions and endowment bundles. With a small enough value for I , the process will converge for just about any parameter values and any strictly positive initial prices. Of course, when you run your program you should note whether it does converge to the equilibrium price-list.

Choose some specific parameter values and initial price-list, and plot (by hand) the price-line and the chosen bundles in the Edgeworth Box for several iterations of the process. Note that if the prices aren't sufficiently close to the equilibrium prices, the chosen bundles may not lie within the confines of the box. This is an important point to understand: each individual consumer simply takes the prices as given and chooses his or her best bundle within the resulting budget set. The consumer takes no account of the total resources available, nor of the other consumers' preferences or choices, because the consumer isn't assumed to have that information.