GENERAL EQUILIBRIUM EXAMPLE

(TWO PERSONS, LATER GENERALIZED TO n; TWO GOODS; PURE EXCHANGE; COBB-DOUGLAS UTILITIES.)

INDIVIDUAL DEMAND:

(x,y) MAXIMIZES $u(x,y) = x^{\alpha} y^{\beta}$ s.t. $x, y \geq 0$ AND TO $p_x x + p_y y \leq M$, WHERE $M = p_x \mathring{x} + p_y \mathring{y}$.

SOLUTION (INDIVIDUAL DEMAND FUNCTION): [WLOG, $\alpha + \beta = 1$]

$$x = \alpha \frac{M}{p_x} \quad \text{AND} \quad y = \beta \frac{M}{p_y}$$

$$\text{LET } \rho := \frac{p_x}{p_y}; \quad x = \alpha\left(\mathring{x} + \frac{1}{\rho}\mathring{y}\right), \quad y = \beta\left(\mathring{y} + \rho \mathring{x}\right).$$

NET DEMAND:

$$\overset{\Delta}{x} := x - \mathring{x} = (\alpha - 1)\mathring{x} + \alpha \frac{1}{\rho} \mathring{y} = -\beta \mathring{x} + \alpha \frac{1}{\rho} \mathring{y}$$

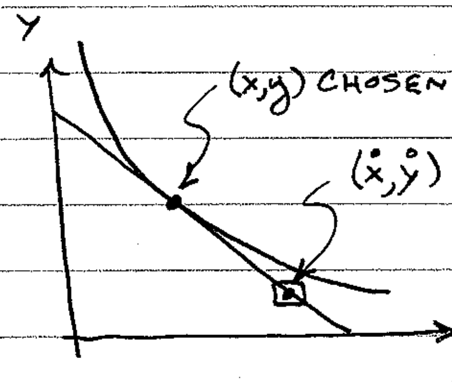
$$\overset{\Delta}{y} := y - \mathring{y} = (\beta - 1)\mathring{y} + \beta \rho \mathring{x} = \beta \rho \mathring{x} - \alpha \mathring{y}$$

NOTE: $\boxed{\overset{\Delta}{y} = -\rho \overset{\Delta}{x}.}$ (*) "WALRAS' LAW" WHEN AGGREGATED

i.e., $\rho \overset{\Delta}{x} + \overset{\Delta}{y} = 0$

$$\rho(x - \mathring{x}) + (y - \mathring{y}) = 0$$

$$p_x(x - \mathring{x}) + p_y(y - \mathring{y}) = 0$$



THE VALUE OF AN INDIVIDUAL'S NET DEMAND IS ZERO; HE CHOOSES ON HIS BUDGET CONSTRAINT.

(AGGREGATED ACROSS PERSONS, NOT ACROSS GOODS)

AGGREGATE DEMAND:

$$X = x_1 + x_2 = \alpha_1 \left(\mathring{x}_1 + \frac{1}{p}\mathring{y}_1\right) + \alpha_2\left(\mathring{x}_2 + \frac{1}{p}\mathring{y}_2\right)$$

$$= \alpha_1 \mathring{x}_1 + \alpha_2 \mathring{x}_2 + (\alpha_1 \mathring{y}_1 + \alpha_2 \mathring{y}_2)\frac{1}{p}$$

$$\overset{\Delta}{X} := X - \mathring{X} = (\alpha_1 - 1)\mathring{x}_1 + (\alpha_2 - 1)\mathring{x}_2 + (\alpha_1 \mathring{y}_1 + \alpha_2 \mathring{y}_2)\frac{1}{p}$$

$$= -(\beta_1 \mathring{x}_1 + \beta_2 \mathring{x}_2) + (\alpha_1 \mathring{y}_1 + \alpha_2 \mathring{y}_2)\frac{1}{p}.$$

$\overset{\Delta}{X}(p)$, A FUNCTION

SIMILARLY FOR THE y-GOOD.

EQUILIBRIUM:

$$X = \mathring{x}_1 + \mathring{x}_2 \quad \text{AND} \quad Y = \mathring{y}_1 + \mathring{y}_2$$

i.e., $\overset{\Delta}{X}(p) = 0$ AND $\overset{\Delta}{Y}(p) = 0$. ← [TWO EQUIL'M CONDITIONS (EQUATIONS)]

BUT BY SUMMING THE BUDGET-BALANCE CONDITION (*) ACROSS ALL PERSONS, WE KNOW THAT $\overset{\Delta}{Y}(p) = -p\overset{\Delta}{X}(p)$.

THUS, $\boxed{\overset{\Delta}{X}(p) = 0 \Leftrightarrow \overset{\Delta}{Y}(p) = 0}$ —

THE TWO EQUIL'M CONDITIONS ARE NOT INDEPENDENT. THERE IS REALLY ONLY ONE EQUILIBRIUM CONDITION:

$$\overset{\Delta}{X}(p) = 0 \; ; \qquad (**)$$

i.e., $$-(\beta_1 \mathring{x}_1 + \beta_2 \mathring{x}_2) + (\alpha_1 \mathring{y}_1 + \alpha_2 \mathring{y}_2)\frac{1}{p} = 0 \qquad (**)$$

THE CLOSED-FORM SOLUTION:

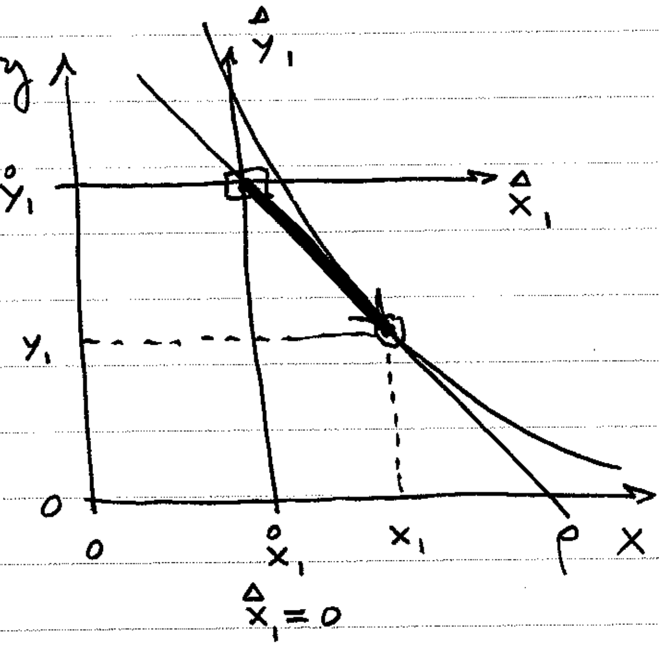
$$\rho = \frac{\alpha_1 \mathring{y}_1 + \alpha_2 \mathring{y}_2}{\beta_1 \mathring{x}_1 + \beta_2 \mathring{x}_2},$$

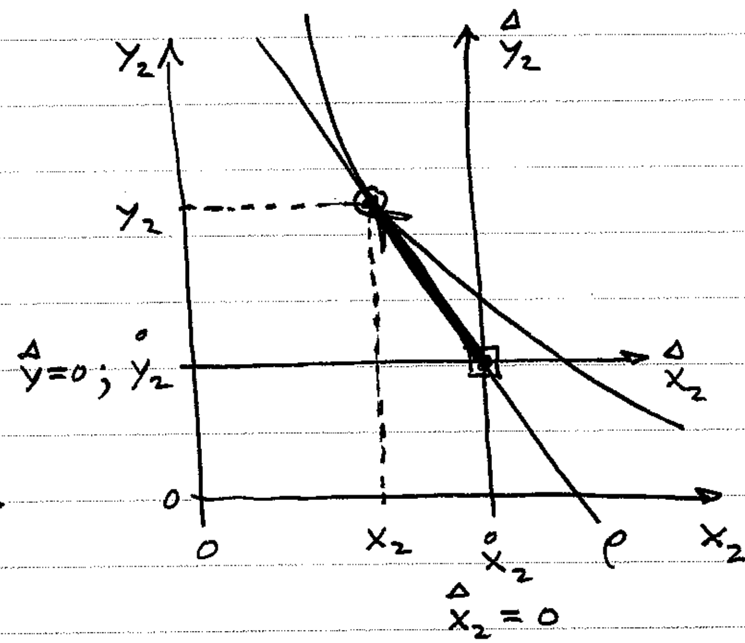
RATIO OF WEIGHTED SUMS OF ENDOWMENT AMOUNTS.

$$= \frac{\sum_{i=1}^{n} \alpha_i \mathring{y}_i}{\sum_{i=1}^{n} \beta_i \mathring{x}_i},$$

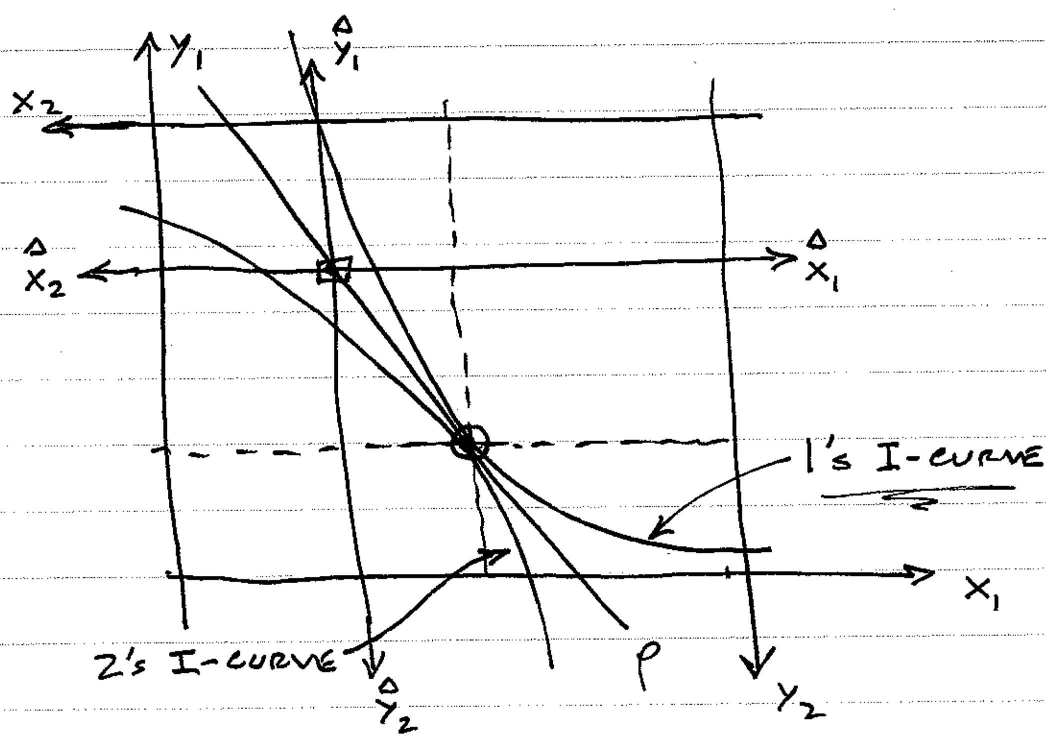
WHICH IS THE SOLUTION FOR n PERSONS (EVERYTHING WE'VE DONE CAN BE WRITTEN THE SAME WAY FOR n AS FOR 2).

THE GEOMETRY, INCLUDING THE EDGEWORTH BOX:





THE "PRICE LINE" IN BOTH INDIVIDUAL DIAGRAMS HAS $-$SLOPE $= \rho$. $\left(\rho = \frac{P_x}{P_y}\right)$.



A Numerical Example

DATA
$$\begin{cases} (\mathring{x}_1, \mathring{y}_1) = (40, 80), & \alpha_1 = \frac{7}{8} \\ (\mathring{x}_2, \mathring{y}_2) = (80, 40), & \alpha_2 = \frac{1}{2}. \end{cases}$$

$$\left(\therefore MRS_1 = 7\frac{y_1}{x_1}\right)$$
$$\left(\therefore MRS_2 = \frac{y_2}{x_2}\right)$$

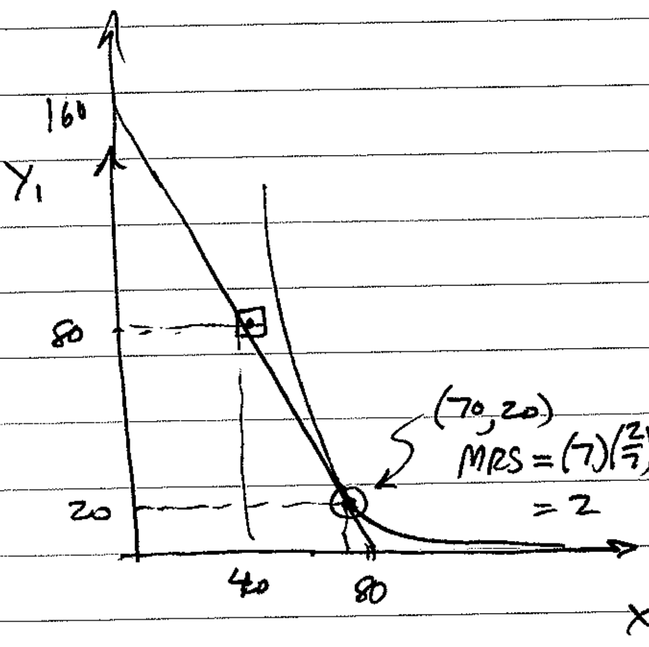
Equilibrium:

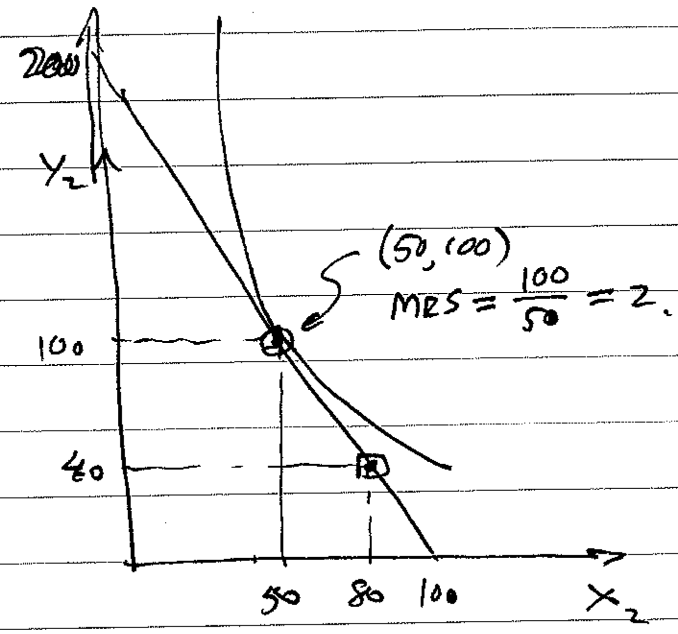
$$p = \frac{\frac{7}{8}(80) + \frac{1}{2}(40)}{\frac{1}{8}(40) + \frac{1}{2}(80)} = \frac{70+20}{5+40} = \frac{90}{45} = 2.$$

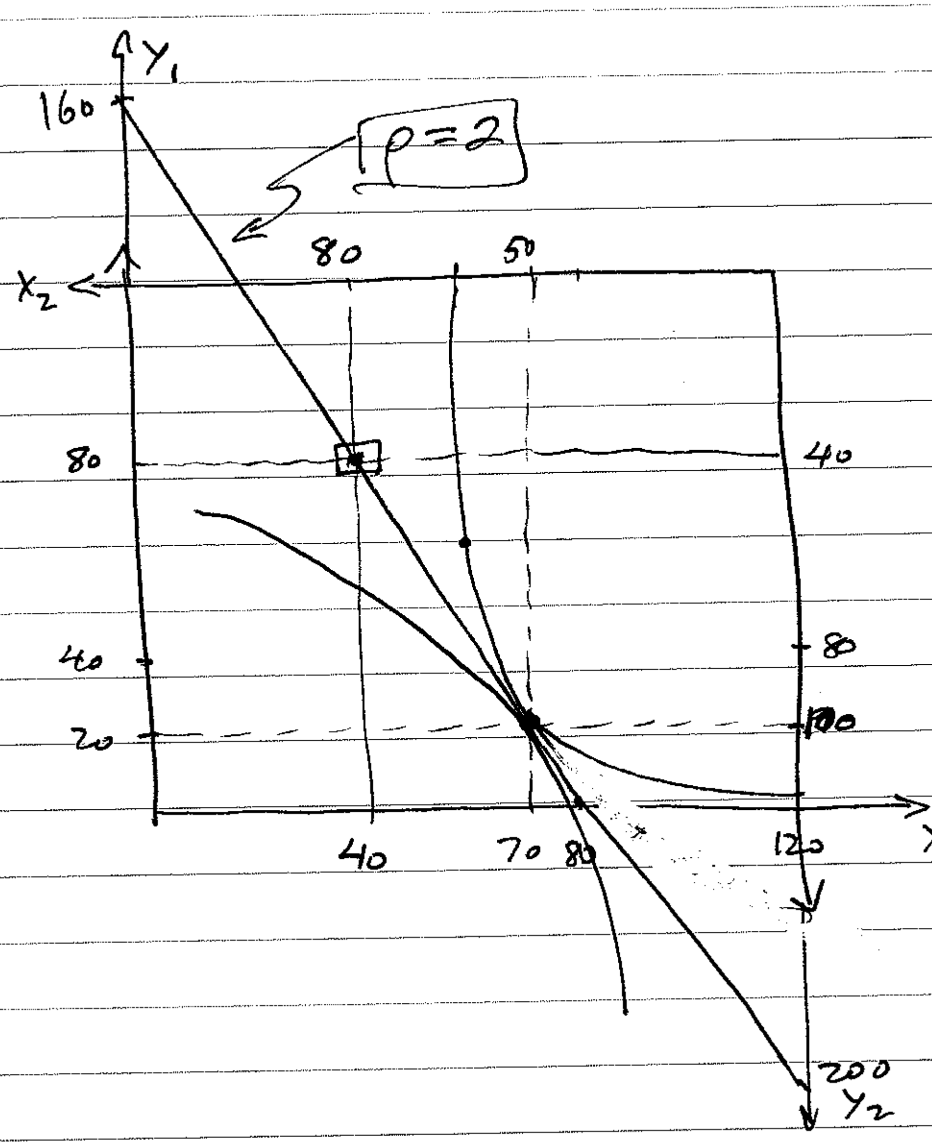
If, for example, $p_x = 2$ and $p_y = 1$, then

$$m_1 = (2)(40) + 80 = 160$$
$$m_2 = (2)(80) + 40 = 200.$$

$$\begin{array}{ll} x_1 = 70 & y_1 = 20 \\ x_2 = 50 & y_2 = 100 \\ \hline X = 120 & Y = 120 \end{array} \qquad \begin{array}{ll} \hat{x}_1 = +30 & \hat{y}_1 = -60 \\ \hat{x}_2 = -30 & \hat{y}_2 = +60 \\ \hline \hat{X} = 0 & \hat{Y} = 0. \end{array}$$


160
Y_1
80
20
40
80
X_1
(70,20)
MRS = (7)(2/7)
= 2


200
Y_2
(50,100)
MRS = 100/50 = 2.
100
40
50
80
100
X_2


Y_1
160
$p=2$
80
50
X_2
80
40
40
20
80
100
40
70
80
120
X_1
200
Y_2

ANOTHER NUMERICAL COBB-DOUGLAS EXAMPLE

DATA:

$n = 2$

$$
\begin{aligned}
(\mathring{x}_1, \mathring{y}_1) &= (30, 60) & \alpha_1 &= \tfrac{2}{3} \\
(\mathring{x}_2, \mathring{y}_2) &= (60, 30) & \alpha_2 &= \tfrac{2}{3} \\
\hline
(\mathring{X}, \mathring{Y}) &= (90, 90)
\end{aligned}
$$

EQUILIBRIUM:

$$
\rho = \frac{\frac{2}{3}(60) + \frac{2}{3}(30)}{\frac{1}{3}(30) + \frac{1}{3}(60)} = \frac{40 + 20}{10 + 20} = \frac{60}{30} = 2
$$

... FROM THE CLOSED-FORM EXPRESSION WE DERIVED FOR THE TWO-GOOD CASE.

[NOTE THAT IF $p_y = 1$ AND $p_x = 2$ (THIS IS ONE WAY TO HAVE $\rho = 2$), THEN

$$
M_1 = (2)(30) + (1)(60) = 120
$$

$$
M_2 = (2)(60) + (1)(30) = 150
$$

ARE THE VALUES OF THE CONSUMERS' INITIAL HOLDINGS (THEIR "ENDOWMENTS").]

$$
\begin{aligned}
(x_1, y_1) &= (40, 40) & (\overset{\Delta}{x}_1, \overset{\Delta}{y}_1) &= (10, -20) \\
(x_2, y_2) &= (50, 50) & (\overset{\Delta}{x}_2, \overset{\Delta}{y}_2) &= (-10, 20) \\
\hline
(X, Y) &= (90, 90) & (\overset{\Delta}{X}, \overset{\Delta}{Y}) &= (0, 0), \text{ i.e., TOTAL } \underline{\text{NET}} \text{ TRADE IS ZERO FOR EACH GOOD.} \\
&= (\mathring{X}, \mathring{Y})
\end{aligned}
$$

IN EQUILIBRIUM,

MR. 1 CHOOSES TO GIVE UP 20 UNITS OF THE Y-GOOD TO OBTAIN A 10-UNIT ADDITION OF THE X-GOOD;

MR. 2 CHOOSES TO GIVE UP 10 UNITS OF THE X-GOOD (JUST THE AMOUNT MR. 1 WANTS TO BUY) TO OBTAIN AN ADDED 20 UNITS OF THE Y-GOOD (JUST THE AMOUNT MR. 1 WANTS TO SELL).

SOME THINGS TO NOTICE:

(1) ANALOGY WITH OPTIMIZATION: $E(p)=0$ CORRESPONDS TO $F'(x)=0$ (F.O.C.). GRAPH OF EACH FOR COMPARATIVE STATICS.

(2) WITH ℓ GOODS ($\ell > 2$), WE WILL HAVE $\ell - 1$ INDEPENDENT EQUILIBRIUM CONDITIONS, WITH WALRAS' LAW TAKING CARE OF THE REMAINING MARKET, AND WE COULD TAKE ONE GOOD (SAY, GOOD ℓ) AS "NUMERAIRE," USING ONLY THE $\ell - 1$ RELATIVE PRICES $\rho_k = \frac{p_k}{p_\ell}$, BECAUSE OF HOMOGENEITY OF DEGREE ZERO OF FUNCTION(S).

(3) WHAT IF WE CAN'T GET A CLOSED-FORM SOLUTION (i.e., AN EXPLICIT EXPRESSION) FOR STATE VARIABLE(S) IN TERMS OF PARAMETERS? HOW DO WE DO COMPARATIVE STATICS IN THAT CASE? USE ANALOGY WITH OPTIMIZATION: USE IFT.

(4) SOME QUESTIONS THAT NATURALLY ARISE:

(a) WHAT IF THERE IS NO EQUILIBRIUM? WILL THERE GENERALLY BE ONE? UNDER WHAT CONDITIONS?

(b) WHAT IF THERE ARE MULTIPLE EQUILIBRIA? WHEN WILL EQUILIBRIUM BE UNIQUE?

(c) WHAT IF NOT <u>AT</u> EQUILIBRIUM? WHAT ARE STABILITY PROPERTIES OF EQUILIBRIUM?

(d) IS EQUILIBRIUM OUTCOME "GOOD"?

(e) WHAT IF MARKETS & PRICES NOT USED? WHEN WILL THEY BE USED?

(f) WHAT IF NOT EVERYONE IS A PRICE-TAKER?

... AND MORE ...

Comparison of Individual Decision Analysis
and
Equilibrium Analysis

	Analysis of Individual <u>Decision-Making (501A)</u>	Analysis of Interaction Among <u>Multiple Economic Units (501B)</u>
Typical unit:	Household; Firm	Market, industry, economy, firm, club, organization, society electorate
Assumption:	Unit's choice is optimal, according to some objective function, perhaps subject to constraint(s)	Unit is in equilibrium
Analytical characterization:	First-order conditions -- i.e., equations	Equilibrium condition(s) -- i.e., equations
Typical question:	What is effect of an exogenous change -- e.g., in price of another good, weather, etc. ("comparative statics")	What is effect of exogenous change -- e.g., in expectations, weather, etc. ("comparative statics")
Method:	IFT applied to FOC	IFT on equilibrium conditions
Other questions:		Existence, optimality, stability. is it "the right model"?