The Definition of Market Equilibrium

The concept of market equilibrium, like the notion of equilibrium in just about every other context, is supposed to capture the idea of a state of the system in which there are no forces tending to cause the state to change to a different state. For a market system, we think some prices are likely to change if there is excess demand or supply for any of the goods; and conversely that if all markets clear — *i.e.*, if no good is in excess demand or supply — then the prices will not change. And since the quantities that are transacted depend on the prices, the quantities should not change, either. So the natural definition of a general equilibrium of all markets is that all the markets clear — *i.e.*, that the price-list $\mathbf{p} \in \mathbb{R}_+^l$ satisfies the **equilibrium condition**

$$\overset{\triangle}{\mathbf{X}}(\mathbf{p}) = \mathbf{0} \qquad i.e., \ \overset{\triangle}{X}_k(\mathbf{p}) = 0 \, , \ k = 1, \ldots, l. \tag{$*$}$$

Provisional Definition: Let $E = ((u^i, \mathring{\mathbf{x}}^i))_{i=1}^n$ be an economy consisting of n consumers $(u^i, \mathring{\mathbf{x}}^i)$. Let $\mathbf{x}^i(\cdot) : \mathbb{R}_+^l \to \mathbb{R}^l$ denote the demand function of consumer $(u^i, \mathring{\mathbf{x}}^i)$, and let $\overset{\triangle}{\mathbf{X}}(\cdot) : \mathbb{R}_+^l \to \mathbb{R}^l$ denote the market net demand function $\overset{\triangle}{\mathbf{X}}(\mathbf{p}) := \sum_{i=1}^n (\mathbf{x}^i(\mathbf{p}) - \mathring{\mathbf{x}}^i)$. A **market equilibrium** of E is a price-list $\mathbf{p} \in \mathbb{R}_+^l$ that satisfies the equilibrium condition $(*)$.

There are many situations where this definition works just fine, but there are also many situations where it's not satisfactory. For example,

(1) If a price p_k is zero and there is excess supply of good k — *i.e.*, $\overset{\triangle}{X}_k(\mathbf{p}) < 0$ — it seems unlikely that this would lead to a change in any of the prices.

(2) What if the demand function $\mathbf{x}^i(\cdot)$ is not well-defined at some price-lists $\mathbf{p}$ for one or more consumers $(u^i, \mathring{\mathbf{x}}^i)$? For example, if $p_k = 0$, the CMP for some consumers may not have a solution.

(3) What if some consumer's demand function $\mathbf{x}^i(\cdot)$ is not single-valued at some price-lists? For example, a utility function u^i might have an indifference curve with a "flat spot" — an extreme example is a linear utility function $u(x, y) = ax + by$.

The following definition explicitly avoids issues (2) and (3) by including only situations in which all demand functions are well-defined and single-valued for every price-list $\mathbf{p} \in \mathbb{R}_+^l$. The definition takes account of issue (1) by allowing that excess supply of some goods is consistent with equilibrium if those goods have a price of zero.

Definition: Let $E = ((u^i, \mathring{\mathbf{x}}^i))_{i=1}^n$ be an economy consisting of n consumers, all of whose demand functions $\mathbf{x}^i(\cdot) : \mathbb{R}_+^l \to \mathbb{R}^l$ are well-defined and single-valued on $\mathbb{R}_+^l$, and let $\overset{\Delta}{\mathbf{X}}(\cdot) : \mathbb{R}_+^l \to \mathbb{R}^l$ denote the corresponding market net demand function. A **market equilibrium** of E is a price-list $\mathbf{p} \in \mathbb{R}_+^l$ that satisfies the equilibrium condition

$$\forall k = 1, \ldots, l : \quad \overset{\Delta}{X}_k(\mathbf{p}) \leqq 0 \quad \text{and} \quad \overset{\Delta}{X}_k(\mathbf{p}) = 0 \text{ if } p_k > 0. \tag{Clr}$$

We'll also refer to a price-list that satisfies (**Clr**) as an equilibrium of the net demand function $\overset{\Delta}{X}(\cdot)$.

We'll use this equilibrium condition throughout the course, so we give it a name that we'll use to refer to it: (**Clr**), which is an abbreviation for *Clear*, since the condition says that all markets clear.

A market equilibrium is also called a **Walrasian equilibrium**. An essential feature of this equilibrium concept is the assumption — implicit in the definition — that all consumers are **price takers**. Each consumer, in solving his consumer maximization problem, treats the prices as *parameters* that will be unaffected by his decision about which consumption bundle he will choose.