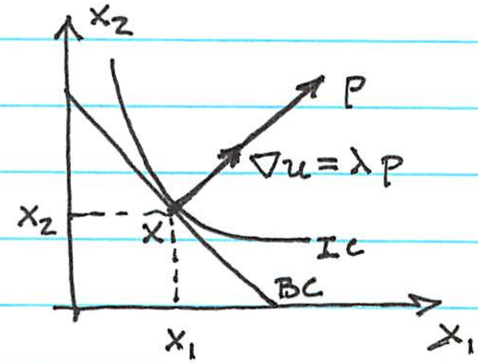
Examples for First 501A Lecture

(CMP) $\max_{x \in \mathbb{R}^n_+} u(x)$ s.t. $p \cdot x \leqq w$

(UMH) The consumer chooses a bundle x that is a solution of (CMP).

Implication: (if $u(\cdot)$ is "nice")

FOC
$$\left\{ \begin{array}{l} \exists \lambda > 0 : \nabla u = \lambda p \\ \quad \text{IC \& BC are tangent at } x \\ \text{i.e. } \dfrac{\partial u}{\partial x_k} = \lambda p_k, \quad k = 1, \ldots, n \\ \text{And also } p \cdot x = w. \end{array} \right.$$



Examples:

① $u(x,y) = xy \quad p_x = 2, \ p_y = 1, \ w = 12$

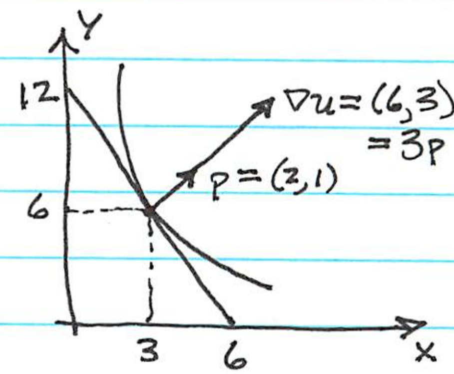
$u_x = y, \ u_y = x; \ MRS = \dfrac{u_x}{u_y} = \dfrac{y}{x}$

FOC: $\exists \lambda > 0$:

$$\left. \begin{array}{ll} u_x = \lambda p_x & \text{i.e., } y = \lambda \cdot 2 \\ u_y = \lambda p_y & \text{i.e., } x = \lambda \cdot 1 \end{array} \right\} y = 2x$$

$p_x x + p_y y = w$ i.e., $2x + y = 12 \rightarrow 2x + 2x = 12; \ 4x = 12$

$\therefore \boxed{x = 3, \ y = 6} \quad \therefore \lambda = 3.$



Note: When $n = 2$ we can use the fact that $MRS = \dfrac{u_x}{u_y}$ to write the FOC this way:

(M) $MRS = \dfrac{p_x}{p_y}$

(C) $p_x x + p_y y = w$

In Example 1: $\left\{ \begin{array}{l} \dfrac{y}{x} = \dfrac{2}{1} = 2; \ \text{i.e., } y = 2x \\ 2x + y = 12 \end{array} \right.$

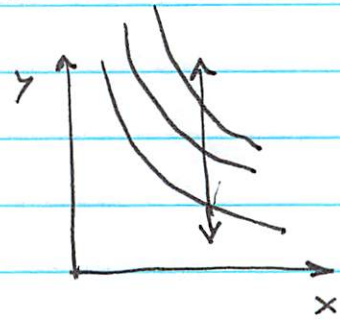
... then as above.

SAME B.C. AS IN (1)

(2) $u(x,y) = y + 6\log x$ $\quad P_x = 2,\ P_y = 1,\ W = 12$

$u_x = \dfrac{6}{x},\ u_y = 1;\ MRS = \dfrac{6}{x}$

NOTE THAT MRS DEPENDS ONLY ON x, SO I-CURVES ARE VERTICAL SHIFTS OF ONE ANOTHER.



<u>FOC:</u>

$$\left.\begin{array}{ll} u_x = \dfrac{6}{x} & \dfrac{6}{x} = \lambda \cdot 2 \\ u_y = 1 & 1 = \lambda \cdot 1 \end{array}\right\} \lambda = 1,\ x = 3$$

$P_x x + P_y y = W \longrightarrow (2)(3) + y = 12;\ \therefore y = 6$

$\boxed{x=3,\ y=6}$, SAME AS IN (1); $\lambda = 1$.

THIS IS A COINCIDENCE; TRY DIFFERENT VALUES FOR P_x, P_y, AND/OR W AND YOU WILL FIND THAT THE UTILITY FUNCTIONS IN (1) AND (2) LEAD TO DIFFERENT BUNDLES BEING CHOSEN.

FOR EXAMPLE, TRY $P_x = 2,\ P_y = 1,\ W = 18$
(PRICES UNCHANGED, BUT BUDGET INCREASED BY 50%)

OR TRY $P_x = 1.50,\ P_y = 1,\ W = 12$
(BUDGET AND P_y UNCHANGED, BUT P_x REDUCED BY 25%).

③ $u(x,y) = x^{\alpha} y^{\beta}$ (GENERALIZES ① TO GENERAL COBB-DOUGLAS)

WE'LL LEAVE p_x, p_y, w AS UNSPECIFIED PARAMETERS.

WE WANT TO SOLVE FOR x AND y AS FUNCTIONS OF p_x, p_y, w

— i.e., WE WANT TO DERIVE THIS CONSUMER'S <u>DEMAND FUNCTION</u> $(x(p_x,p_y,w),\ y(p_x,p_y,w))$.

$$u_x = \alpha x^{\alpha-1} y^{\beta} = \frac{\alpha}{x} x^{\alpha} y^{\beta} = \frac{\alpha}{x} u(x,y)$$

$$u_y = \beta x^{\alpha} y^{\beta-1} = \frac{\beta}{y} x^{\alpha} y^{\beta} = \frac{\beta}{y} u(x,y)$$

$$MRS = \frac{u_x}{u_y} = \frac{\frac{\alpha}{x} u(x,y)}{\frac{\beta}{y} u(x,y)} = \left(\frac{\alpha}{\beta}\right)\frac{y}{x}$$

<u>FOC:</u>

CONSTANT EXPENDITURE SHARES

$$MRS = \frac{p_x}{p_y}: \quad \left(\frac{\alpha}{\beta}\right)\frac{y}{x} = \frac{p_x}{p_y}; \text{ i.e., } \beta p_x x = \alpha p_y y$$

$$p_x x + p_y y = w: \quad p_x x + \frac{\beta}{\alpha} p_x x = w; \text{ i.e., } \left(1 + \frac{\beta}{\alpha}\right) p_x x = w$$

$$\text{i.e., } \frac{\alpha+\beta}{\alpha} p_x x = w; \quad \text{i.e., } p_x x = \frac{\alpha}{\alpha+\beta} w$$

$$\text{SIMILARLY, } \quad p_y y = \frac{\beta}{\alpha+\beta} w.$$

$$\therefore \quad x = \frac{\alpha}{\alpha+\beta}\left(\frac{w}{p_x}\right) \text{ AND } y = \frac{\beta}{\alpha+\beta}\left(\frac{w}{p_y}\right) \leftarrow$$ CONSUMER'S DEMAND FUNCTIONS