# DEMAND THEORY

(THEORY OF CONSUMER BEHAVIOR; CONSUMER CHOICE)

WE WANT A MODEL, OR THEORY, OF HOW THE CONSUMER CHOOSES WHAT HE WILL BUY AND CONSUME.

THE THEORY WE DEVELOP WILL BE VERY TYPICAL — i.e., IT WILL HAVE THE SAME STRUCTURE AS OTHER MODELS WE WILL DEVELOP (SUCH AS THEORY OF THE FIRM; THEORY OF DECISION-MAKING UNDER UNCERTAINTY; ETC.), AND OUR ANALYSIS USING THE MODEL WILL ASK THE SAME QUESTIONS AND USE THE SAME TECHNIQUES AS IN SUBSEQUENT MODELS.

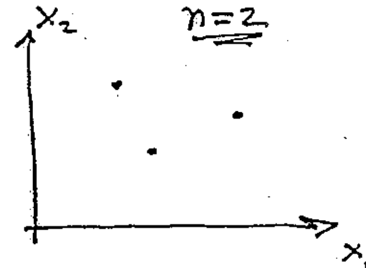
THE GENERAL STRUCTURE: (OF A DECISION THEORY)

(1) ALTERNATIVES
(2) CONSTRAINTS
(3) CRITERION FOR CHOOSING (OBJECTIVE FUNCTION, PREFERENCE, ETC.)

WE WILL EMPHASIZE THE RELATION BETWEEN...

(a) REALITY (REAL CHOICES)
(b) ANALYSIS (SYMBOLIC; ALGEBRAIC)
(c) GEOMETRY.

# THE CONSUMER'S PROBLEM:




(1) ALTERNATIVES:

CONSUMPTION BUNDLES, i.e., AMOUNTS (QUANTITIES) OF VARIOUS GOODS OR COMMODITIES.

GEOMETRIC: EACH ALTERNATIVE IS A POINT.

ALGEBRAIC: EACH ALTERNATIVE IS AN $n$-TUPLE, OR LIST, OF NON-NEGATIVE NUMBERS — i.e., $x = (x_1, \ldots, x_n) \in \mathbb{R}^n_+$

→ THE ANALYST HAS TO DECIDE HOW TO DIVIDE THINGS UP INTO "COMMODITIES." FOR EXAMPLE, ARE HAMBURGER AND STEAK DISTINCT COMMODITIES, OR BOTH INSTANCES OF BEEF? IS GASOLINE AT SPEEDWAY & PARK THE SAME COMMODITY AS AT RIVER & CAMPBELL?

(2) CONSTRAINTS:

WE'VE ALREADY INTRODUCED $n$ CONSTRAINTS WITHOUT MENTIONING THEM: THE NON-NEGATIVITY CONSTRAINTS $x_1 \geqq 0, x_2 \geqq 0, \ldots, x_n \geqq 0$.

AN ADDITIONAL CONSTRAINT IS IMPOSED ON THE CONSUMER BY HIS BUDGET — WHAT HE CAN AFFORD. IF THE COMMODITIES' PRICES (PER UNIT, IN \$) ARE $p_1, \ldots, p_n$, AND IF HIS BUDGET (INCOME, WEALTH) IS $w$ (DOLLARS), THEN HE CAN ONLY CHOOSE BUNDLES $x$ THAT SATISFY HIS BUDGET CONSTRAINT $p \cdot x \leqq w$.

(BC) $p \cdot x \leqq w$ ; i.e., $p_1 x_1 + \cdots + p_n x_n \leqq w$

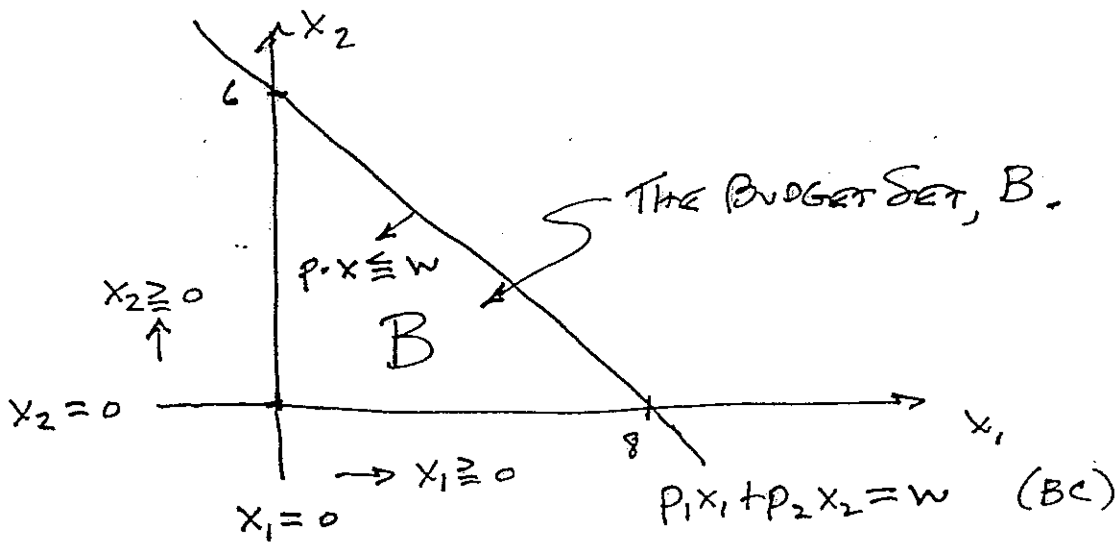
i.e., $\sum_{i=1}^{n} p_i x_i \leqq w$

THE GEOMETRY:

THE BUDGET SET, OR FEASIBLE SET, IS THE SET OF ALL BUNDLES $x$ THAT SATISFY (BC) AND THE NON-NEGATIVITY CONSTRAINTS — i.e.,

$$B = \{x \in \mathbb{R}^n_+ \mid p \cdot x \leqq w\}$$
$$= \{x \in \mathbb{R} \mid p \cdot x \leqq w \ \& \ x_i \geqq 0 \ (\forall i)\}.$$

BECAUSE $p \cdot x \leqq w$ IS A LINEAR INEQUALITY, THE BUDGET SET IS A HALF-SPACE IN $\mathbb{R}^n$ BOUNDED BY THE HYPERPLANE $p \cdot x = w$, AND TRUNCATED BY $x_1 \geqq 0, \ldots, x_n \geqq 0$.




e.g., $3x_1 + 4x_2 = 24$

(3) <u>CRITERION, OR OBJECTIVE:</u>

HERE AT THE BEGINNING WE WILL ASSUME THAT THE CONSUMER HAS A <u>UTILITY FUNCTION</u> $u: \mathbb{R}^n_+ \to \mathbb{R}$ WHICH TELLS HIM (OR US) WHICH BUNDLES HE LIKES MORE AND WHICH ONES LESS. WE INTERPRET

$u(\tilde{x}) > u(x)$ TO MEAN HE <u>PREFERS</u> BUNDLE $\tilde{x}$ TO BUNDLE $x$ (i.e., IF OFFERED A CHOICE BETWEEN JUST $\tilde{x}$ AND $x$, HE WOULD DEFINITELY CHOOSE $\tilde{x}$);

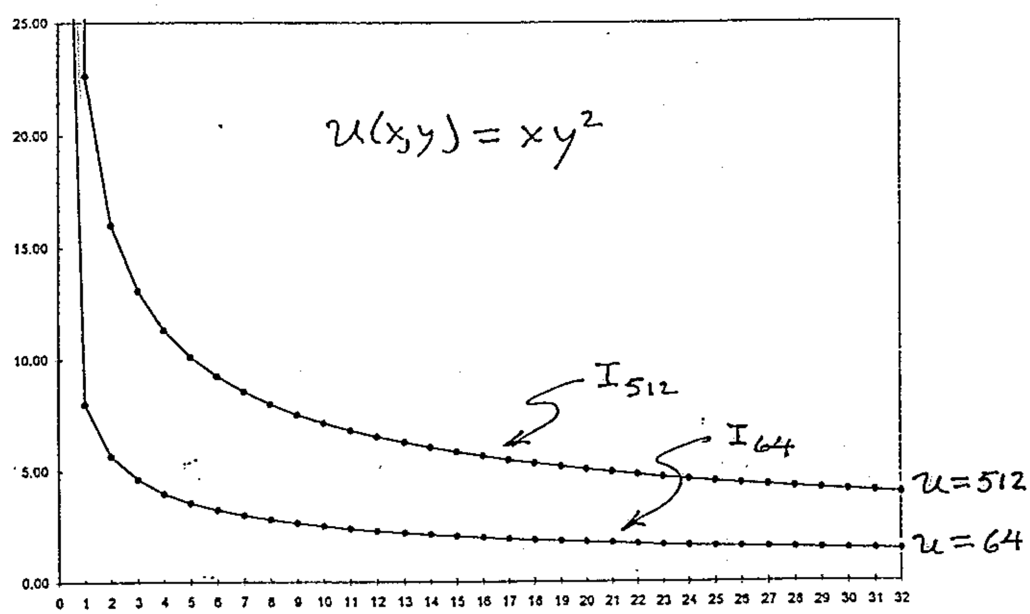
AND $u(\tilde{x}) = u(x)$ TO MEAN HE IS <u>INDIFFERENT</u> BETWEEN THE TWO BUNDLES $\tilde{x}$ AND $x$ (i.e., IF OFFERED ~~THE CHOICE~~ OF $\tilde{x}$ OR $x$, WE CANNOT SAY WHICH HE WOULD CHOOSE).

→ NOTE: ULTIMATELY, WE WILL MOVE TO A WEAKER AND MORE INTUITIVE ASSUMPTION, THAT THE CONSUMER'S CHOICES CAN BE DESCRIBED BY A <u>PREFERENCE RELATION</u> (ESSENTIALLY AN INDIFFERENCE MAP) THAT IS WELL-BEHAVED, AND WE WILL DETERMINE CONDITIONS UNDER WHICH THAT DESCRIPTION OF BEHAVIOR IS EQUIVALENT TO THE UTILITY-FUNCTION DESCRIPTION WE ARE USING HERE.

NOTE THAT THE FUNCTION $u(\cdot)$, IF IT IS CONTINUOUS, CAN BE DESCRIBED BY ITS <u>LEVEL CURVES</u>. A LEVEL CURVE OF $u(\cdot)$ (AS FOR ANY FUNCTION) IS A SET OF THE FORM

$$I_c := \{x \in \mathbb{R}^n_+ \mid u(x) = c\} \text{ FOR SOME } c \in \mathbb{R}$$

— i.e., THE SET OF ALL BUNDLES $x$ THAT YIELD UTILITY $c$.



# THE CONSUMER'S MAXIMIZATION PROBLEM:

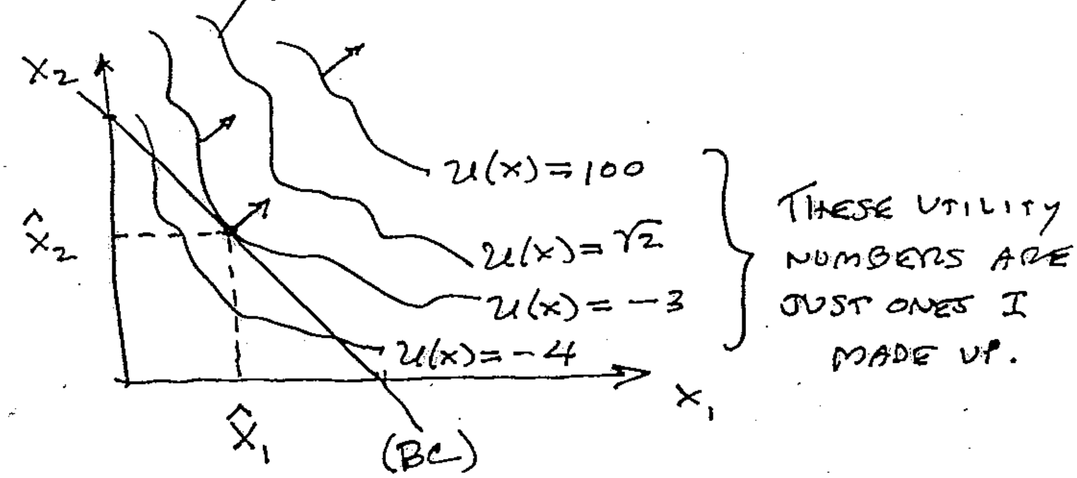
WE CAN SUMMARIZE EVERYTHING WE'VE SAID ABOUT THE CONSUMER'S CHOICE PROBLEM BY SIMPLY STATING THE <u>CONSUMER'S MAXIMIZATION PROBLEM</u> (CMP): [ALSO: UTILITY MAXIMIZATION PROBLEM (UMP)]

(CMP) CHOOSE $x \in \mathbb{R}^n$ TO MAXIMIZE $u(x)$

SUBJECT TO $x \geqq \underline{0}$ AND TO

$$p \cdot x \leqq w. \quad \text{(BC)}$$

GEOMETRICALLY, THE (CMP) LOOKS LIKE THIS:




AND A SOLUTION TO (CMP) IS A BUNDLE, SAY $\hat{x}$, THAT YIELDS THE GREATEST VALUE OF $u(x)$ AMONG ALL THE BUNDLES IN THE BUDGET SET $B$.

## THINGS WE WANT TO KNOW ABOUT THE SOLUTION:

(1) DOES A SOLUTION EXIST? $\Big\{$ OR "UNDER WHAT CONDITIONS DOES A SOLUTION EXIST?"

(2) IS THE SOLUTION UNIQUE? $\Big\{$ ~~OR "UNDER WHAT CONDITIONS IS THE SOLUTION UNIQUE?"~~

(3) HOW DOES THE SOLUTION CHANGE AS THE CONSUMER'S ENVIRONMENT CHANGES?

IN THE (CMP):

THE CONSUMER IS REPRESENTED BY THE UTILITY FUNCTION $u(\cdot)$;

THE ENVIRONMENT IS REPRESENTED BY THE PARAMETERS $p_1, \ldots, p_n$ AND $w$.

IN OTHER WORDS, WE WOULD LIKE TO KNOW THE SOLUTION FUNCTION THAT GIVES $x$ AS A FUNCTION OF $p$ AND $w$:

$$x = f(p, w)$$

i.e., THE DEMAND FUNCTION; EQUIVALENTLY, THE DEMAND FUNCTIONS FOR THE $n$ GOODS:

$$\begin{aligned} x_1 &= f_1(p, w) = f_1(p_1, \ldots, p_n, w) \\ &\vdots \\ x_n &= f_n(p, w) = f_n(p_1, \ldots, p_n, w). \end{aligned}$$

FOR EXAMPLE, WHEN THERE ARE ONLY TWO GOODS:
(LET'S USE $x$ AND $y$ INSTEAD OF $x_1$ AND $x_2$)

$$(\text{CMP}) \quad \max_{(x,y)\in\mathbb{R}^2_+} u(x,y) \text{ s.t. } p_x x + p_y y \leq w$$

FOC: (INTERIOR)

$$\left.\begin{aligned} u_x(x,y) &= \lambda p_x \\ u_y(x,y) &= \lambda p_y \\ p_x x + p_y y &= w \end{aligned}\right\}$$

WE WANT TO SOLVE THESE THREE EQUATIONS FOR THE THREE VARIABLES $x, y, \lambda$:

$$(x,y,\lambda) = f(p_x, p_y, w).$$

NOTE THAT IN THE TWO-GOOD CASE WE CAN ELIMINATE THE $\lambda$ AND EXPRESS THE TWO MARGINAL CONDITIONS AS A SINGLE EQUATION WITH THE MRS:

$$\frac{u_x}{u_y} = \frac{\lambda p_x}{\lambda p_y} = \frac{p_x}{p_y}\,; \text{ i.e., } MRS = \frac{p_x}{p_y}$$

DEPENDS ON $(x,y)$

LET'S CONSIDER THE UTILITY FUNCTION
$u(x,y) = x^2 y$:

(1) FOR SPECIFIC VALUES OF THE PARAMETERS, WE CAN SOLVE FOR THE CONSUMER'S CHOSEN BUNDLE.
FOR EXAMPLE, SUPPOSE $W=36$ AND $P_x=3$, $P_y=2$:

$$\max_{(x,y)\in\mathbb{R}^2_+} u(x,y) = x^2 y \quad \text{s.t. } 3x+2y \leq 36$$

FOC:

$u_x = \lambda P_x$: $2xy = 3\lambda$

$u_y = \lambda P_y$: $x^2 = 2\lambda$

$3x+2y=36$

SOLVING THE EQUATIONS:

$2\frac{y}{x} = \frac{3}{2}$; $4y=3x$; $y=\frac{3}{4}x$

$\therefore\ 3x + 2\left(\frac{3}{4}x\right) = 36$

$3x + \frac{3}{2}x = 36$

$x=8,\ y=6$

(2) BUT WE CAN ALSO SOLVE FOR THE CHOSEN BUNDLE AS A <u>FUNCTION</u> OF THE PARAMETER VALUES — i.e., THE CONSUMER'S DEMAND FUNCTION:

$$\max_{(x,y)\in\mathbb{R}^2_+} u(x,y) = x^2 y \quad \text{s.t. } P_x x + P_y y \leq W$$

FOC:

$u_x = \lambda P_x$

$u_y = \lambda P_y$

$P_x x + P_y y = W$

SOLVING:

$\frac{u_x}{u_y} = \frac{P_x}{P_y}$; i.e., $2\frac{y}{x} = \frac{P_x}{P_y}$

i.e., $2P_y y = P_x x$

$\therefore\ P_x x + \frac{1}{2}P_x x = W$; $\frac{3}{2}P_x x = W$

i.e., $P_x x = \frac{2}{3}W$, $P_y y = \frac{1}{3}W$

$\therefore\ x = \frac{2}{3}\left(\frac{W}{P_x}\right)$, $y = \frac{1}{3}\left(\frac{W}{P_x}\right)$