Scalable and Fully Distributed Localization with Mere Connectivity

Miao Jin[1], Su Xia[1], Hongyi Wu[1], and Xianfeng Gu[2]

1. University of Louisiana at Lafayette

2. Stony Brook University

Presenter: Hongyi Wu

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MOTIVATION

- Location information is imperative to a variety of applications in wireless sensor networks.
- Location system based on mere connectivity is ideal for large-scale sensor networks, for cost-effectiveness.

RELATED WORK

- Previous connectivity-based localization methods:
  - Multi-Dimensional Scaling (MDS)
    - Low scalability
    - Essentially centralized
  - Neural network-based
    - Numerically unstable
  - Graph rigidity theory
    - Low localization accuracy

OUR PROPOSED METHOD

- A planar sensor network is modeled as a discrete surface with the distance between two nodes approximated by hop counts.
- Due to the approximation error, the surface is curved and cannot be embedded directly on a flat plane.
- We compute the provably optimal flat metric which introduces the least distortion from estimated metric to isometrically embed the surface to plane.

MAIN CONTRIBUTIONS

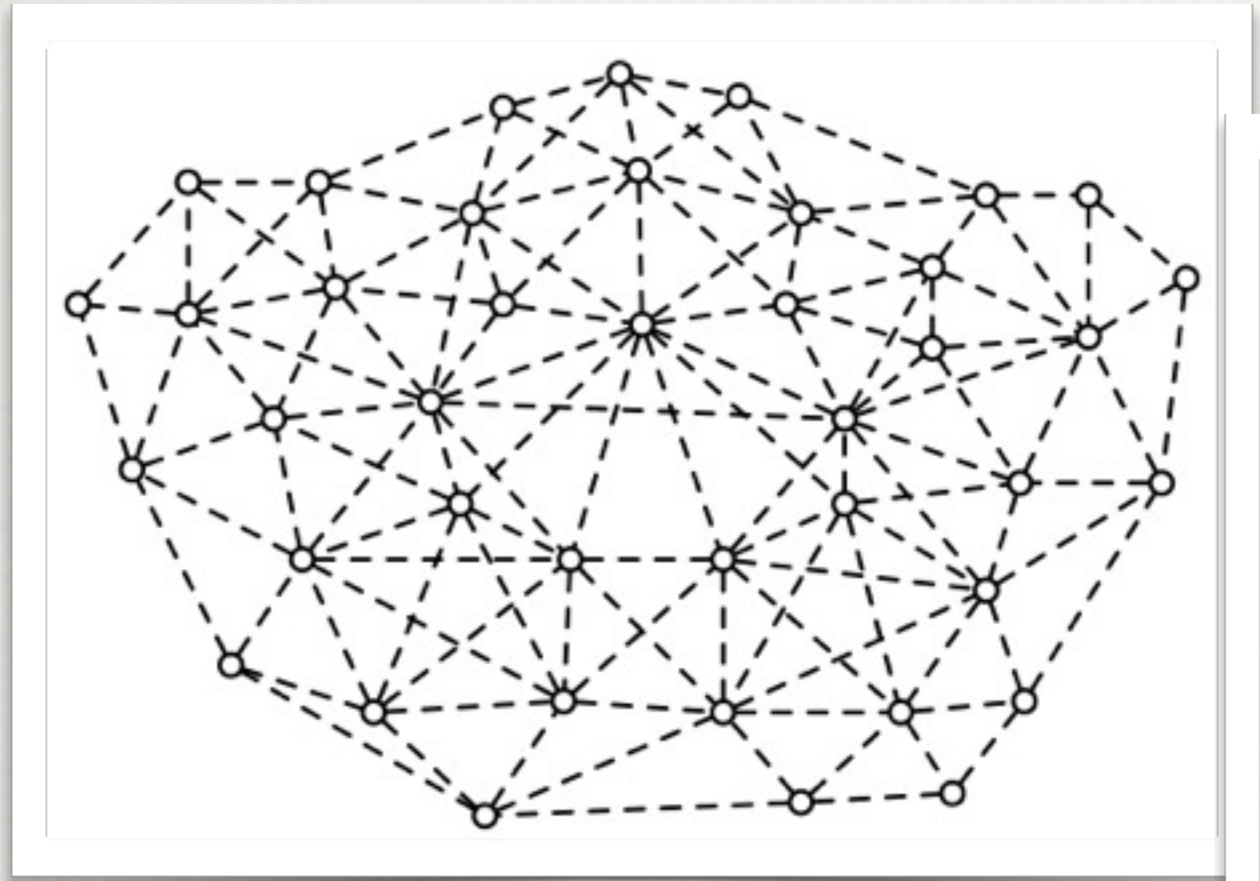
- Fully distributed: each node only requires information from its direct neighbors
- Scalable
  - Computation and communication cost are both linear to the size of the network
  - Limited error propagation
- Theoretically sound: provably optimal
- Numerically stable : free of the choice of initial values and local minima with theoretical guarantee

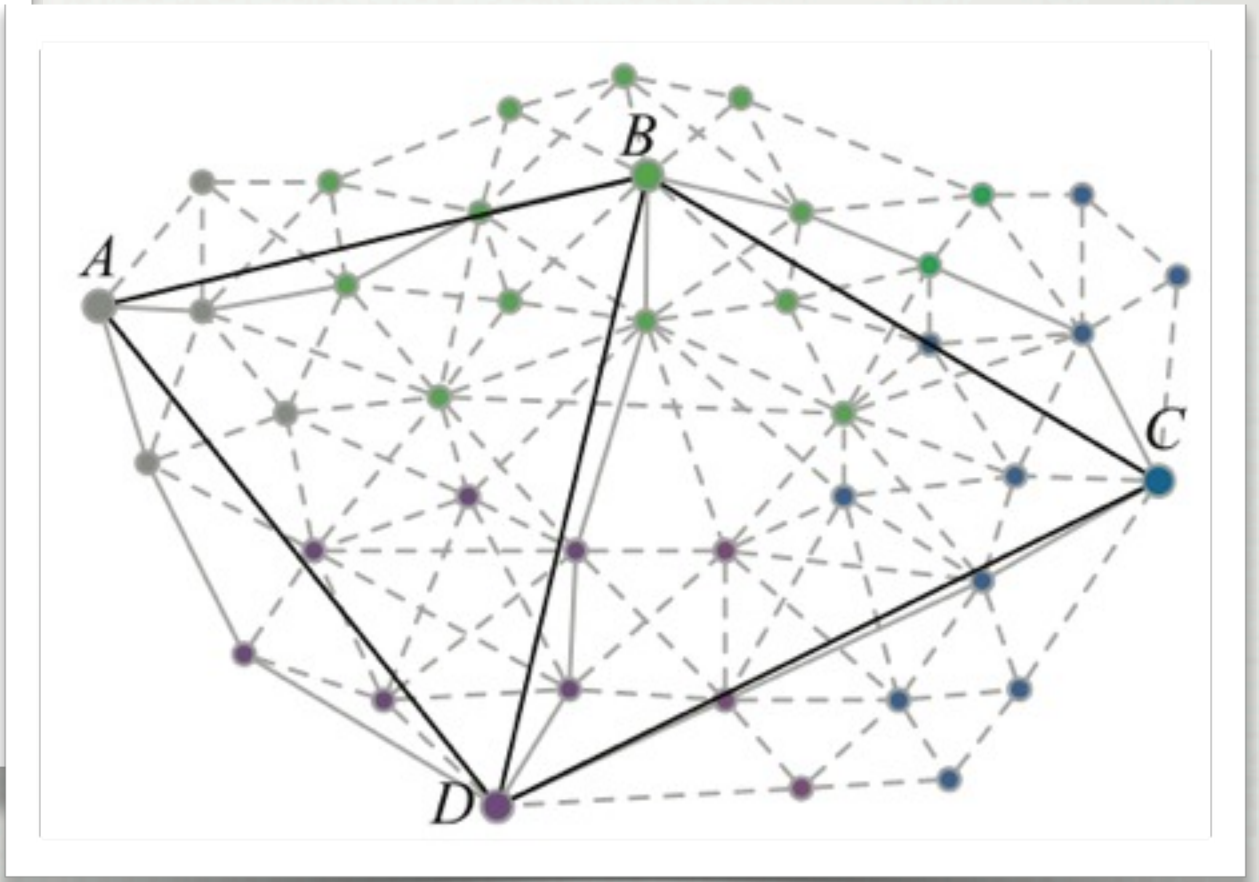
TALK OUTLINE

- ☐ Theory of flat metric
- ☐ Distributed algorithm for sensor networks
- ☐ Discussions on cost and error propagation
- ☐ Simulations and comparison

DISCRETE METRIC AND DISCRETE GAUSSIAN CURVATURE

- Triangulation mesh

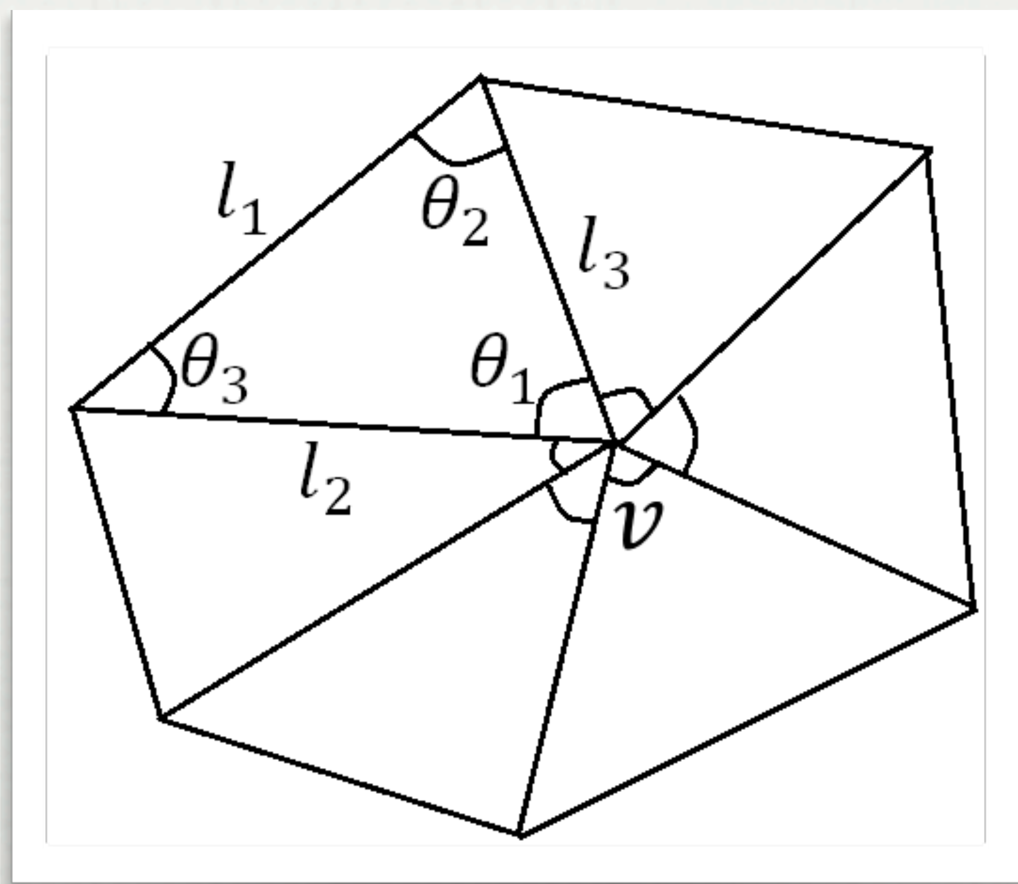




INFOCOM'11: "A Distributed Triangulation Algorithm for Wireless Sensor Networks on 2D and 3D Surface"

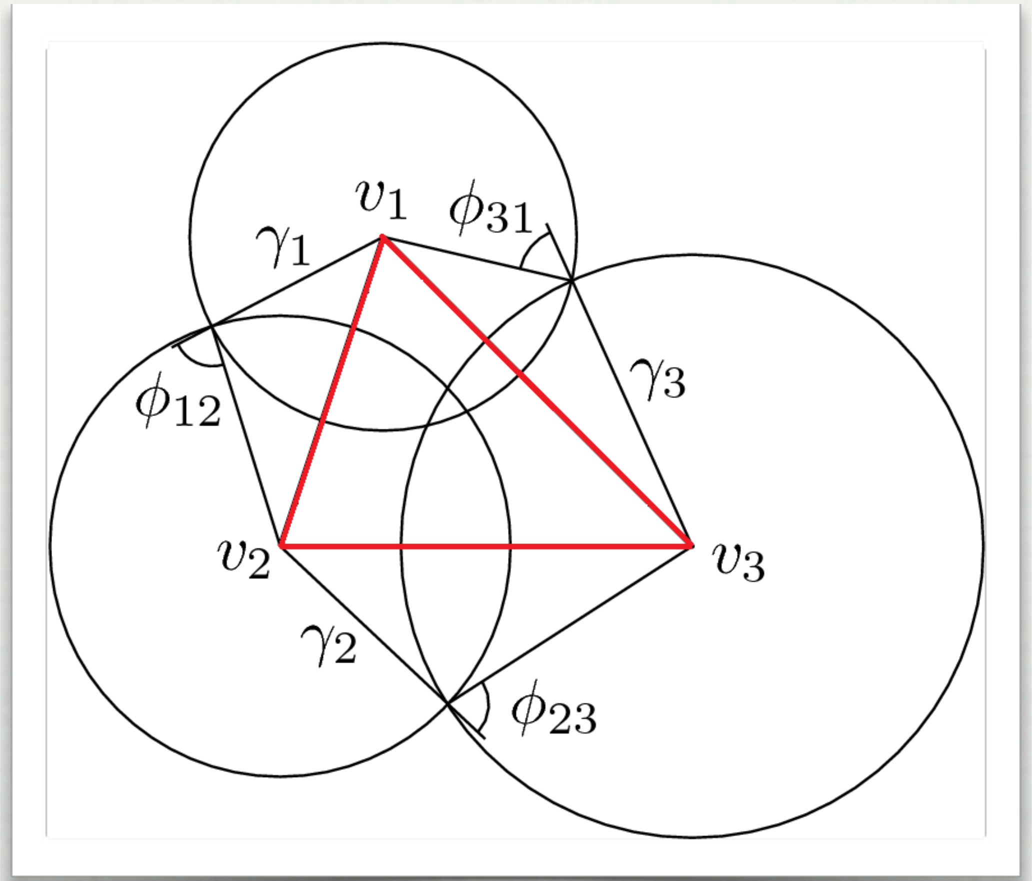
DISCRETE METRIC AND DISCRETE GAUSSIAN CURVATURE

- Discrete metric: edge length of the triangulation
- Discrete Gaussian curvature: induced by metric, measured as angle deficit



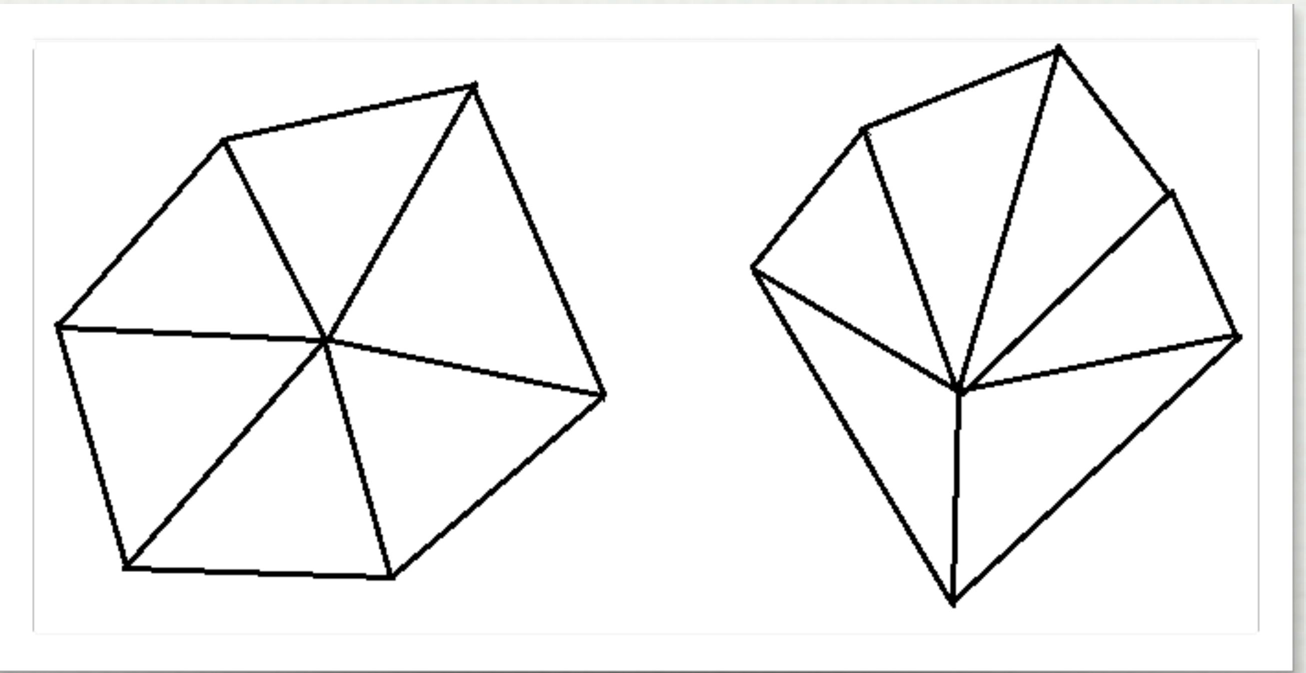
CIRCLE PACKING METRIC

- ☐ A vertex is associated with a circle with radius of γ_i
- ☐ Two circles have an intersection angle of $\mathbf{\Phi}_{ij}$
- ☐ Circle packing metric induces discrete metric



FLAT METRIC

- Flat metric: a set of edge lengths which induce zero Gaussian curvature for all inner vertices such that the triangulation is isometrically embedded to plane.
- Infinite number of flat metrics for a given connectivity-based triangulation.



OPTIMAL FLAT METRIC

- Question: given a triangulation mesh, which flat metric introduces the least distortion from the estimated discrete metric?
- Theory (Theorem 2): the flat metric with the least distortion is unique and satisfies:

$$u_j = const, \forall v_j \in \partial M$$

where $u_j = \log \Upsilon_j$ and v_j is a boundary vertex

TOOL TO COMPUTE OPTIMAL FLAT METRIC

- Discrete Ricci flow
  - [Hamilton 1982]: Ricci flow on closed surfaces of non-positive Euler characteristic
  - [Chow 1991]: Ricci flow on closed surfaces of positive Euler characteristic
  - [Chow and Luo 2003]: Discrete Ricci flow including existence of solutions, criteria, and convergence.
  - [Jin et al. 2008]: a unified framework of computational algorithms for discrete Ricci flow

COMPUTE OPTIMAL FLAT METRIC

- Discrete Ricci flow is an efficient tool to decide metrics based on user predefined curvature

$$\frac{du_i(t)}{dt} = (\bar{K}_i - K_i)$$

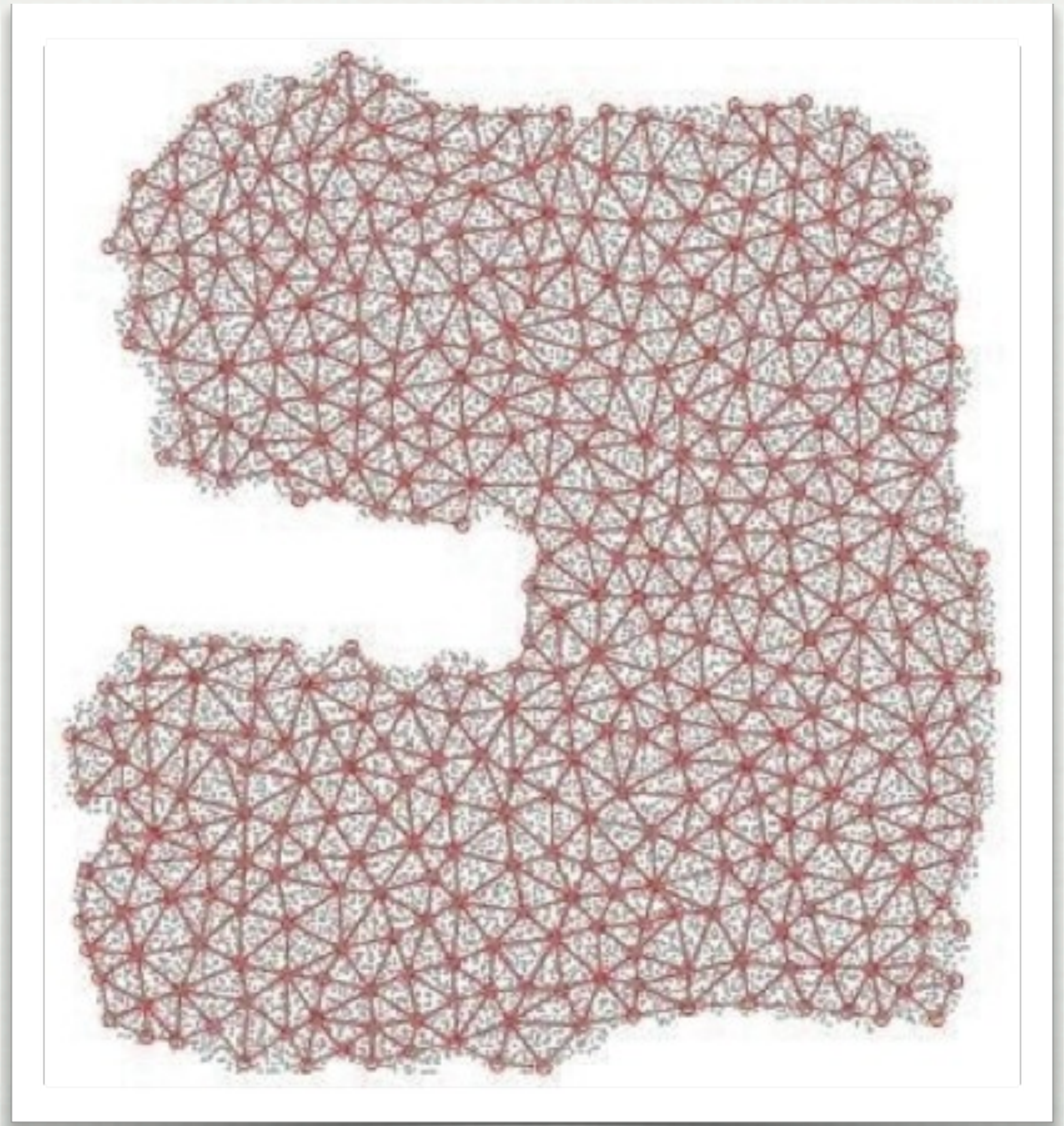
where u_i is the circle packing metric, deforming according to the difference between the target Gaussian curvature and the current Gaussian curvature.

TALK OUTLINE

- ☐ Theory of flat metric
- ☐ <u>Distributed algorithm for sensor networks</u>
- ☐ Discussions on cost and error propagation
- ☐ Experiments and comparison

DISTRIBUTED ALGORITHM (1) PRE-PROCESSING

- Build a triangulation from network graph



INFOCOM'11: "A Distributed Triangulation Algorithm for Wireless Sensor Networks on 2D and 3D Surface"

DISTRIBUTED ALGORITHM (2) COMPUTE FLAT METRIC

- All edge lengths initialization: unit
  - The curvature of a vertex is nonzero
  - The initial triangulation surface can't be embedded on a plane.
- Find the flat metric, such that
  - The triangulation surface can be isometrically embedded on a plane
  - The introduced localization error is minimal

DISTRIBUTED ALGORITHM (2) COMPUTE FLAT METRIC

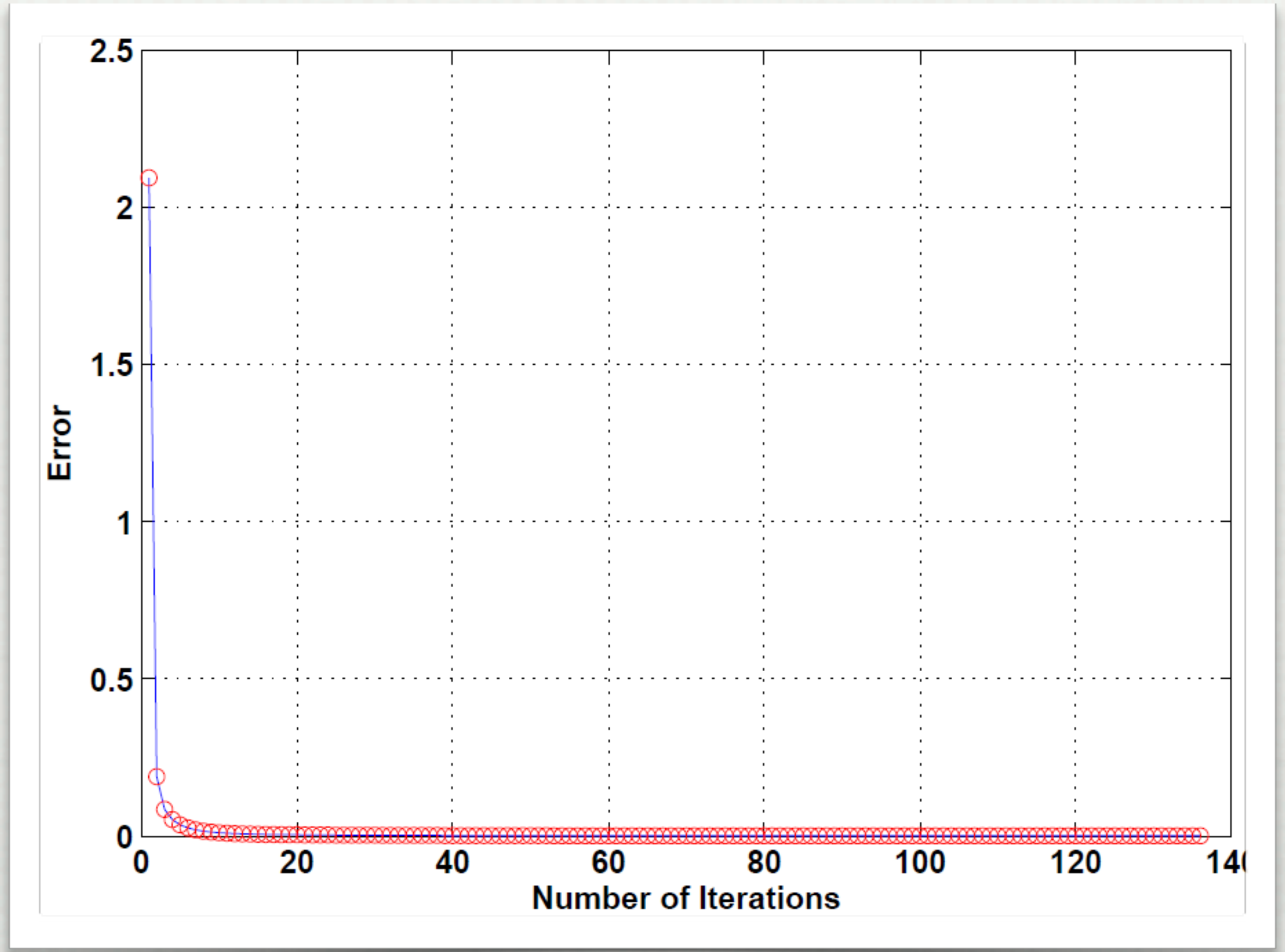
- Determine flat metric based on Ricci flow
  - Set the target Gaussian curvature of all inner nodes to zero
  - Update the circle packing metric u_i on vertex v_i by the difference of v_i's target and current Gaussian curvature

$$u_i = u_i + \delta(\bar{K}_i - K_i)$$

  - Stop until the curvature error is less than ϵ. The final circle packing metric induces flat metric

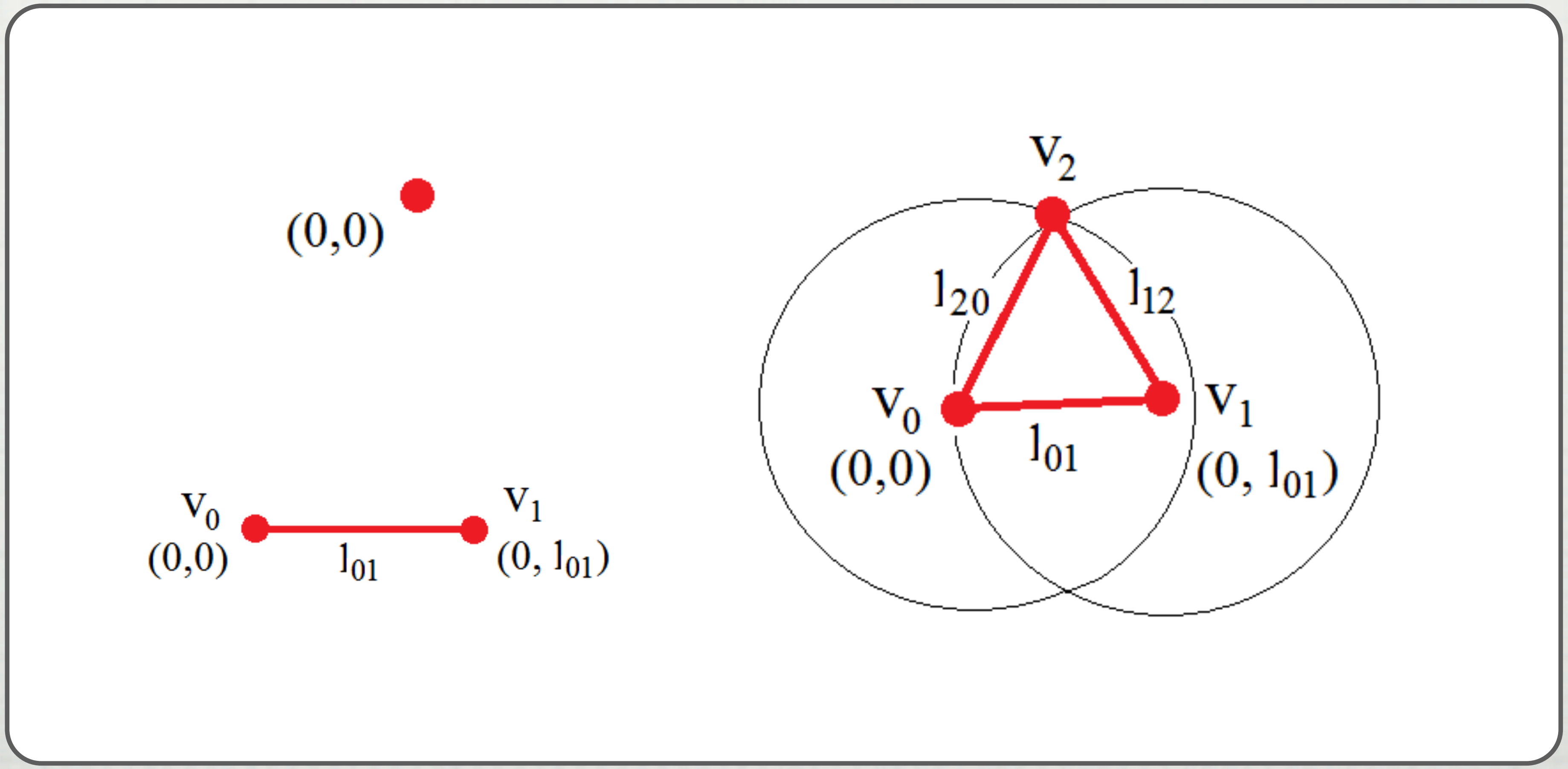
CONVERGENCE RATE

- Number of iterations: O(-logϵ/δ)



DISTRIBUTED ALGORITHM (3) ISOMETRIC EMBEDDING

- Start embedding from one vertex
- Continuously propagate to the whole triangulation



COST

- Step 1: Preprocessing step - building triangulation
  - Time complexity: $O(n)$
  - Communication cost: $O(n)$
- Step 2: Compute flat metric
  - Time complexity: $O(-\log\epsilon/\delta)$
  - Communication cost: $O(-g\log\epsilon/\delta)$
- Step 3: compute isometric embedding
  - Time complexity: $O(m)$
  - Communication cost: $O(m)$

ERROR PROPAGATION

- The impact of error to our proposed scheme can be modeled as a discrete Green function:

$$G(p,q) = \frac{1}{|p-q|}$$

  where p and q are two points far away from the boundary.

- The impact of a measurement error at Point p to Point q decreases quickly with their distance.

TALK OUTLINE

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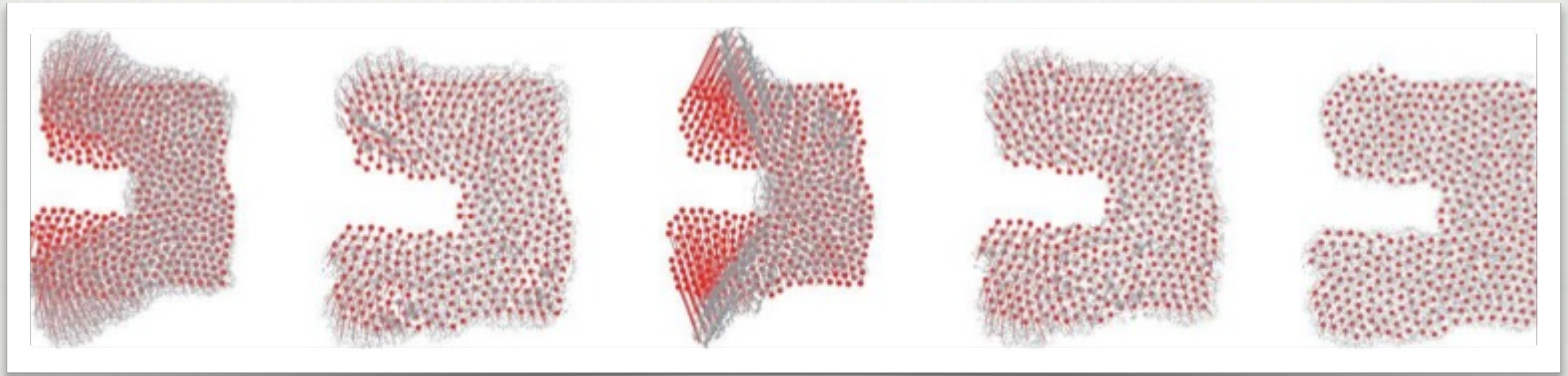
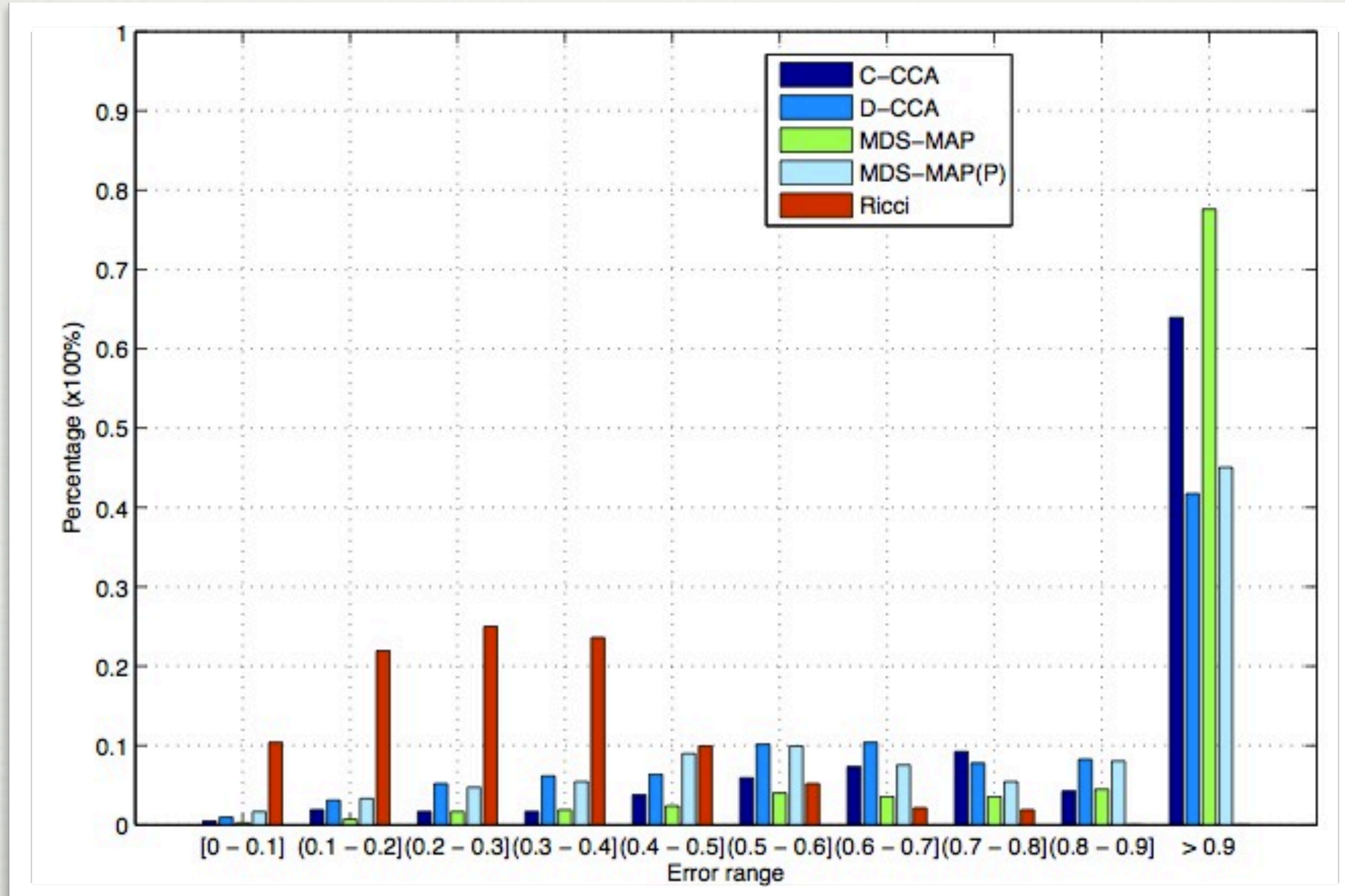
- ☐ <u>Experiments and comparison</u>

ERROR COMPARISON

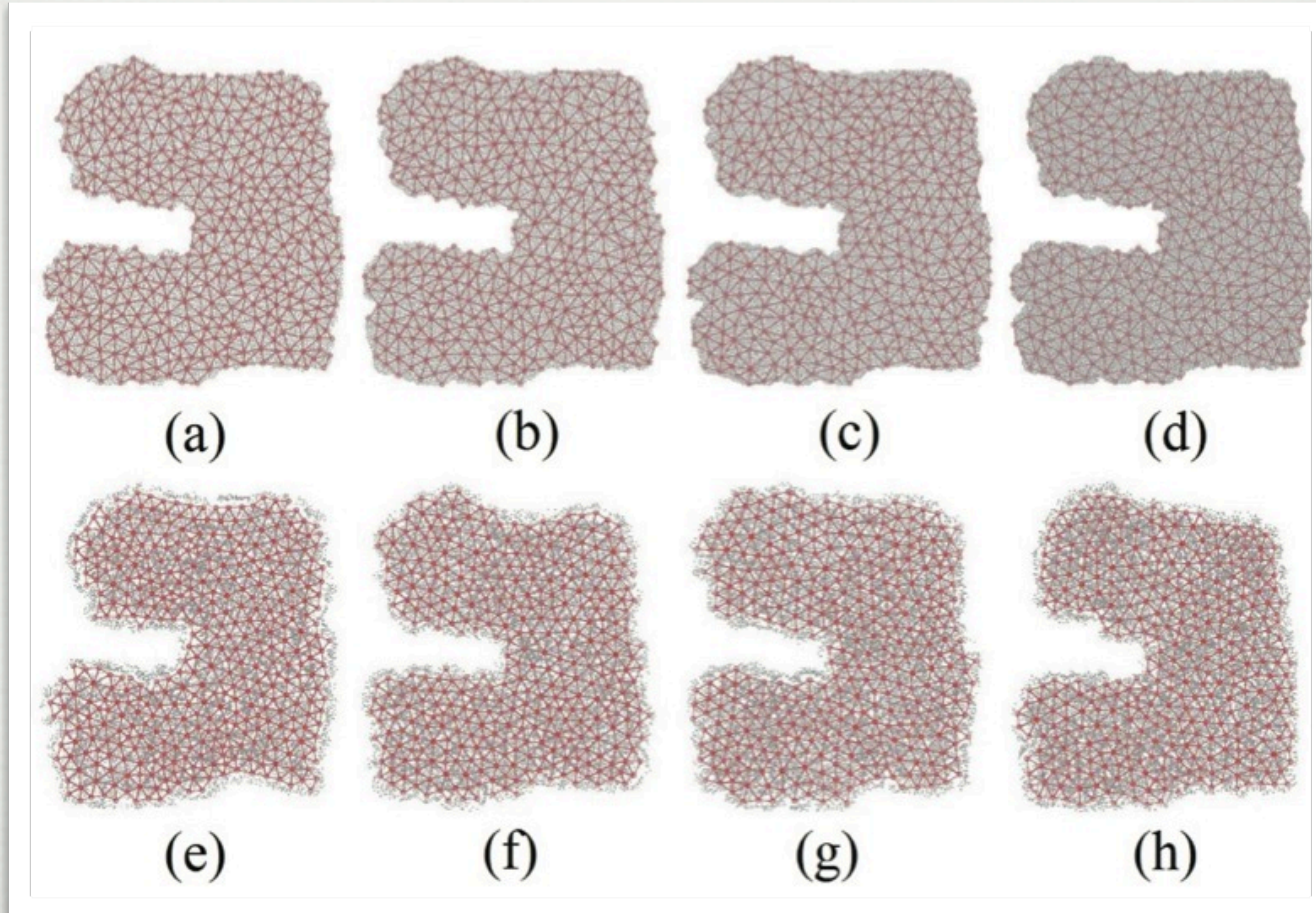


Scenario	C-CCA	D-CCA	MDS-MAP	MDS-MAP(P)	Ricci
Topology 1 (Fig. 6 a(1)-e(1))	2.10	0.88	2.52	0.89	0.29
Topology 2 (Fig. 6 a(2)-e(2))	0.71	0.69	0.56	0.68	0.32
Topology 3 (Fig. 6 a(3)-e(3))	0.72	0.64	0.62	0.75	0.48
Topology 4 (Fig. 6 a(4)-e(4))	0.78	0.70	1.18	0.61	0.55
Topology 5 (Fig. 6 a(5)-e(5))	1.17	0.8	1.27	0.99	0.63

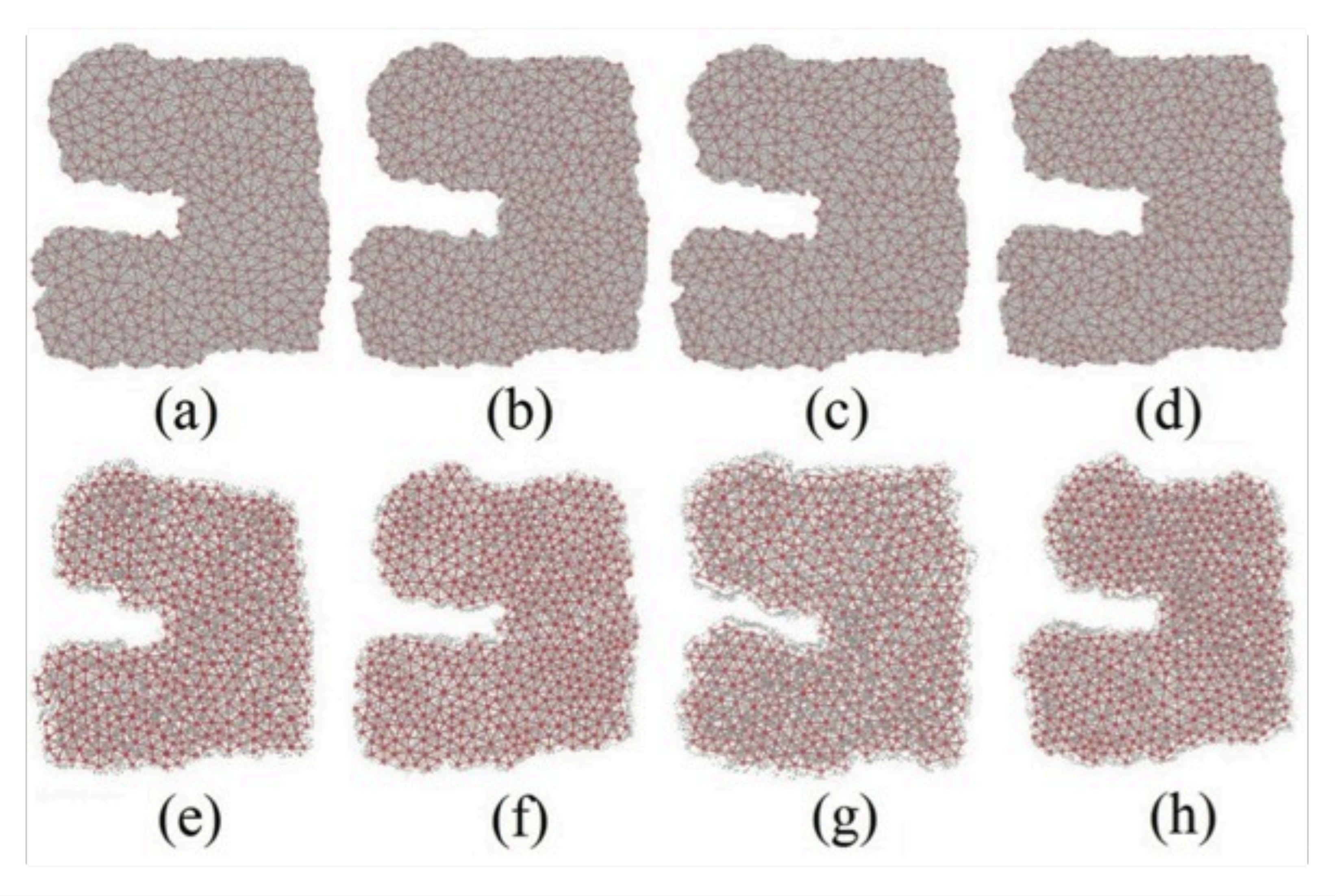
DISTRIBUTION OF LOCALIZATION ERRORS



VARIANT DENSITY



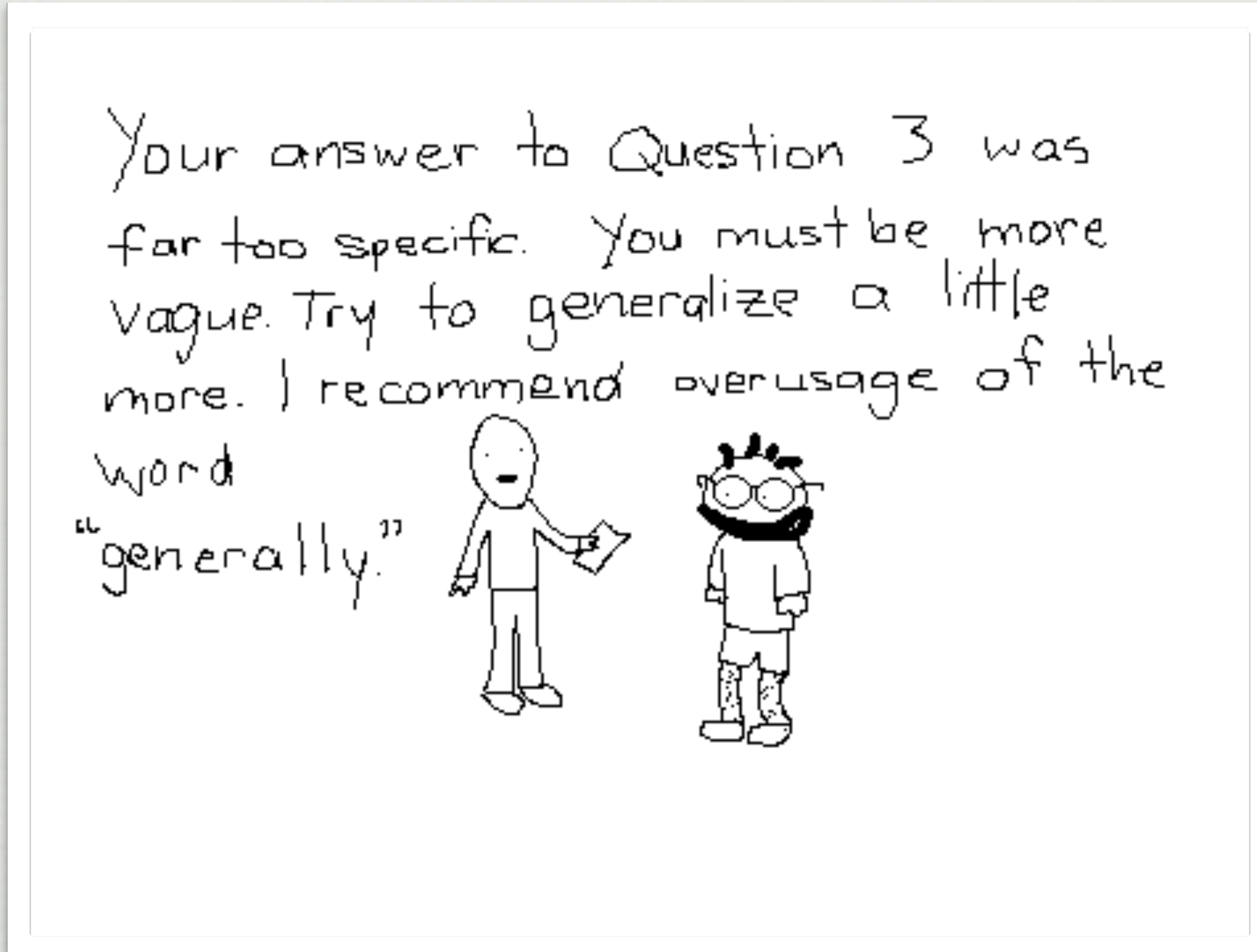
DIFFERENT TRANSMISSION MODELS



Trans. models	UDG	Quasi-UDG	Log-Norm	Probability
Localization error	0.25	0.34	0.42	0.43

SUMMARY AND FUTURE WORK

- Mere connectivity-based localization method
  - Theoretically guaranteed and numerically stable
  - Fully distributed and highly scalable with linear computation time and communication cost and local error propagation
- Future work: localization for sensor nodes deployed on general 3D surface


Your answer to Question 3 was far too specific. You must be more vague. Try to generalize a little more. I recommend overusage of the word "generally."

QUESTIONS ?