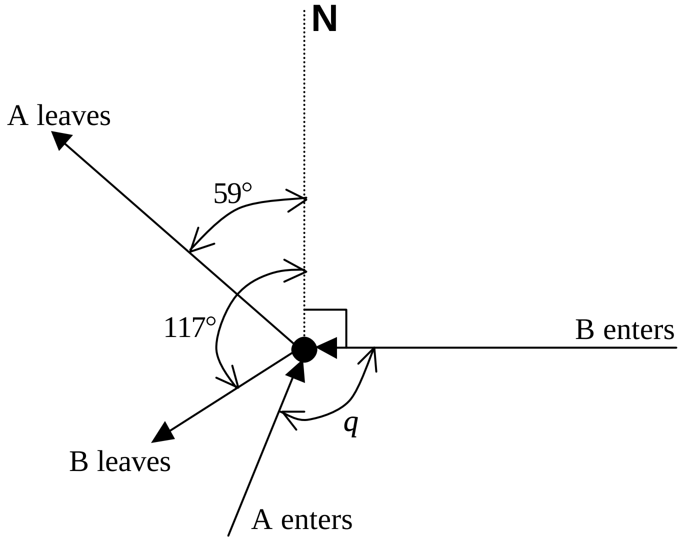
**CE 467 / 567 Highway Safety and Operations**
**Homework 7 Solutions**

We can begin this problem with some preliminaries. First, a diagram with associated angles could include:




We are given some pertinent information about the post-crash movement of vehicles A and B. Vehicle A travels 33 ft with a coefficient of drag of 0.80. At an angle of 59° from North, the grade over which vehicle A travels is given by: (+1%)*cos(90° – 59°) = (+1%)*cos(31°)= +0.86%. With the skid distance formula:

$$d_{A-skid} = \frac{v_{A-i}^2}{2g(f+G)}$$

$$v_{A-i}^2 = 2g(f+G)\cdot d_{A-skid} = 2\cdot(32.2\text{ ft/s}^2)\cdot(0.80+0.0086)\cdot(33\text{ ft}) = 1718.4\ \text{ft}^2/\text{sec}^2$$

$$v_{A-i} = 41.5\text{ ft/sec}$$

On the other hand, vehicle B travels 21 ft with a coefficient of drag of 0.55. At an angle of 117° from North, the grade over which vehicle B travels is given by: (+1%)*cos(117°–90°) = (+1%)*cos(27°)= +0.89%. Then:

$$d_{B-skid} = \frac{v_{B-i}^2}{2g(f+G)}$$

$$v_{B-i}^2 = 2g(f+G)\cdot d_{B-skid} = 2\cdot(32.2\text{ ft/s}^2)\cdot(0.55+0.0089)\cdot(21\text{ ft}) = 755.9\ \text{ft}^2/\text{sec}^2$$

$$v_{B-i} = 27.5\text{ ft/sec}$$

This gives us the two vehicles' speeds upon departing the crash location.

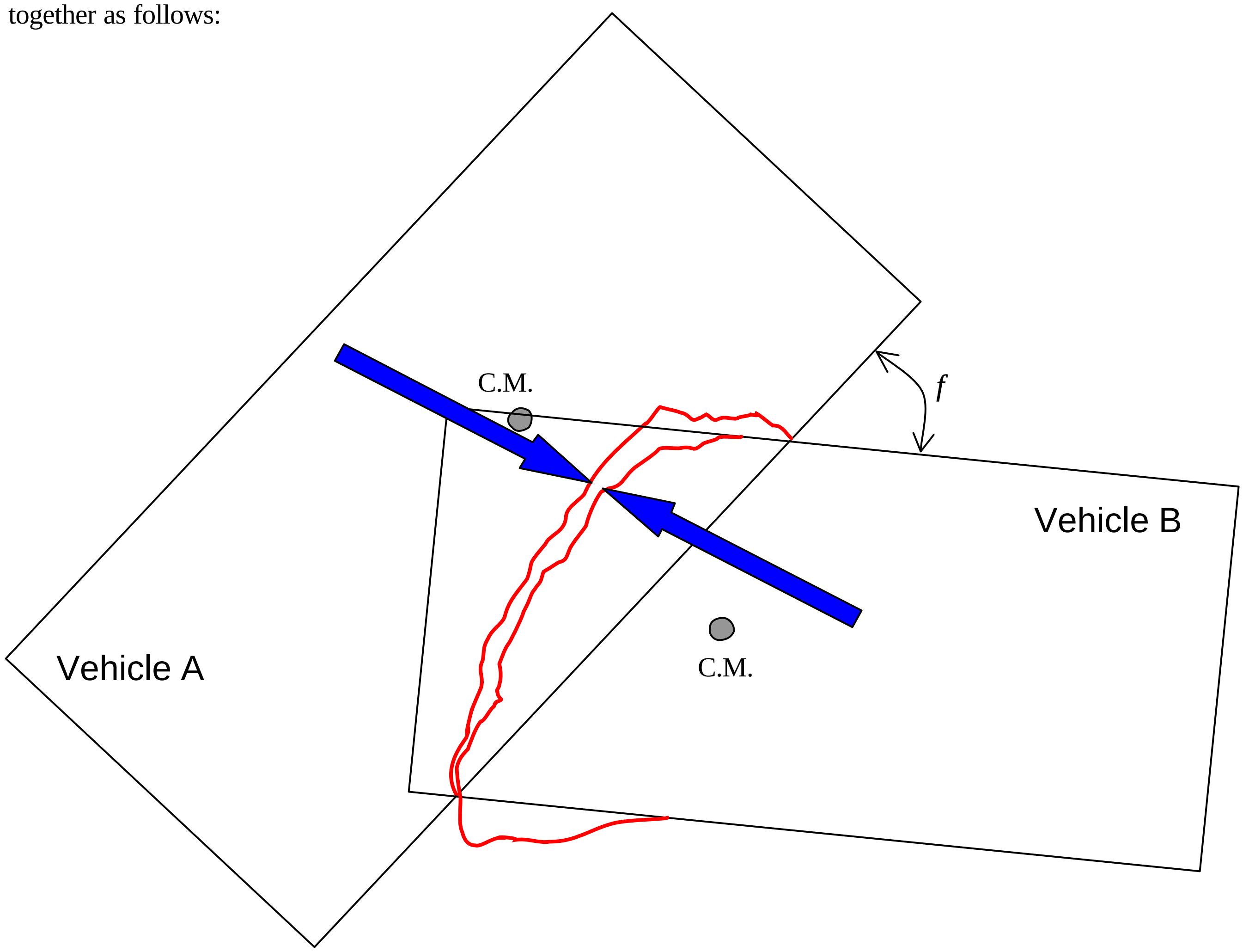
We can then analyze the momentum transfer during the crash. To start, we can note that vehicle A had a male passenger (guessing, 180 lbs) and a female passenger (guessing, 130 lbs). This brings the total weight of vehicle A to 2650 + 180 + 130 = 2960 lbs, or a mass of 91.9 slugs. For vehicle B with a single male passenger (guessing, 180 lbs), the total weight is 3175 + 180 = 3355 lbs, or a mass of 104.2 slugs. Then, taking the y (North) and x (West) components of the momentum vector:

$$P_x = (91.9\text{ slugs})\cdot(415\text{ ft/sec})\cdot\cos(90°-59°) + (104.2\text{ slugs})\cdot(27.5\text{ ft/sec})\cdot\cos(117°-90°) = 5822.3\text{ lb-sec}$$

$$P_x = (91.9\text{ slugs})\cdot(415\text{ ft/sec})\cdot\sin(90°-59°) - (104.2\text{ slugs})\cdot(27.5\text{ ft/sec})\cdot\sin(117°-90°) = 663.4\text{ lb-sec}$$

With this information, we can try to solve for the speed of each vehicle just at the point of impact. We do not know the angle $q$ but we may be able to estimate it, based on the deformation of vehicle A and vehicle B.

If we were to cut out vehicle A and vehicle B (from the second page of the homework), we could piece them together as follows:




The angle $f$ in this case is the angle ***at maximum engagement***. From this graph, using a protractor, this appears to be about 53°. The likely maximum force acts at the location and direction indicated, based on the fact that both vehicles spun clockwise after the crash (based on their final orientation after the crash). The forces in these locations would produce the necessary clockwise moment on each vehicle.

As the initial force (at initial impact) was at the front right corner of vehicle B, and the front right bumper of vehicle A, we might expect that vehicle B may have rotated slightly clockwise before the point of maximum engagement. We also might expect that vehicle A may have rotated slightly counter-clockwise (or not at all) between the initial impact and the point of maximum engagement during the crash. So, the angle of the vehicles at initial impact is likely less than the angle $f$, or perhaps 51°. [If in doubt, we could always go back and do some sensitivity analysis to the assumed angle at initial impact.]

Then, we can proceed with the momentum analysis. For conservation of momentum, we have:

$$P_x = -(91.9 \text{ slugs})\cdot(v_{A-crash})\cdot\cos(51°) + (104.2 \text{ slugs})\cdot(v_{B-crash})\cdot\cos(0°) = 5822.3 \text{ lb-sec}$$
$$P_y = (91.9 \text{ slugs})\cdot(v_{A-crash})\cdot\sin(51°) + (104.2 \text{ slugs})\cdot(v_{B-crash})\cdot\sin(0°) = 663.4 \text{ lb-sec}$$

Solving these two equations for $v_{A\text{-}crash}$ and $v_{B\text{-}crash}$ gives:

$$P_x = -57.8\cdot(v_{A-crash}) + 104.2\cdot(v_{B-crash}) = 5822.3 \text{ lb-sec}$$
$$P_y = 71.4\cdot(v_{A-crash}) = 663.4 \text{ lb-sec}$$

$v_{A\text{-}crash}$ = 9.3 ft/sec (slow!) $\qquad$ $v_{B\text{-}crash}$ = 61.0 ft/sec

Finally, we can solve for vehicle B's speed before starting to skid using:

$$d_{B-skid\,1} = \frac{v_{B-start}^2 - v_{B-crash}^2}{2g(f+G)}$$
$$v_{B-start}^2 = 2g(f+G)\cdot d_{B-skid\,1} + v_{B-crash}^2 = 2\cdot(32.2 \text{ ft/s}^2)\cdot(0.68+0.01)\cdot(74 \text{ ft}) + (61.0 \text{ ft/sec})^2$$
$$v_{B-start}^2 = 7009.3 \text{ ft}^2/\text{sec}^2 \quad \text{or} \quad v_{B-start} = 83.7 \text{ ft/sec (fast!)}$$

Now, answering the questions that were given:

(a) At first impact, vehicle B was headed due west, with a speed of $v_{B\text{-}crash}$ = 61.0 ft/sec.

Vehicle A was headed about 51° North of East (or 39° East of North), with a speed of $v_{A\text{-}crash}$ = 9.3 ft/sec.

(b) The orientation of each vehicle is shown in the diagram on the previous page, with both vehicles rotated a slight amount from the initial impact. Vehicle A is headed about 53° North of East (or 37° East of North), and vehicle B is headed 2° North of West (or 88° West of North).

(c) The approach speed of vehicle B before going into the skid is $v_{B\text{-}start}$ = 83.7 ft/sec.