CE 467 / 567 Highway Safety and Operations
Homework 5 Solutions

Problem 1

(a) A simple comparison gives 18 crashes per year in the before period, and 14 crashes per year afterwards. The net safety benefit is a reduction of 4 crashes per year.

(b) A simple calculation gives the mean and variance of the comparison sites of
$E[l_{pop}]$ = 82.9 crashes/5 years = 16.58 crashes/yr
$Var[l_{pop}]$ = 47.8 (crashes/5yrs)^2 = 1.91 (crashes/yr)^2

With Empirical Bayes,

$$a = \frac{1}{1+\frac{Var[l_{pop}]}{E[l_{pop}]}} = \frac{1}{1+\frac{1.91}{16.58}} = 0.8966$$

$$E[l_{pop} \mid \hat{l}_i] = a\,E[l_{pop}] + (1-a)\,\hat{l}_i$$

$$E[l_{pop} \mid \hat{l}_i] = (0.8966)\cdot(16.58 \text{ crashes/yr}) + (1-0.8966)\cdot(18 \text{ crashes/yr})$$

$$E[l_{pop} \mid \hat{l}_i] = \underline{16.73 \text{ crashes/yr}}$$

This is the expected number of crashes without any treatment, so we can compare this with the number of 14 crashes per year afterwards. In this case, the effect is lower, with a savings of only 2.73 crashes per year.

(c) The critical issue hear is that the Empirical Bayes accounts for randomness in the distribution of crashes before the treatment. By controlling for this, we avoid a "regression-to-the-mean" effect, where the crashes would simply tend back toward the mean anyway. If the crashes were tending back toward the mean later, this would suggest that we would over-estimate the safety benefits with a "simple" evaluation.

Problem 2

(a) In the model, those factors that increase crash frequency include AADT on both the major and minor roads, and the existence of trucks in the peak hour. Certainly, I would expect higher AADT to result in more crashes, simply from greater exposure to risk in the intersection. The peak truck effect is very difficult to understand – since trucks have both positive and negative effects on crashes (truck drivers are better trained, but may cause conflicts with drivers who are dealing with slow truck speeds).

(b)-(c) Those factors decreasing crash frequency include protected left turn bays, the percentage of left turns from the minor street (which is very difficult to explain – I would not expect this variable to have much of an effect, or a positive one at that). Also, the increase in grade change tends to reduce crashes, which again seems counter-intuitive. I'm happy neither of these is statistically significant

(d) The crash frequency does not increase linearly with the AADT. Instead, it is *multiplicative* with the AADT – so as AADT increases, the crash frequency rises to the factor of AADT. Mathematically, taking the exponent of both sides results in

$$l = \ldots \cdot AADT1^{+0.7249} \cdot AADT2^{+0.3110} \cdot \ldots$$

which is definitely non-linear. At least for these intersections, the use of a crash *rate* in million vehicle entries is probably inappropriate, because the true relationship appears to be non-linear.

Continued→

(e) The simple answer here is, based on the same math trick as in part (d), the crash frequency drops with the dummy (0 or 1) variable *PROTLT* as:

$$l = \ldots \cdot e^{-0.7381 \cdot PROTLT} \cdot \ldots$$

As a result, a jump from *PROTLT* = 0 (meaning the term $e^{-0.7381*PROTLT} = 1$) to *PROTLT* = 1 (meaning a factor of $e^{-0.7381*PROTLT} = 0.4780$) will reduce the crash frequency by a factor of 0.478. Or, it provides a 52.2% reduction in the crash frequency at the intersection!

As an example, using the average value given in the problem, with the initial λ=5.9, the subsequent crash frequency would be 0.478*5.9 = 2.82 crashes per year. Wow!