

Frustration and Anger in Games*

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Abstract

Frustration, anger, and blame have important consequences for economic and social behavior, concerning for example monopoly pricing, contracting, bargaining, violence, and politics. Drawing on insights from psychology, we develop a formal approach to exploring how frustration and anger, via blame and aggression, shape interaction and outcomes in strategic settings.

KEYWORDS: frustration, anger, blame, belief-dependent preferences, psychological games.

JEL codes: C72, D03

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1 Introduction

Anger can shape economic outcomes. Consider three cases:

Case 1: In 2015 Turing Pharmaceuticals raised the price of Daraprim, a therapeutic drug, from \$12 to \$750 per dose. The company was subsequently accused of price gouging. Should Turing have considered the consequences of customer anger before determining the new price for the drug?

Case 2: When local football teams favored to win instead lose, the police get more reports of husbands assaulting wives (Card & Dahl 2011). Do unexpected losses spur vented frustration?

Case 3: Following the sovereign debt crises that began in 2009, some EU countries embarked on austerity programs. Was it because citizens lost benefits that some cities experienced riots?

Pricing, domestic violence, political landscapes: these are important themes, and we propose that others (involving—say—recessions, contracting, arbitration, terrorism, road rage, or support for populist political candidates) could plausibly be imagined. However, to carefully assess the impact of anger on social and economic interactions, one needs a theory that predicts outcomes based on the decision-making of anger-prone individuals and that accounts for strategic considerations. We develop such a theory.

Insights from psychology about the triggers and repercussions of anger are evocative. The behavioral consequences of emotions are called “action tendencies,” and the action tendency associated with anger is aggression and the urge to retaliate. Angry players may be willing to forgo material gains to punish others, or be predisposed to aggression when this serves as a credible threat. But while insights of this nature can be gleaned from psychologists’ writings, their analysis usually stops with the individual rather than going on to assess overall economic and social implications. We take the basic insights about anger that psychology has produced as input and inspiration for our theory.¹

We study the strategic interaction of decision makers who become angry when they are frustrated.² Frustration occurs when someone is unexpectedly denied something he or she

¹The psychology literature is huge. One source of inspiration for us is the *International Handbook of Anger* (Potegal, Spielberger & Stemmler 2010) which offers cross-disciplinary perspective reflecting “affective neuroscience, business administration, epidemiology, health science, linguistics, political science, psychology, psychophysiology, and sociology” (p. 3). The absence of “economics” in the list may indicate that our approach is original!

²A large body of work in psychology connects frustration, anger, and aggression, beginning with Dollard *et al.* (1939). See, for example, Averill (1982), Berkowitz (1989), and the (*op. cit.*) *Handbook*, especially the chapters by Lewis, Wranik & Scherer, and Berkowitz.

cares about. We assume that people are frustrated when they get less material rewards than expected.^{3,4} They then become hostile towards whomever they blame for their frustration. Because a player's frustration depends on his beliefs about others' choices, and the blame a player attributes to another may depend on his beliefs about others' choices or belief, all our models find their intellectual home in the framework of psychological game theory; see Geanakoplos, Pearce & Stacchetti (1989), and Battigalli & Dufwenberg (2009).

There are a number of ways to assign blame.⁵ We present three approaches, captured by distinct utility functions. Players motivated by simple anger (SA) become generally hostile when frustrated. In contrast, those motivated by anger from blaming behavior (ABB) or anger from blaming intentions (ABI) are more discriminating, asking who caused, or who intended to cause, their frustration. SA captures the well-known psychological phenomenon of *displaced aggression*, where an angry person takes out frustration on a blameless bystander.⁶ However, for some authors, blame or other-responsibility is a prerequisite for anger.⁷ ABB and ABI capture this view.

We develop a modeling framework in which players have beliefs about both others' beliefs and actions as well as their own actions. In any game form first-movers are never frustrated, and therefore behave (at the root) as if to maximize their material payoffs given their beliefs. We establish the existence of two types of equilibria in games where players are motivated by SA, ABB, or ABI. The first is an extension of the sequential equilibrium for psychological games developed in Battigalli & Dufwenberg (2009), adapted to our framework. In pure-strategy sequential equilibria, frustration arises only off the equilibrium path, and furthermore in perfect information games with unique material-payoff equilibria, there is always an equilibrium with anger that is realization equivalent to the material-payoff equilibrium, though anger may also result in additional equilibria. In two-player *leader-follower* games, followers with SA, ABB, or ABI always do at least as well, materially, as players who are not anger-prone. This result is consistent with the work of Sell, Cosmides, & Tooby (2009), who argue that anger is the result of a process of natural selection for behaviors that resolve bargaining conflicts in favor of the anger-prone individual.

We show via examples how our models can capture Cases 1 and 2 above (for Case 3, see Passarelli & Tabellini 2017). Case 1 is captured by an ultimatum minigame, where SA and ABB allow for a pure SE involving rejection of the greedy offer, and all concepts allow for an

³The assumption that frustration depends on expectations is well supported in the psychology literature. See e.g. Berkowitz (1978, p. 697), who says "Unlike deprivations, frustrations can only be surprising (to a greater or lesser extent). People who do not expect to reach their goals are not anticipating the pleasure these goals would bring. Their hopes are not dashed if they have no hopes."

⁴Our focus on material rewards is admittedly restrictive. See the Discussion in Section 8.

⁵See e.g. Alicke (2000), and Halpern (2016, Chapter 6).

⁶See Marcus-Newhall *et al.* (2000).

⁷See e.g. the chapter by Wranik & Scherer in the (*op. cit.*) *Handbook*.

SE where rejection occurs with positive probability. Case 2 is captured by SA in an example that we call “Hammering one’s thumb.”⁸ In contrast, incorporating notions of blame into our analysis of this situation via either ABB and ABI eliminates displaced aggression.

SE requires that each player i is certain and never changes his mind about the true beliefs and plans, hence intentions, of his co-players. We find this feature questionable, and therefore we also develop a notion of polymorphic sequential equilibrium (PSE), where players correctly anticipate how others behave on average, yet different (epistemic) types of the same player may have different plans in equilibrium. In a PSE, once an agent observes some moves of co-players, he makes inferences about their intentions. We show that in leader-followers games with SA, ABB, or ABI, an SE involving non-degenerate behavioral strategies of the leader will correspond to a PSE where the leader’s strategy is purified.

Finally, we illustrate how the effects of anger are sensitive to assumptions about the reference expectation that determines frustration, and how rapidly it is updated. We show that our model can be interpreted either via the notion of “fast play”, where the reference belief is fixed at the initial history, or “slow play”, where the reference expectation changes each period. Slow play formalizes the phenomenon of cooling-off, whereby anger subsides over time (Grimm & Mengel, 2011; Oechssler *et al.*, 2015).

A small literature examines the role of anger in economic behavior. Early work exploring the role of anger in solving commitment problems includes Hirshleifer (1987), Frank (1988), and Elster (1998). Most recent studies are empirical or experimental, indicative of hostile action occurring in economic situations, based on either observational data or experimental data.⁹ A few studies do present theory, including Rotemberg (2005, 2008, 2011); Brams (2011); Winter (2014, 2016); Akerlof (2016); and Passarelli & Tabellini (2017) (*cf.* the reference to riots in Case 3). We compare and contrast our approach with these theories and with models of distributional preferences and reciprocity in Section 7.

Our approach differs from the previous literature in that we do not start with data, but with notions from psychology which we incorporate into general games, and we are led to use assumptions which differ substantially from previous theoretical work. We develop most of our analysis for a two-stage setting described in Section 2. Sections 3 and 4 define our notions of frustration, blame, anger, and utility. Section 5 examines equilibria. Section 6 generalizes

⁸The example is inspired by Frijda (1993), who says “Many experiences or responses of anger... are elicited by events that involve no blameworthy action” and suggests that simple frustrations such as “one’s car refusing to start, finding one’s bicycle has a flat tyre, rain on the fifth day of one’s holiday after four previous days of rain... hitting one’s head on the kitchen shelf, dropping a needle for the third time in a row” or “hammering one’s thumb” may result in anger. He goes on to say that “the target of anger may be a person who has fallen ill on the day of one’s party, or one who just happened to be present when a plan failed.”

⁹See Anderson & Simester (2010) and Rotemberg (2005, 2011) on pricing, Card & Dahl (2011) and Munyo & Rossi (2013) on violence, and Carpenter & Matthews (2012), Gurdal, Miller & Rustichini (2014), Gneezy & Imas (2014), Persson (2017), and van Leeuwen *et al.* (2017) for experiments.

our approach to multistage games and develops the notions of fast and slow play. Section 7 compares our approach to other related models. Section 8 concludes. Proofs are collected in the Appendix.

2 Setup

In this section we develop a framework that is well-suited to the study of frustration, anger and blame. We begin by describing the rules of interaction (the game form). We then define first- and second-order conditional belief systems that capture not only a player's system of beliefs regarding the behavior of other players, but also her system of beliefs about her own actions, which we refer to as her plan.

2.1 Game form

Consider a finite two-stage game form describing the rules of interaction and the consequences of players' actions. The set of players is I . To ease notation, we assume that all players take actions simultaneously at each stage. Thus, nodes are histories h of action profiles $a^t = (a_i^t)_{i \in I}$; $h = \emptyset$ is the empty history (the root), $h = (a^1)$ a history of length one, which may be terminal or not, and $h = (a^1, a^2)$ a history of length 2, which is terminal. H is the set of nonterminal histories and Z is the set of terminal histories (end nodes). The set of feasible actions of i given $h \in H$ is $A_i(h)$. This set is a singleton if i is not active given h . Thus, for all $h \in H$, $I(h) = \{i \in I : |A_i(h)| > 1\}$ is the set of active players given h . In a perfect information game, $I(h)$ is a singleton for each $h \in H$. We omit parentheses whenever no confusion may arise. For example, we may write $h = a^1$ instead of $h = (a^1)$, and $h = (a_i^1, a_j^2)$ if i (resp. j) is the only first (resp. second) mover. Finally, we let $A(h) = \times_{i \in I} A_i(h)$ and $A_{-i}(h) = \times_{j \neq i} A_j(h)$.

We assume that the material consequences of players' actions are determined by a profile of monetary payoff functions $(\pi_i : Z \rightarrow \mathbb{R})_{i \in I}$. This completes the description of the game form, if there are no chance moves. If the game contains chance moves, we augment the player set with a dummy player c (with $c \notin I$), who selects a feasible action at random. Thus, we consider an augmented player set $I_c = I \cup \{c\}$, and the sets of first and second movers may include c : $I(\emptyset), I(a^1) \subseteq I_c$. If the chance player is active at $h \in H$, its move is described by a probability mass function $\sigma_c(\cdot|h) \in \Delta(A_c(h))$.

The following example, to which we will return in our discussion of blame, is here employed to illustrate our notation:

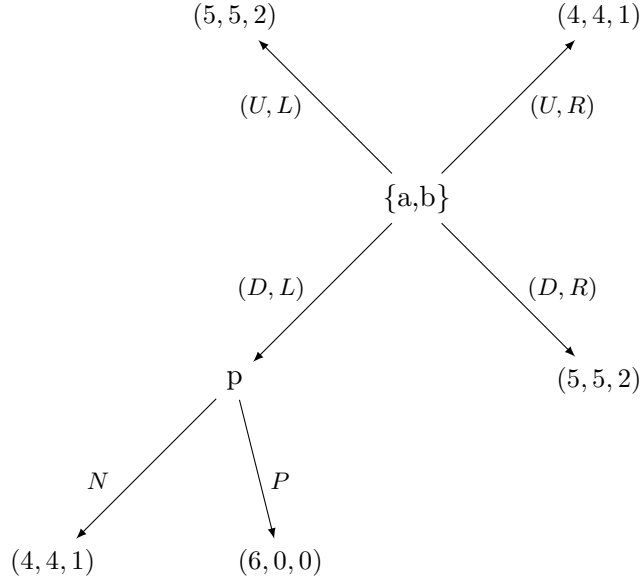


Figure A. Asymmetric punishment.

Example 1 Ann and Bob (a and b in Fig. A) move simultaneously in the first stage. Penny the punisher (p) may move in the second stage; by choosing P she increases π_a and decreases π_b . Profiles of actions and monetary payoffs are listed in players' alphabetical order. We have:

$$\begin{aligned}
 H &= \{\emptyset, (D, L)\}, \quad Z = \{(U, L), (U, R), (D, R), ((D, L), N), ((D, L), P)\}, \\
 I(\emptyset) &= \{a, b\}, \quad I((D, L)) = \{p\}, \\
 A_a(\emptyset) &= \{U, D\}, \quad A_b(\emptyset) = \{L, R\}, \quad A_p((D, L)) = \{N, P\}. \quad \blacktriangle
 \end{aligned}$$

2.2 Beliefs

It is conceptually useful to distinguish three aspects of a player's beliefs: beliefs about co-players' actions, beliefs about co-players' beliefs, and the player's plan, which we represent as beliefs about own actions. Beliefs are defined conditional on each history. Abstractly denote by Δ_{-i} the space of co-players' beliefs (the formal definition is given below). Player i 's beliefs can be compactly described as conditional probability measures over paths and beliefs of others, *i.e.*, over $Z \times \Delta_{-i}$. Events, from i 's point of view, are subsets of $Z \times \Delta_{-i}$. Events about behavior take form $Y \times \Delta_{-i}$, with $Y \subseteq Z$; events about beliefs take form $Z \times E_{\Delta_{-i}}$, with $E_{\Delta_{-i}} \subseteq \Delta_{-i}$.¹⁰

¹⁰ Δ_{-i} turns out to be a compact metric space. Events are Borel measurable subsets of $Z \times \Delta_{-i}$. We do not specify terminal beliefs of i about others' beliefs, as they are not relevant for the models in this paper.

Personal histories To model how i determines the subjective value of feasible actions, we add to the commonly observed histories $h \in H$ also personal histories of the form (h, a_i) , with $a_i \in A_i(h)$. In a game with perfect information, $(h, a_i) \in H \cup Z$. But if there are simultaneous moves at h , then (h, a_i) is not a history in the standard sense. As soon as i irreversibly chooses action a_i , he observes (h, a_i) , and can determine the value of a_i using his beliefs conditional on this event (i knows in advance how he is going to update his beliefs conditional on what he observes). We denote by H_i the set of histories of i —standard and personal—and by $Z(h_i)$ the set of terminal successors of h_i .¹¹ The standard precedence relation \prec for histories in $H \cup Z$ is extended to H_i in the obvious way: for all $h \in H$, $i \in I(h)$, and $a_i \in A_i(h)$, it holds that $h \prec (h, a_i)$ and $(h, a_i) \prec (h, (a_i, a_{-i}))$ if i is not the only active player at h . Note that $h \prec h'$ implies $Z(h') \subseteq Z(h)$, with strict inclusion if at least one player (possibly chance) is active at h .

First-order belief systems For each $h_i \in H_i$, player i holds beliefs $\alpha_i(\cdot|Z(h_i)) \in \Delta(Z(h_i))$ about the actions that will be taken in the continuation of the game. The system of beliefs $\alpha_i = (\alpha_i(\cdot|Z(h_i)))_{h_i \in H_i}$ must satisfy two properties. First, the rules of conditional probabilities hold whenever possible: if $h_i \prec h'_i$ then for every $Y \subseteq Z(h'_i)$

$$\alpha_i(Z(h'_i)|Z(h_i)) > 0 \Rightarrow \alpha_i(Y|Z(h'_i)) = \frac{\alpha_i(Y|Z(h_i))}{\alpha_i(Z(h'_i)|Z(h_i))}. \quad (1)$$

We use obvious abbreviations to denote conditioning events and the conditional probabilities of actions: for all $h \in H$, $a = (a_i, a_{-i}) \in A_i(h) \times A_{-i}(h)$,

$$\begin{aligned} \alpha_i(a|h) &= \alpha_i(Z(h, a)|Z(h)), \\ \alpha_{i,i}(a_i|h) &= \sum_{a'_{-i} \in A_{-i}(h)} \alpha_i(a_i, a'_{-i}|h), \\ \alpha_{i,-i}(a_{-i}|h) &= \sum_{a'_i \in A_i(h)} \alpha_i(a'_i, a_{-i}|h). \end{aligned}$$

Note that $\alpha_{i,i}(a_i|h) = \alpha_i(Z(h, a_i)|Z(h))$, and that (1) implies $\alpha_i(a^1, a^2|\emptyset) = \alpha_i(a^2|a^1) \alpha_i(a^1|\emptyset)$.

With this, we can write in a simple way our second requirement, that i 's beliefs about the actions simultaneously taken by the co-players are independent of i 's action: for all $h \in H$, $i \in I$, $a_i \in A_i(h)$, and $a_{-i} \in A_{-i}(h)$,

$$\alpha_{i,-i}(a_{-i}|h) = \alpha_{i,-i}(a_{-i}|h, a_i). \quad (2)$$

¹¹That is, $H_i = H \cup \{(h, a_i) : h \in H, i \in I(h), a_i \in A_i(h)\}$. The definition of $Z(h_i)$ is standard for $h_i \in H$; for $h_i = (h, a_i)$ we have $Z(h, a_i) = \bigcup_{a_{-i} \in A_{-i}(h)} Z(h, (a_i, a_{-i}))$.

Properties (1)–(2) imply

$$\alpha_i(a_i, a_{-i}|h) = \alpha_{i,i}(a_i|h)\alpha_{i,-i}(a_{-i}|h).$$

Thus, α_i is made of two parts, what i believes about his own behavior and what he believes about the behavior of others. The array of probability measures $\alpha_{i,i} \in \times_{h \in H} \Delta(A_i(h))$ is—technically speaking—a behavioral strategy, and we interpret it as the **plan** of i . The reason is that the result of i 's contingent planning is precisely a system of conditional beliefs about what action he would take at each history. If there is only one co-player, also $\alpha_{i,-i} \in \times_{h \in H} \Delta(A_{-i}(h))$ corresponds to a behavioral strategy. With multiple co-players, $\alpha_{i,-i}$ corresponds instead to a “correlated behavioral strategy.” Whatever the case, $\alpha_{i,-i}$ gives i 's conditional beliefs about others' behavior, and these beliefs may not coincide with the plans of others. We emphasize: a player's plan does not describe actual choices, actions on the path of play are the only actual choices.

A system of conditional probability measures $\alpha_i = (\alpha_i(\cdot|Z(h_i)))_{h_i \in H_i}$ satisfying (1)–(2) is a **first-order** belief of i . Let Δ_i^1 denote the space of such beliefs. It can be checked that Δ_i^1 is a compact metric space, so the same holds for $\Delta_{-i}^1 = \times_{j \neq i} \Delta_j^1$, the space of co-players' first-order beliefs profiles.

Second-order belief systems Players do not only hold beliefs about paths, they also hold beliefs about the beliefs of co-players. In the following analysis, the only co-players' beliefs affecting the values of actions are their first-order beliefs. Therefore, we limit our attention to **second-order** beliefs, *i.e.*, systems of conditional probability measures $(\beta_i(\cdot|h_i))_{h_i \in H_i} \in \times_{h_i \in H_i} \Delta(Z(h_i) \times \Delta_{-i}^1)$ that satisfy properties analogous to (1)–(2).¹² First, if $h_i \prec h'_i$ then

$$\beta_i(h'_i|h_i) > 0 \Rightarrow \beta_i(E|h'_i) = \frac{\beta_i(E|h_i)}{\beta_i(h'_i|h_i)} \quad (3)$$

for all $h_i, h'_i \in H_i$ and every event $E \subseteq Z(h'_i) \times \Delta_{-i}^1$. Second, i realizes that his choice cannot influence the first-order beliefs of co-players and their simultaneous choices, so i 's beliefs satisfy an independence property:

$$\beta_i(Z(h, (a_i, a_{-i})) \times E_\Delta | (h, a_i)) = \beta_i(Z(h, (a'_i, a_{-i})) \times E_\Delta | (h, a'_i)), \quad (4)$$

for every $h \in H$, $a_i, a'_i \in A_i(h)$, $a_{-i} \in A_{-i}(h)$, and event $E_\Delta \subseteq \Delta_{-i}^1$ about co-players' first-order beliefs. The space of i 's second-order beliefs is denoted by Δ_{-i}^2 .

It can be checked that starting from $\beta_i \in \Delta_i^2$ and letting $\alpha_i(Y|h_i) = \beta_i(Y \times \Delta_{-i}^1|h_i)$ for all $h_i \in H_i$ and $Y \subseteq Z$, we obtain a system α_i satisfying (1)–(2), *i.e.*, an element of Δ_i^1 . This

¹²We use obvious abbreviations, such as writing h for event $Z(h) \times \Delta_{-i}^1$, whenever this causes no confusion.

α_i is the first-order belief implicit in β_i . Whenever we write in a formula beliefs of different orders for a player, we assume that first-order beliefs are derived from second-order beliefs, otherwise beliefs of different orders would not be mutually consistent. Also, we write initial beliefs omitting the empty history, as in $\beta_i(E) = \beta_i(E|\emptyset)$ or $\alpha_i(a) = \alpha_i(a|\emptyset)$, whenever this causes no confusion.

Conditional expectations Let ψ_i be any real-valued measurable function of variables that i does not know, *e.g.*, the terminal history or the co-players' first-order beliefs. Then i can compute the expected value of ψ_i conditional on any history $h_i \in H_i$ by means of his belief system β_i , denoted $\mathbb{E}[\psi_i|h_i; \beta_i]$. If ψ_i depends only on actions, *i.e.*, on the path z , then $\mathbb{E}[\psi_i|h_i; \beta_i]$ is determined by the α_i derived from β_i , and we can write $\mathbb{E}[\psi_i|h_i; \alpha_i]$. In particular, α_i gives the conditional expected material payoffs:

$$\begin{aligned}\mathbb{E}[\pi_i|h; \alpha_i] &= \sum_{z \in Z(h)} \alpha_i(z|h) \pi_i(z), \\ \mathbb{E}[\pi_i|(h, a_i); \alpha_i] &= \sum_{z \in Z(h, a_i)} \alpha_i(z|h, a_i) \pi_i(z)\end{aligned}$$

for all $h \in H$, $a_i \in A_i(h)$. $\mathbb{E}[\pi_i|h; \alpha_i]$ is what i expects to get conditional on h given α_i , which also specifies i 's plan. $\mathbb{E}[\pi_i|(h, a_i); \alpha_i]$ is i 's expected payoff of action a_i . If a_i is what i planned to choose at h , $\alpha_{i,i}(a_i|h) = 1$, and then $\mathbb{E}[\pi_i|h; \alpha_i] = \mathbb{E}[\pi_i|(h, a_i); \alpha_i]$. For initial beliefs, we omit $h = \emptyset$ from such expressions; in particular, the initially expected material payoff is $\mathbb{E}[\pi_i; \alpha_i]$.

3 Frustration

Anger is triggered by frustration. While we focus on anger as a social phenomenon—frustrated players blame, become angry with, and care for the payoffs of *others*—our account of frustration refers to own payoffs only. In Section 8 (in hindsight of definitions to come) we discuss this approach in depth.

We define player i 's **frustration** at history h , given his first-order belief system α_i as

$$F_i(h; \alpha_i) = \left[\mathbb{E}[\pi_i; \alpha_i] - \max_{a_i \in A_i(h)} \mathbb{E}[\pi_i|(h, a_i); \alpha_i] \right]^+,$$

where $[x]^+ = \max\{x, 0\}$. In words, frustration is given by the gap, if positive, between i 's initially expected payoff and the currently best expected payoff he believes he can obtain.

Diminished expectation— $\mathbb{E}[\pi_i|h; \alpha_i] < \mathbb{E}[\pi_i; \alpha_i]$ —is only a necessary condition for frustration. For i to be frustrated it must also be the case that i cannot close the gap.

At the root, frustration must always be zero, because nothing has yet happened and so expectations cannot be diminished:

Remark 1 For every player $i \in I$ and system of first-order beliefs $\alpha_i \in \Delta_i^1$, frustration must equal 0 at the initial history $h = \emptyset$, since (1) and (2) imply

$$\mathbb{E}[\pi_i; \alpha_i] = \sum_{a_i^1 \in A_i(\emptyset)} \alpha_{i,i}(a_i^1|\emptyset) \mathbb{E}[\pi_i|a_i^1; \alpha_i] \leq \max_{a_i^1 \in A_i(\emptyset)} \mathbb{E}[\pi_i|a_i^1; \alpha_i].$$

Frustration is possible at the end nodes, but can't influence subsequent choices as the game is over. One might allow the anticipation of frustration to be felt at end nodes to influence earlier decisions; however, the assumptions we make in the analysis below rule this out. Furthermore, a player is influenced by the frustrations of co-players only insofar as their behavior is affected. Therefore, in our 2-stage setting, the relevant frustration occurs in the second stage and is given by

$$F_i(a^1; \alpha_i) = \left[\mathbb{E}[\pi_i; \alpha_i] - \max_{a_i^2 \in A_i(a^1)} \mathbb{E}[\pi_i|(a^1, a_i); \alpha_i] \right]^+.$$

Example 2 Return to Fig. A. Suppose Penny initially expects \$2: $\alpha_p(U, L) + \alpha_p(D, R) = 1$, $\mathbb{E}[\pi_p; \alpha_p] = 2$. After (D, L) we have

$$F_p((D, L); \alpha_p) = [\mathbb{E}[\pi_p; \alpha_p] - \max\{\pi_p((D, L), N), \pi_p((D, L), P)\}]^+ = 2 - 1 = 1.$$

This is independent of her plan, because she is initially certain she will not move. If instead $\alpha_p((U, L)|\emptyset) = \alpha_p((D, L)|\emptyset) = \frac{1}{2}$ then

$$F_p((D, L); \alpha_p) = \frac{1}{2} \cdot 2 + \frac{1}{2} \alpha_p(N|(D, L)) \cdot 1 - 1 = \frac{1}{2} \alpha_p(N|(D, L));$$

Penny's frustration is highest if she initially plans not to punish Bob. \blacktriangle

4 Anger

A player's preferences over actions at a given node—his action tendencies—depend on expected material payoffs and frustration. A frustrated player tends to hurt others, if this is not too costly (cf. Dollard *et al.* 1939, Averill 1983, Berkowitz 1989). We consider different versions of this frustration-aggression hypothesis related to different cognitive appraisals of blame. In

general, player i moving at history h chooses action a_i to maximize the expected value of a belief-dependent “decision utility” of the form

$$u_i(h, a_i; \beta_i) = \mathbb{E}[\pi_i | (h, a_i); \alpha_i] - \theta_i \sum_{j \neq i} B_{ij}(h; \beta_i) \mathbb{E}[\pi_j | (h, a_i); \alpha_i], \quad (5)$$

where α_i is the first-order belief system derived from second-order belief β_i , and $\theta_i \geq 0$ is a sensitivity parameter. $B_{ij}(h; \beta_i) \geq 0$ measures how much of i 's frustration is blamed on j , and the presence of $\mathbb{E}[\pi_j | (h, a_i); \alpha_i]$ in the formula translates this into a tendency to hurt j .

We assume that

$$B_{ij}(h; \beta_i) \leq F_i(h; \alpha_i). \quad (6)$$

Therefore, first-movers behave at the root (given their beliefs) as if they are motivated only by material self-interest, because there is no frustration in the first stage:

Remark 2 *The decision utility of a first-mover coincides with the first-mover's expected material payoff:*

$$u_i(\emptyset, a_i; \beta_i) = \mathbb{E}[\pi_i | a_i; \alpha_i].$$

When i is the only active player at $h = a^1$, he determines the terminal history with his choice $a_i = a^2$, and decision utility has the form

$$u_i(h, a_i; \beta_i) = \pi_i(h, a_i) - \theta_i \sum_{j \neq i} B_{ij}(h; \beta_i) \pi_j(h, a_i).$$

We assume throughout that players' utilities are commonly known among them. This is partly a modeling choice made for reasons of analytical tractability, but we note that the approach is supported by recent intriguing experimental evidence reported by van Leeuwen *et al.* (2017). They find that (pre-play) “facial cues provide a credible signal of destructive behavior,” and that subjects are to a degree capable of recognizing such “angry buttons.”

We next consider functional forms that capture different notions of blame.

4.1 Simple Anger (SA)

Our most rudimentary hypothesis, **simple anger (SA)**, is that i 's tendency to hurt others is proportional to i 's frustration. This affect heuristic is unmodulated by the cognitive appraisal of blame, so $B_{ij}(h; \beta_i) = F_i(h; \alpha_i)$:

$$u_i^{SA}(h, a_i; \alpha_i) = \mathbb{E}[\pi_i | (h, a_i); \alpha_i] - \theta_i \sum_{j \neq i} F_i(h; \alpha_i) \mathbb{E}[\pi_j | (h, a_i); \alpha_i]. \quad (7)$$

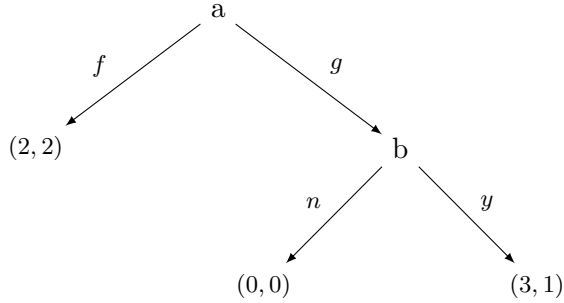


Figure B. Ultimatum Minigame.

Example 3 (*Ultimatum Minigame*) Ann and Bob (a and b in Fig. B) negotiate: Ann can make fair offer f , which is automatically accepted, or greedy offer g , which Bob accepts or rejects.¹³ His frustration following g is

$$F_b(g; \alpha_b) = [(1 - \alpha_b(g)) \cdot 2 + \alpha_b(g)\alpha_b(y|g) \cdot 1 - 1]^+.$$

Therefore

$$u_b^{SA}(g, n; \alpha_b) - u_b^{SA}(g, y; \alpha_b) = 3\theta_b [2(1 - \alpha_b(g)) + \alpha_b(g)\alpha_b(y|g) - 1]^+ - 1.$$

For Bob to be frustrated he must not expect g with certainty. If frustrated, the less he expects g , and—interestingly—the less he plans to reject, the more prone he is to reject once g materializes. The more resigned Bob is to getting a low payoff, the less frustrated and prone to aggression he is. Furthermore, it is readily seen from the example how our model captures non-consequential behavior (cf. the references in footnote 3). Holding beliefs and other payoffs constant, increasing Bob’s payoff from the fair offer f will lead to greater frustration after g . This greater frustration will increase the disutility Bob receives from Ann’s material payoff, making rejection of the greedy offer (and punishment of Ann) more attractive. ▲

4.2 Anger from blaming behavior (ABB)

Action tendencies may depend on a player’s cognitive appraisal of how to blame others. When a frustrated player i blames co-players for their behavior, he examines the actions chosen in stage 1, without considering others’ intentions. How much i blames j is determined by a continuous function $B_{ij}(a^1; \alpha_i)$ that depends only on first-order belief α_i and is such that

$$B_{ij}(a^1; \alpha_i) = \begin{cases} 0, & \text{if } j \notin I(\emptyset), \\ F_i(a^1; \alpha_i), & \text{if } \{j\} = I(\emptyset). \end{cases} \quad (8)$$

¹³Vis-à-vis Case 1 in Section 1, the setup can be interpreted as capturing a monopoly seller (Ann) who can offer Bob either a high or a low split of the gains from trade.

According to (8), if j is not active in the first stage, he cannot be blamed by i . If instead j is the *only* active player, he is fully blamed.¹⁴ We consider below specific versions of $B_{ij}(h; \alpha_i)$ that satisfy (6) and (8). With this, i 's decision utility with **anger from blaming behavior (ABB)** is

$$u_i^{ABB}(h, a_i; \alpha_i) = \mathbb{E}[\pi_i | (h, a_i); \alpha_i] - \theta_i \sum_{j \neq i} B_{ij}(h; \alpha_i) \mathbb{E}[\pi_j | (h, a_i); \alpha_i].$$

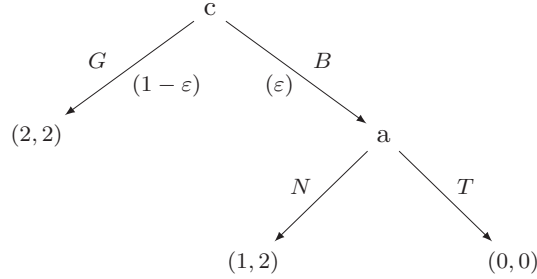


Figure C. Hammering one's thumb.

Example 4 (*Inspired by Frijda 1993*) To illustrate the difference between SA and ABB, consider Fig. C. Andy the handyman (a) uses a hammer. His apprentice, Bob (b), is inactive. On a bad day (determined by chance, c) Andy hammers his thumb and can then take it out on Bob, or not. Assuming $\alpha_a(B) = \varepsilon < 1/2$, we have

$$F_a(B; \alpha_a) = (1 - \varepsilon) \cdot 2 + \varepsilon \alpha_a(N|B) \cdot 1 - 1 > 0.$$

With SA and with θ_a sufficiently high, on a bad day Andy chooses T in a fit of displaced aggression. But, since Bob is passive, with ABB Andy chooses N regardless of θ_a . \blacktriangle

SA and ABB yield the same behavior in the Ultimatum Minigame and similar game forms. Say that a game form is a **leader-followers game** if there is only one active player in the first stage, who does not move in stage two: $I(\emptyset) = \{j\}$ and $I(\emptyset) \cap I(a^1) = \emptyset$ for some $j \in I$ and every a^1 . If a leader-followers game has exactly two active players, one in the first, and one in the second stage, we say that it is a **leader-follower game**. Let us write u_{i,θ_i} to make the dependence of u_i on θ_i explicit; then (Equation 8) implies:

Remark 3 *In leader-followers games, SA and ABB coincide, that is, $u_{i,\theta_i}^{SA} = u_{i,\theta_i}^{ABB}$ for all θ_i .*

Next, we contrast two specific functional forms for ABB.

¹⁴Recall that $I(h)$ is the set of active players at h , possibly including chance. For example, $I(\emptyset) = \{c\}$ in the game form of Figure C.

Could-have-been blame When frustrated i considers, for each j , what he would have obtained at most, in expectation, had j chosen differently:

$$\max_{a'_j \in A_j(\emptyset)} \mathbb{E} [\pi_i | (a_{-j}^1, a'_j); \alpha_i].$$

If this could-have-been payoff is more than what i currently expects (that is, $\mathbb{E}[\pi_i | a^1; \alpha_i]$), then i blames j , up to i 's frustration (so (6) holds):

$$B_{ij}(a^1; \alpha_i) = \min \left\{ \left[\max_{a'_j \in A_j(\emptyset)} \mathbb{E} [\pi_i | (a_{-j}^1, a'_j); \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i] \right]^+, F_i(a^1; \alpha_i) \right\}. \quad (9)$$

Blame function (9) satisfies (8); in particular, (9) implies that j cannot be blamed if he is not active in the first stage (cf. Proposition 5 below).

Example 5 Consider Penny at $a^1 = (D, L)$ in Fig. A. Her could-have-been payoff—wrt both Ann and Bob—is $2 \geq \mathbb{E}[\pi_p; \alpha_p]$, her updated expected payoff is $\mathbb{E}[\pi_p | (D, L); \alpha_p] \leq 1$, and her frustration is $[\mathbb{E}[\pi_p; \alpha_p] - 1]^+$. Therefore

$$\begin{aligned} B_{pa}((D, L); \alpha_p) &= B_{pb}((D, L); \alpha_p) = \\ &= \min \{ [2 - \mathbb{E}[\pi_p | (D, L); \alpha_p]]^+, [\mathbb{E}[\pi_p; \alpha_p] - 1]^+ \} = [\mathbb{E}[\pi_p; \alpha_p] - 1]^+, \end{aligned}$$

i.e., each of Ann and Bob is fully blamed by Penny for her frustration. \blacktriangle

Blaming unexpected deviations When frustrated after a^1 , i assesses, for each j , how much he would have obtained had j behaved as expected:

$$\sum_{a'_j \in A_j(\emptyset)} \alpha_{ij}(a'_j) \mathbb{E} [\pi_i | (a_{-j}^1, a'_j); \alpha_i],$$

where $\alpha_{ij}(a'_j)$ is the marginal probability of action a'_j according to i 's belief α_i . With this, the blame formula is

$$\begin{aligned} B_{ij}(a^1; \alpha_i) &= \\ &= \min \left\{ \left[\sum_{a'_j \in A_j(\emptyset)} \alpha_{ij}(a'_j) \mathbb{E} [\pi_i | (a_{-j}^1, a'_j); \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i] \right]^+, F_i(a^1; \alpha_i) \right\}. \quad (10) \end{aligned}$$

If j is not active in the first stage, we get

$$B_{ij}(a^1; \alpha_i) = \min \left\{ [\mathbb{E}[\pi_i | a^1; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]]^+, F_i(a^1; \alpha_i) \right\} = 0;$$

that is, j cannot have deviated and cannot be blamed. If, instead, j is the only active player in the first stage, then

$$\sum_{a'_j \in A_j(\emptyset)} \alpha_{ij}(a'_j) \mathbb{E} [\pi_i | (a_{-j}^1, a'_j); \alpha_i] = \sum_{a' \in A(\emptyset)} \alpha_i(a') \mathbb{E} [\pi_i | a'; \alpha_i] = \mathbb{E} [\pi_i; \alpha_i],$$

and (10) yields

$$B_{ij}(a^1; \alpha_i) = \min \left\{ [\mathbb{E}[\pi_i; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]]^+, F_i(a^1; \alpha_i) \right\} = F_i(a^1; \alpha_i).$$

Therefore, like blame function (9), also (10) satisfies (8).

If a_j^1 is what i expected j to do in the first stage ($\alpha_{ij}(a_j^1) = 1$) then

$$B_{ij}(a^1; \alpha_i) = \min \left\{ [\mathbb{E}[\pi_i | a^1; \alpha_i] - \mathbb{E}[\pi_i | a^1; \alpha_i]]^+, F_i(a^1; \alpha_i) \right\} = 0.$$

In other words, j did not deviate from what i expected and j is not blamed by i . This is different from “could-have-been” blame (9).

Example 6 Suppose that, in Fig. A, Penny is initially certain of (U, L) : $\alpha_p(U, L) = 1$ and $\mathbb{E}[\pi_p; \alpha_p] = 2$. Upon observing (D, L) her frustration is $F_p((D, L); \alpha_p) = [\mathbb{E}[\pi_p; \alpha_p] - 1]^+ = 1$. Using eq. (10), at $a^1 = (D, L)$, Penny fully blames Ann, who deviated from U to D . Since

$$\sum_{a'_a \in A_a(\emptyset)} \alpha_{pa}(a'_a) \mathbb{E} [\pi_p | (a_{-a}^1, a'_a); \alpha_p] = \pi_p(U, L) = 2,$$

we get that Penny’s blame of Ann equals Penny’s frustration

$$B_{pa}((D, L); \alpha_p) = \min \left\{ [2 - \mathbb{E}[\pi_p | a^1; \alpha_p]]^+, 1 \right\} = 1.$$

On the other hand, Penny does not blame Bob, who played L as expected. To verify this, note that when frustrated after (D, L) Penny assesses how much she would have obtained had Bob behaved as expected:

$$\sum_{a'_b \in A_b(\emptyset)} \alpha_{pb}(a'_b) \mathbb{E} [\pi_p | (a_{-b}^1, a'_b); \alpha_p] = \mathbb{E}[\pi_p | (D, L); \alpha_p]$$

and

$$B_{pb}((D, L); \alpha_p) = \min \left\{ [\mathbb{E}[\pi_p | (D, L); \alpha_p] - \mathbb{E}[\pi_p | (D, L); \alpha_p]]^+, 1 \right\} = 0,$$

in contrast to could-have-been blame under which, as we saw, Penny fully blames Bob (Example 5). \blacktriangle

Formulae (9) and (10) each credit the full frustration on the first-mover of a leader-followers game, because each satisfies (8) (see Remark 3).

4.3 Anger from blaming intentions (ABI)

A player i prone to **anger from blaming intentions (ABI)** asks himself, for each co-player j , whether j intended to give him a low expected payoff. Since such intention depends on j 's first-order beliefs α_j (which include j 's plan, $\alpha_{j,j}$), how much i blames j depends on i 's second-order beliefs β_i , and the decision utility function has the form (5).

The maximum payoff that j , initially, can expect to give to i is

$$\max_{a_j^1 \in A_j(\emptyset)} \sum_{a_{-j}^1 \in A_{-j}(\emptyset)} \alpha_{j,-j}(a_{-j}^1) \mathbb{E} [\pi_i | (a_j^1, a_{-j}^1); \alpha_j].$$

Note that

$$\begin{aligned} & \max_{a_j^1 \in A_j(\emptyset)} \sum_{a_{-j}^1 \in A_{-j}(\emptyset)} \alpha_{j,-j}(a_{-j}^1) \mathbb{E} [\pi_i | (a_j^1, a_{-j}^1); \alpha_j] \\ & \geq \sum_{a^1 \in A(\emptyset)} \alpha_j(a^1) \mathbb{E} [\pi_i | a^1; \alpha_j] = \mathbb{E} [\pi_i | \alpha_j], \end{aligned}$$

where the inequality holds by definition, and the equality is implied by the chain rule (3). Note also that $\alpha_j(\cdot | a^1)$ is kept fixed under the maximization; we focus on what j initially believes he could achieve, taking the view that at the root he cannot control a_j^2 but predicts how he will choose in stage 2. We assume that i 's blame on j at a^1 equals i 's expectation, given second-order belief β_i and conditional on a^1 , of the difference between the maximum payoff that j can expect to give to i and what j actually plans/expects to give to i , capped by i 's frustration:

$$\begin{aligned} & B_{ij}(a^1; \beta_i) = \tag{11} \\ & \min \left\{ \mathbb{E} \left[\max_{a_j^1} \sum_{a_{-j}^1} \alpha_{j,-j}(a_{-j}^1) \mathbb{E} [\pi_i | (a_j^1, a_{-j}^1); \alpha_j] - \mathbb{E} [\pi_i; \alpha_j] \middle| a^1; \beta_i \right], F_i(a^1; \alpha_i) \right\}, \end{aligned}$$

where α_i is derived from β_i . The expression is nonnegative as per the previously highlighted inequality. Now, i 's decision utility after $h = a^1$ is

$$u_i^{ABI}(h, a_i; \beta_i) = \mathbb{E} [\pi_i | (h, a_i); \alpha_i] - \theta_i \sum_{j \neq i} B_{ij}(h; \beta_i) \mathbb{E} [\pi_j | (h, a_i); \alpha_i],$$

with $B_{ij}(a^1; \beta_i)$ given by (11).

Example 7 Return to Fig. B. The maximum payoff Ann can expect to give to Bob is 2, independently of α_a . Suppose Bob, upon observing g , is certain Ann “randomized” and

planned to offer g with probability p : $\beta_b(\alpha_a(g) = p|g) = 1$, with $p < 1$. Also, Bob is certain after g that Ann expected him to accept with probability q , *i.e.*, $\beta_b(\alpha_a(y|g) = q|g) = 1$. Finally, suppose Bob initially expected to get the *fair* offer ($\alpha_b(f) = 1$), so his frustration after g is $F_b(a^1; \alpha_b) = 2 - 1 = 1$. Bob’s blame of Ann’s intentions is

$$B_{ba}(g; \beta_b) = \min \{2 - [2(1 - p) + qp], 1\} = \min \{p(2 - q), 1\}.$$

If p is low enough, or q high enough, Bob does not blame all his frustration on Ann. He gives her some credit for the initial intention to make the fair offer with probability $1 - p > 0$, and the degree of credit depends on q . ▲

5 Equilibrium analyses

While in this paper we depart from traditional game-theoretic analysis in using belief-dependent decision utility, our analysis is otherwise traditional.

We consider two equilibrium notions. The first is an adaptation of Battigalli & Dufwenberg’s (2009) sequential equilibrium (SE) concept.¹⁵ For simplicity, we adopt a complete information framework where the rules of the game and players’ (psychological) preferences are common knowledge.¹⁶ We interpret a (sequential) equilibrium as a profile of strategies and beliefs representing a “commonly understood” way to play the game by rational (utility maximizing) agents. Our approach allows us to investigate the implications of our belief-dependent utility model within a standard framework. This is a choice of focus more than an endorsement of sequential equilibrium as a solution concept.¹⁷

SE requires that each player i is certain and never changes his mind about the true beliefs and plans, hence intentions, of his co-players. We find this feature questionable. Therefore, we also explore a generalization — “polymorphic sequential equilibrium” (PSE)— that allows for meaningful updating about others’ intentions.

5.1 Sequential Equilibrium (SE)

The SE concept gives equilibrium conditions for infinite hierarchies of conditional probability systems. In our particular application, utility functions only depend on first- or second-order

¹⁵Battigalli & Dufwenberg (2009) extend Kreps & Wilson’s (1982) classic notion of sequential equilibrium to psychological games; we here consider the version (Section 6) for preferences with own-plan dependence and “local” psychological utility functions.

¹⁶For an equilibrium analysis of incomplete-information psychological games see Attanasi, Battigalli & Manzoni (2016), for a non-equilibrium analysis see Battigalli, Charness & Dufwenberg (2013).

¹⁷Battigalli & Dufwenberg (2009, Sections 2, 5) argue that, with belief-dependent preferences, alternatives to SE like rationalizability, forward induction, and self-confirming equilibrium are even more plausible than with standard preferences.

beliefs, so we define SE for assessments comprising beliefs up to only the second order. Since, technically, first-order beliefs are features of second-order beliefs (see 2.2), we provide definitions that depend only on second-order beliefs, which give SEs for games where psychological utility functions depend only of first-order beliefs as a special case. Finally, although so far we have restricted our analysis of frustration and anger to two-stage game forms, our abstract definitions of equilibrium for games with belief-dependent preferences (and the associated existence theorem) apply to all multistage game forms.

Fix a game form and decision utility functions $u_i(h, \cdot; \cdot) : A_i(h) \times \Delta_i^2 \rightarrow \mathbb{R}$ ($i \in I$, $h \in H$). This gives a **psychological game** in the sense of Battigalli & Dufwenberg (2009, Section 6). An **assessment** is a profile of behavioral strategies and beliefs $(\sigma_i, \beta_i)_{i \in I} \in \times_{i \in I} (\Sigma_i \times \Delta_i^2)$ such that $\Sigma_i = \times_{h \in H} \Delta(A_i(h))$ and σ_i is the plan $\alpha_{i,i}$ entailed by second-order belief β_i :

$$\sigma_i(a_i|h) = \alpha_{i,i}(a_i|h) = \beta_i(Z(h, a_i) \times \Delta_{-i}^1|h) \quad (12)$$

for all $i \in I$, $h \in H$, $a_i \in A_i(h)$. Eq. (12) implies that the behavioral strategies contained in an assessment are implicitly determined by players' beliefs about paths; therefore, they could be dispensed with. We follow Battigalli & Dufwenberg (2009) and make behavioral strategies explicit in assessments only to facilitate comparisons with the equilibrium refinements literature.

Definition 1 An assessment $(\sigma_i, \beta_i)_{i \in I}$ is **consistent** if, for all $i \in I$, $h \in H$, and $a = (a_j)_{j \in I(h)} \in A(h)$,
(a) $\alpha_i(a|h) = \prod_{j \in I(h)} \sigma_j(a_j|h)$,
(b) $\text{marg}_{\Delta_{-i}^1} \beta_i(\cdot|h) = \delta_{\alpha_{-i}}$,
where α_i is derived from β_i and $\delta_{\alpha_{-i}}$ is the Dirac probability measure that assigns probability 1 to the singleton $\{\alpha_{-i}\} \subseteq \Delta_{-i}^1$.

Condition (a) requires that players' beliefs about actions satisfy independence across co-players (on top of own-action independence), and—conditional on each h —each i expects each j to behave in the continuation as specified by j 's plan $\sigma_j = \alpha_{j,j}$, even though j has previously deviated from $\alpha_{j,j}$. All players thus have the same first-order beliefs. Condition (b) requires that players' beliefs about co-players' first-order beliefs (hence their plans) are correct and never change, on or off the path. Thus all players, essentially, have the same second-order beliefs (considering that they are introspective and therefore know their own first-order beliefs). These conditions capture the “trembling-hand” interpretation of deviations implicit in Kreps & Wilson's (1982) original definition of equilibrium.

Definition 2 An assessment $(\sigma_i, \beta_i)_{i \in I}$ is a **sequential equilibrium (SE)** if it is consistent and satisfies the following sequential rationality condition: for all $h \in H$ and $i \in I(h)$, $\text{Supp} \sigma_i(\cdot|h) \subseteq \arg \max_{a_i \in A_i(h)} u_i(h, a_i; \beta_i)$.

It can be checked that this definition of SE is equivalent to the traditional one when players have standard preferences, *i.e.*, when there is a profile of utility functions $(v_i : Z \rightarrow \mathbb{R})_{i \in I}$ such that $u_i(h, a_i; \beta_i) = \mathbb{E}[v_i | (h, a_i); \alpha_i]$.¹⁸ A special case is the **material-payoff game**, where $v_i = \pi_i$ for each $i \in I$.

Theorem 1 *If $u_i(h, a_i; \cdot)$ is continuous for all $i \in I$, $h \in H$ and $a_i \in A_i(h)$, then there is at least one SE.*

Battigalli & Dufwenberg (2009) prove a version of this existence result where first-order beliefs are modeled as belief systems over pure strategy profiles. Our setting adds personal histories, and here first-order beliefs are modeled as beliefs about paths to capture beliefs about a players' own plan as well as others' behavior. Given those modifications, the “trembling-hand” technique used in Theorem 9 Battigalli & Dufwenberg (2009) can be applied to establish the general existence of psychological sequential equilibria in our framework. We omit the details.¹⁹

What we said so far about equilibrium does not assume specific functional forms. From now on, we focus on u_i^{SA} , u_i^{ABB} , and u_i^{ABI} . Since frustration and blame are continuous in beliefs, decision utility is also continuous, and we obtain existence in all cases of interest:

Theorem 2 *Every game with SA, ABB, or ABI has at least one SE.*

5.2 Properties of equilibria with frustration and anger

Next, we derive some general properties of how decision-makers with SA, ABB, or ABI behave. First, in pure-strategy SE's, players are never frustrated on the equilibrium path:

Proposition 1 *Let $(\sigma_i, \beta_i)_{i \in I}$ be an SE assessment of a game with SA, ABB, or ABI; if a history $h \in H$ has probability 1 under profile $(\sigma_i)_{i \in I}$, then*

$$F_i(h'; \alpha_i) = 0 \text{ and } \text{Supp}\sigma_i(\cdot | h') \subseteq \arg \max_{a'_i \in A_i(h')} \mathbb{E}[\pi_i | h'; \alpha_i]$$

for all $h' \preceq h$ and $i \in I$, where α_i is derived from β_i . Therefore, an SE strategy profile of a game with SA, ABB, or ABI with randomization (if any) only in the last stage is also a Nash equilibrium of the agent form of the corresponding material-payoff game.

¹⁸According to the standard definition of SE, sequential rationality is given by global maximization over (continuation) strategies at each $h \in H$. By the One-Shot-Deviation principle, in the standard case this is equivalent to “local” maximization over actions at each $h \in H$.

¹⁹A similar technique is used in the proof of Proposition 2 (first part) in the appendix.

To illustrate, in the Ultimatum Minigame (Fig. B), (f, n) can be an SE under ABB, and is a Nash equilibrium of the agent form²⁰ with material-payoff utilities. With (counterfactual) anger, n becomes a credible threat. Proposition 1 also holds for the multistage extension of Section 6.

Recall that two assessments are **realization-equivalent** if the corresponding strategy profiles yield the same probability distribution over terminal histories:

Proposition 2 *In every perfect-information (two-stage) game form with no chance moves and a unique SE of the material-payoff game, this unique material-payoff equilibrium is realization-equivalent to an SE of the psychological game with ABI, ABB, or—with only two players—SA.*

If unique, the material-payoff SE of a perfect-information game must be in pure strategies. By Proposition 1, players must maximize their material payoff on the path even if they are prone to anger. As for off-equilibrium path decision nodes, deviations from the material-payoff SE strategies can only be due to the desire to hurt the first-mover, which can only increase his incentive to stick to the material-payoff SE action.

It is quite easy to show by example that without perfect information, or with chance moves, a material-payoff SE need not be an SE with frustration and anger. The same holds for some multistage game forms (cf. Section 6). Disregarding chance moves, randomization, and ties, the common feature of material-payoff equilibria that are not realization-equivalent to equilibria with frustration and anger is this (see Fig. A and Ex. 8 below): Start with a material-payoff equilibrium and add anger to decision utility; now, at an off-path node after Ann deviates, frustrated Penny wants to hurt Bob, which implies rewarding Ann; this makes it impossible to incentivize both Ann not to deviate and Penny to punish Bob after Ann’s deviation.

Next, we show that our model captures an important aspect of the psychology of anger. Sell, Cosmides, & Tooby (2009) argue that anger “is produced by a neurocognitive program engineered by natural selection to use bargaining tactics to resolve conflicts of interest in favor of the angry individual.” The proposition below demonstrates that our approach is consistent with this view. In leader-follower games, our models predict that anger-sensitive followers will obtain at least as large a material payoff as self-interested ones.

Proposition 3 *In every (two player) leader-follower game with a unique SE of the material-payoff game, the expected material payoff of the follower in any SE with SA, or ABB, or ABI is at least as large as the material payoff of the follower in the unique material-payoff SE.*

²⁰In the agent form of a game, each h where player i is active corresponds to a copy (i, h) of i with strategy set $A_i(h)$ and the same utility function as i .

To see why this is the case, consider an SE of the psychological game that yields a different expected material payoff to the follower than the material-payoff equilibrium. Since a first mover cannot be frustrated, all the actions chosen by the leader with positive probability in this SE must maximize his expected material payoff. Thus, if in this SE the leader deviates from the material-payoff equilibrium action, it must be the case that he correctly expects this action to be “punished” by the follower with a deviation from the follower’s material-payoff maximizing reply. This in turn requires that the follower is frustrated by the material-payoff equilibrium action of the leader, hence, that his expected payoff in this SE is higher than in the material-payoff equilibrium.

With more than two followers, Proposition 3 need not hold. For example, there may exist equilibria where one follower behaves as-if self-interested, while the other is frustrated by the material-payoff equilibrium outcome. This can result in the first follower getting less in the material-payoff equilibrium, depending for example upon which follower is active in the second stage.

5.3 Examples

We next illustrate via three examples, how the SE concept works and how SA, ABB (both versions), and ABI may alter material incentives and produce different predictions.

Example 8 Consider Fig. A., Asymmetric Punishment. Can material-payoff equilibrium outcome (U, L) be part of an SE with frustration and anger? The answer is yes under ABI and the blaming-unexpected-deviations version of ABB. To see this note as first movers, Ann and Bob act as-if selfish (as they are not frustrated - see Remarks ?? and 2). Hence, they would deviate if they could gain materially. In the SE, they would expect 5 if not deviating, making Ann the sole deviation candidate (she would get $6 > 5$ were Penny to choose P ; for Bob, 5 is the best he could hope for). But Ann deviating can be dismissed since if (D, L) were reached Penny would not blame Bob (her only punishable co-player) under either relevant blame function, so she would choose N (regardless of θ_p). Under SA and the could-have-been version of ABB, however, it may be impossible to sustain an SE with (U, L) ; at (D, L) Penny would blame each of Ann and Bob (as explained). By choosing P she hurts Bob more than she helps Ann and would do so if

$$\begin{aligned} u_p^{ABB}((D, L), P; \alpha_p) &> u_p^{ABB}((D, L), N; \alpha_p) \\ &\iff \\ 0 - 6\theta_p B_{pa}((D, L); \alpha_p) &> 1 - 8\theta_p B_{pa}((D, L); \alpha_p). \end{aligned}$$

The rhs of the last inequality uses $B_{pb}((D, L); \alpha_p) = B_{pa}((D, L); \alpha_p)$. Since $B_{pa}((D, L); \alpha_p) = F_p((D, L); \alpha_p) = 1 > 0$, Penny would choose P if $-6\theta_p > 1 - 8\theta_p \iff \theta_p > 1/2$, so Ann would want to deviate and choose D . \blacktriangle

Example 9 Consider Fig. B., the Ultimatum Minigame. By Proposition 2, every utility function discussed admits (g, y) as an SE, regardless of anger sensitivity. To check this directly, just note that, if Bob expects g , he cannot be frustrated, so—when asked to play—he maximizes his material payoff. Under SA and ABB (both versions), (f, n) qualifies as another SE if $\theta_b \geq 1/3$; following g , Bob would be frustrated and choose n , so Ann chooses f . Under ABI (f, n) cannot be a SE. To verify, assume it were, so $\alpha_a(f) = 1$. Since the SE concept does not allow for players revising beliefs about beliefs, we get $\beta_b(\alpha_a(f) = 1|g) = 1$ and $B_{ba}(g; \beta_b) = 0$; Bob maintains his belief that Ann planned to choose f , hence she intended to maximize Bob’s payoff. Hence, Bob would choose y , contradicting that (f, n) is an SE. Next, note that (g, n) is not an SE under any concept: Given SE beliefs Bob would not be frustrated and so he would choose y . The only way to observe rejected offers with positive probability in an SE is with non-deterministic plans. To find such an SE, note that we need $\alpha_a(g) \in (0, 1)$; if $\alpha_a(g) = 0$ Bob would not be reached and if $\alpha_a(g) = 1$ he would not be frustrated, and hence, he would choose y . Since Ann uses a non-degenerate plan she must be indifferent, so $\alpha_b(y) = 2/3$, implying that Bob is indifferent too. In SE, Bob’s frustration is $[2(1 - \alpha_a(g)) + \frac{2}{3}\alpha_a(g) - 1]^+ = [1 - \frac{4}{3}\alpha_a(g)]^+$, which equals his blame of Ann under SA and ABB. Hence we get the indifference condition

$$1 - \theta_b \left[1 - \frac{4}{3}\alpha_a(g)\right]^+ \cdot 3 = 0 - \theta_b \left[1 - \frac{4}{3}\alpha_a(g)\right]^+ \cdot 0$$

$$\iff$$

$$\alpha_a(g) = \frac{3}{4} - \frac{1}{4\theta_b},$$

where $\theta_b \geq 1/3$. The more prone to anger Bob is the more likely he is to get the low offer, so Bob’s initial expectations, and hence his frustration and blame, is kept low. Under ABI we get another indifference condition:

$$1 - \theta_b B_{ba}(g; \beta_b) \cdot 3 = 0 - \theta_b B_{ba}(g; \beta_b) \cdot 0$$

$$\iff$$

$$1 - \theta_b \min \left\{ 1 - \frac{4}{3}\alpha_a(g), \frac{4}{3}\alpha_a(g) \right\} \cdot 3 = 0.$$

The left term in braces is Bob’s frustration, while

$$\frac{4}{3}\alpha_a(g) = 2 - \left[2(1 - \alpha_a(g)) + \frac{2}{3}\alpha_a(g) \right]$$

is the difference between the maximum payoff Ann could plan for Bob and that actually planned. The first term is lower if $\alpha_a(g) \geq 3/8$; so, if we can solve the equation for such a

number, we duplicate the SA/ABB-solution; again, this is doable if $\theta_b > 1/3$. If $\theta_b \geq 2/3$, with ABI, there is a second non-degenerate equilibrium plan with $\alpha_a(g) \in (0, \frac{3}{8})$ such that $\alpha_a(g) = 1/4\theta_b$; to see this, solve the ABI indifference condition assuming $\frac{4}{3}\alpha_a(g) \leq 1 - \frac{4}{3}\alpha_a(g)$. This SE exhibits starkly different comparative statics: The higher is θ_b , the less likely Bob is to get a low offer and the less he blames Ann following g in light of her intention to choose f with higher probability. \blacktriangle

The reason why (f, n) in Example 9 cannot be an SE under ABI is that if Bob initially expects Ann to choose f , and she doesn't, so that Bob is frustrated, then he would rate her choice an unintended mistake and not blame her. We emphasize that this is due to assumptions that underlie the SE concept, *i.e.*, the “trembling-hand” interpretation of deviations, not to the formulation of ABI utility. According to our notion of polymorphic sequential equilibrium (see below), Bob may revise his belief about Ann's plan.

Example 10 Consider Fig. C, Hammering one's thumb. With u_a^{ABB} (either version), or u_a^{ABI} , Andy will not blame Bob so his SE-choice is the material-payoff equilibrium, N . But with u_a^{SA} Andy may choose T . Recall that $F_a(B; \alpha_a) = 2(1 - \varepsilon) + \varepsilon\alpha_a(N|B) - 1$, so the more likely Andy believes it to be that he will take it out on Bob, the less he expects initially and the less frustrated he is after B . Yet, in SE, the higher is θ_a the more likely Andy is to take it out on Bob: Andy's utility from N and T is

$$\begin{aligned} u_a^{SA}(B, N; \alpha_a) &= 1 - \theta_a[2(1 - \varepsilon) + \varepsilon\alpha_a(N|B) - 1] \cdot 2, \\ u_a^{SA}(B, T; \alpha_a) &= 0 - \theta_a[2(1 - \varepsilon) + \varepsilon\alpha_a(N|B) - 1] \cdot 0 = 0. \end{aligned}$$

Sequential rationality of SE implies that one possibility is $\alpha_a(N|B) = 1$ and $u_a^{SA}(B, N; \alpha_a) \geq u_a^{SA}(B, T; \alpha_a)$, implying $\theta_a \leq \frac{1}{2(1-\varepsilon)}$. Another possibility is $\alpha_a(N|B) = 0$ and $u_a^{SA}(B, N; \alpha_a) \leq u_a^{SA}(B, T; \alpha_a)$, implying $\theta_a \geq \frac{1}{2(1-2\varepsilon)}$. That is, if Andy is sufficiently susceptible to simple anger, on bad days he takes his frustration out on Bob. If $\theta_a \in (\frac{1}{2(1-\varepsilon)}, \frac{1}{2(1-2\varepsilon)})$, we can solve for an SE where $u_a^{SA}(B, N; \alpha_a) = u_a^{SA}(B, T; \alpha_a)$ and $\alpha_a(N|B) = \frac{1}{2\varepsilon\theta_a} - \frac{1-2\varepsilon}{\varepsilon} \in (0, 1)$.

The final case, where $\theta_a \in (\frac{1}{2(1-\varepsilon)}, \frac{1}{2(1-2\varepsilon)})$, illustrates how we cannot take for granted the existence of an SE in which players use deterministic plans (a point relevant also for u_i^{ABB} or u_i^{ABI} in other games). Here this happens in a game form with a single active player, highlighting that we deal with a psychological game, as this could not be the case in a standard game. \blacktriangle

5.4 Polymorphic sequential equilibrium (PSE)

Suppose a game is played by agents drawn at random and independently from large populations, one for each player role $i \in I$. Different agents in the same population i have the

same belief-dependent preferences,²¹ but they may have different plans, hence different beliefs about paths, even if their beliefs agree about the behavior and beliefs of co-players $-i$. In this case, we say that the population is “polymorphic.” Once an agent observes some moves of co-players, he makes inferences about their intentions.

Let λ_i be a finite support distribution over $\Sigma_i \times \Delta_i^2$, with $\text{Supp}\lambda_i = \{(\sigma_{t_i^1}, \beta_{t_i^1}), (\sigma_{t_i^2}, \beta_{t_i^2}), \dots\}$, where $t_i = t_i^1, t_i^2, \dots$ is an index we refer to as “type” of i .²² We interpret λ_i as a statistical distribution of plans and beliefs of agents playing in role i .²³ With a slight abuse of notation, we let $\lambda_i(t_i)$ denote the fraction of agents in population i with plan and beliefs $(\sigma_{t_i}, \beta_{t_i})$. Also, we denote by

$$T_i(\lambda_i) = \{t_i : (\sigma_{t_i}, \beta_{t_i}) \in \text{Supp}\lambda_i\}$$

the set of possible types of i in distribution λ_i , and we write $T_{-i}(\lambda_{-i}) = \times_{j \neq i} T_j(\lambda_j)$ for the set of profiles of co-players’ types.

Let us take the perspective of an agent of type t_i who knows that the distribution over co-players’ types is $\lambda_{-i} = \prod_{j \neq i} \lambda_j$ and believes that the behavior of each t_j is indeed described by t_j ’s plan σ_{t_j} (in principle, t_i may otherwise believe that t_j behaves differently from his plan). Then it is possible to derive the conditional probability of a type profile t_{-i} given history h . Given that beliefs satisfy independence across players (everybody knows there is independent random matching), the distribution is independent of t_i and can be factorized. In the current two-stage setting we have $\lambda_{-i}(t_{-i}|\emptyset) = \prod_{j \neq i} \lambda_j(t_j)$ and

$$\begin{aligned} \lambda_{-i}(t_{-i}|a^1) &= \frac{\prod_{j \neq i} \sigma_{t_j}(a_j^1) \lambda_j(t_j)}{\sum_{t'_{-i} \in T_{-i}(\lambda_{-i})} \prod_{j \neq i} \sigma_{t'_j}(a_j^1) \lambda_j(t'_j)} \\ &= \frac{\prod_{j \neq i} \sigma_{t_j}(a_j^1) \lambda_j(t_j)}{\prod_{j \neq i} \sum_{t'_j \in T_j(\lambda_j)} \sigma_{t'_j}(a_j^1) \lambda_j(t'_j)} \\ &= \prod_{j \neq i} \frac{\sigma_{t_j}(a_j^1) \lambda_j(t_j)}{\sum_{t'_j \in T_j(\lambda_j)} \sigma_{t'_j}(a_j^1) \lambda_j(t'_j)}. \end{aligned}$$

for all t_{-i} and a^1 , provided that $\sum_{t'_j} \sigma_{t'_j}(a_j^1) \lambda_j(t'_j) > 0$ for each $j \neq i$. Letting

$$\lambda_j(t_j|a^1) = \frac{\sigma_{t_j}(a_j^1) \lambda_j(t_j)}{\sum_{t'_j \in T_j(\lambda_j)} \sigma_{t'_j}(a_j^1) \lambda_j(t'_j)},$$

we get

$$\lambda_{-i}(t_{-i}|a^1) = \prod_{j \neq i} \lambda_j(t_j|a^1).$$

²¹Recall that we are not modelling incomplete information.

²²These are “types” in the sense of epistemic game theory (*e.g.*, Battigalli, Di Tillio & Samet 2013).

²³The marginal of λ_i on Σ_i is a behavior strategy mixture (see Selten 1975).

We say that λ_j is **fully randomized** if σ_{t_j} is strictly positive for every type $t_j \in T_j(\lambda_j)$. If each λ_j is fully randomized, then, for all $h \in H$, $\lambda_{-i}(\cdot|h)$ is well defined, with $\lambda_{-i}(t_{-i}|h) = \prod_{j \neq i} \lambda_j(t_j|h)$ for all $t_{-i} \in T_{-i}(\lambda_{-i})$.

Definition 3 A *polymorphic assessment* is a profile of finite support probability measures $\lambda = (\lambda_i)_{i \in I} \in \times_{i \in I} \Delta(\Sigma_i \times \Delta_i^2)$ such that, for every $i \in I$ and $t_i \in T_i(\lambda_i)$, σ_{i,t_i} is the behavior strategy obtained from β_{t_i} as per (12). A polymorphic assessment λ is **consistent** if there is a sequence $(\lambda^n)_{n=1}^\infty$ of polymorphic assessments converging to λ such that, for all $j \in I$ and $n \in \mathbb{N}$, λ_j^n is fully randomized, and (a-p) for all $h \in H$, $a \in A(h)$, and $t_i \in T_i(\lambda_i^n)$,

$$\alpha_{t_i, -i}^n(a_{-i}|h) = \prod_{j \neq i} \sum_{t_j \in T_j(\lambda_j^n)} \sigma_{t_j}^n(a_j|h) \lambda_j^n(t_j|h),$$

(b-p) for all $h \in H$ and $t_i \in T_i(\lambda_i^n)$,

$$\text{marg}_{\Delta_{-i}^1} \beta_{t_i}^n(\cdot|h) = \sum_{t_{-i} \in T_{-i}(\lambda_{-i}^n)} \lambda_{-i}^n(t_{-i}|h) \delta_{\alpha_{t_{-i}}^n},$$

where, for all $j \in I$, $t_j \in T_j(\lambda_j^n)$ and $n \in \mathbb{N}$, $\alpha_{t_j}^n$ is the first-order belief system derived from $\beta_{t_j}^n$.

Condition (a-p) extends independence condition (a) of Definition 1 to the multiple-types setting. Condition (b-p) implies that, conditional on the co-players' types, everyone has correct beliefs about the others' beliefs, including their plans. Yet uncertainty about co-players' types allows for uncertainty and meaningful updating about such beliefs. Conditions (a-p) and (b-p) imply that different types of the same player share the same beliefs about co-players, but may have different plans. Definition 3 thus is a minimal departure from the notion of consistent assessment, allowing for uncertainty and meaningful updating about the plans, hence intentions, of co-players.

Definition 4 A polymorphic assessment λ is a **polymorphic sequential equilibrium (PSE)** if it is consistent and satisfies the following sequential rationality condition: for all $h \in H$, $i \in I$, and $t_i \in T_i(\lambda_i)$,

$$\text{Supp} \sigma_{t_i}(\cdot|h) \subseteq \arg \max_{a_i \in A_i(h)} u_i(h, a_i; \beta_{t_i}).$$

Remark 4 Every SE is a degenerate (or monomorphic) PSE. Therefore, Theorem 1 implies that, if every decision-utility function $u_i(h, \cdot; \cdot)$ ($i \in I$, $h \in H$) is continuous, then there is at least one PSE. In particular, every game with SA, ABB, or ABI has at least one PSE.

Finally, we demonstrate how the PSE alters predictions in the Ultimatum Minigame and in leader-followers games more generally.

Example 11 Consider again Figure B. If $|\text{Supp}\lambda_i| = 1$ for all i , then our results for the SE analysis still hold as a special case of the more general PSE analysis. Interesting new possibilities arise if $|\text{Supp}\lambda_a| = 2$. Recall that, in the SE with SA/ABB utility functions and non-degenerate plans (Example 9) we had $\alpha_a(g) = \frac{3}{4} - \frac{1}{4\theta_b}$ (with $\theta_b > 1/3$) to keep Bob indifferent. Suppose instead there are two types of Ann, a fraction of $\frac{3}{4} - \frac{1}{4\theta_b}$ of them planning to choose g while the others plan for f . There is a corresponding PSE where (naming Ann's types by planned choice) $\text{Supp}\lambda_a = \{(f, \beta_f), (g, \beta_g)\}$, $\alpha_f(y|g) = \alpha_g(y|g) = \alpha_b(y|g) = 2/3$, and this holds for also for ABI, not only SA and ABB. The first-order belief of type f of Ann, α_f , is derived from β_f , etc. Bob initially believes Ann is either an f - or a g -type, assigning probability $\lambda_g = \frac{3}{4} - \frac{1}{4\theta_b}$ to the latter possibility. After action g he ceases to assign positive probability to being matched with an f -type, assigning instead probability 1 to the g -type, a form of updating about Ann's intentions implied by consistency (Definition 3). This inference makes ABI work as ABB (and SA). Bob's frustration is as in Example 9, so equal to his blame of Ann for each blaming function. Again Bob is indifferent between y and n , and sequentially rational if $\alpha_b(y|g) = 2/3$. Condition $\alpha_f(y|g) = \alpha_g(y|g) = 2/3$ implies both types of Ann are indifferent, hence sequentially rational. Thus, starting with the non-degenerate SE under ABB (and SA) we obtain a PSE, under every blaming function, where Ann's plan is purified.

▲

The insight of the previous example can be generalized:²⁴

Proposition 4 *Consider a leader-followers game and an arbitrary parameter profile $(\theta_i)_{i \in I}$. Every SE with decision-utility functions $(u_{i,\theta_i}^{ABB})_{i \in I}$ [or $(u_{i,\theta_i}^{SA})_{i \in I}$] where the behavioral strategy of the leader has full support corresponds to a PSE with decision-utility functions $(u_{i,\theta_i}^{ABB})_{i \in I}$ [or $(u_{i,\theta_i}^{SA})_{i \in I}$] and also $(u_{i,\theta_i}^{ABI})_{i \in I}$ where the leader is purified.*

6 Multistage extension

In a multistage game form, a (nonempty) nonterminal history is a sequence of action profiles, $h = (a^1, \dots, a^t)$ where $t \geq 1$. As in the two-stage case, we assume that actions are observable; hence, every non-terminal history is public. Our notation for the multistage setting is essentially the same as before. The set of sequences observable by player i also includes personal histories of the form (h, a_i) : $H_i = H \cup \{(h, a_i) : h \in H, a_i \in A_i(h)\}$.

²⁴Recall that, by Remark 3, in leader-followers games SA is equivalent to both versions of ABB.

A belief system for i over paths and beliefs of others is an array of probability measures $\beta_i = (\beta_i(\cdot|h_i))_{h_i \in H_i}$ satisfying (3) and (4), which apply to the multistage setting as well. Also the notation on beliefs is as before: $\alpha_i \in \Delta_i^1$, $\beta_i \in \Delta_i^2$, and α_i is the first-order belief system derived from β_i when they appear in the same formula.

Therefore, the definition of SE from Section 5 can be applied without modifications, and Theorem 2 implies the existence of SE with SA, ABB, or ABI in the multistage setting.

We distinguish two extreme scenarios according to the behaviorally relevant periodization: In the **slow-play** scenario, stages correspond to periods, and player i 's reference expectation to determine frustration at the beginning of period (stage) $t + 1$ is given by his belief at the beginning of period t . In the **fast-play** scenario, the game's different stages occur in the same period and the relevant reference expectation of i in stage t is given by his initial belief (at the root).²⁵ In either case, we maintain the assumption that blame is continuous in beliefs, capped by frustration, and equal to frustration in the case of SA.

6.1 Slow play

We start with this scenario because it allows for a relatively simple extension of the two-stage setting, with initial beliefs replaced by one-period-lagged beliefs: For any non-terminal history of the form $h = (\bar{h}, a)$ the frustration of i conditional on h given α_i is

$$F_i(h; \alpha_i) = \left[\mathbb{E}[\pi_i|\bar{h}; \alpha_i] - \max_{a_i \in A_i(h)} \mathbb{E}[\pi_i|(h, a_i); \alpha_i] \right]^+.$$

(When $\bar{h} = \emptyset$ and $h = a^1$, we are back to the two-period formula.) The decision utility of action $a_i \in A_i(h)$ has the general form (5), where the blame functions $B_{ij}(h; \beta_i)$ are of the SA, ABB, or ABI type. Specifically: $B_{ij}(h; \beta_i) = F_i(h; \alpha_i)$ for SA, whereas the could-have-been blame, blaming deviations, and blaming intentions can be defined with straightforward adaptations of (9), (10), and (11) respectively; therefore we omit the details.

This extension of the two-stage setting has the stark feature that past frustrations do not affect current behavior. (A more nuanced version of the model might feature a decaying effect of past frustrations.)

A detail in modeling game forms becomes relevant in the slow play scenario: We have to explicitly allow for non-terminal histories after which no player (not even chance) is active,

²⁵Applications may involve intermediate cases, as in alternating-offer bargaining models where a period comprises two stages. The two extremes convey the main ideas.

such as history g in Fig. D.

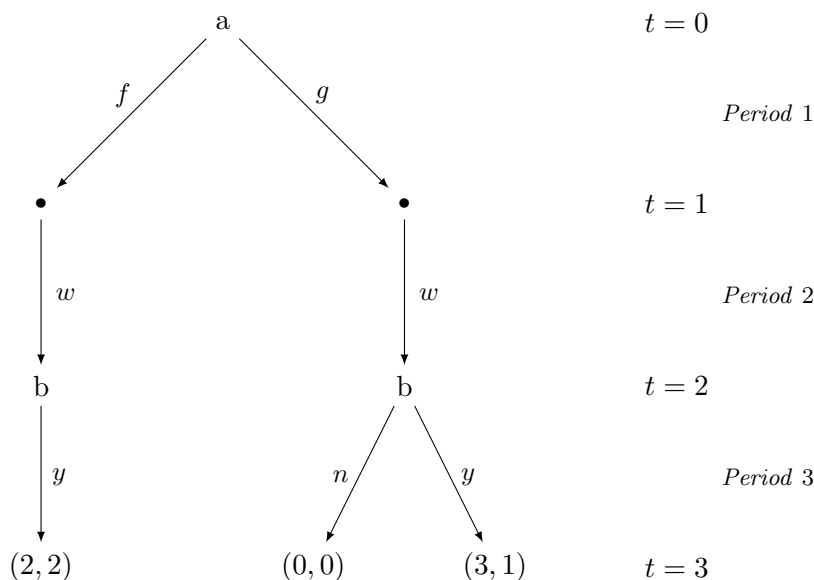


Figure D. Ultimatum Minigame with delayed reply.

At such histories there is only one feasible action profile, as each player has only one feasible action, to wait. In the two-periods setting this detail is irrelevant: If nobody is active at the root, play effectively starts (and ends) in the second period; if nobody is active at a^1 , it can be modeled as a terminal history. With more than two periods, having to wait may affect behavior.

Example 12 Consider Fig. D. Suppose that Bob initially expects f with positive probability. Then, in period 2, after g , he is frustrated; however Bob cannot hurt Ann immediately because he has to wait. In period 3, Bob’s lagged expectation has fully adapted downward, and he is not frustrated. According to our slow-play model, the frustration experienced by Bob in period 2 does not affect his decision utility in period 3: Bob fully “cools off” and behaves as-if selfish.²⁶ Therefore the unique SE and PSE outcome of the game is (g, w, y) , where w denotes waiting. ▲

6.2 Fast play

When play is fast, all stages belong to the same period, therefore the reference belief that determines player i ’s frustration conditional on any history is i ’s initially expected monetary

²⁶See e.g. Grimm & Mengel (2011) and Oechssler *et al.* (2015) for experiments.

payoff. Thus, i 's frustration at h given α_i is calculated with reference to initial expectations, as in Section 3:

$$F_i(h; \alpha_i) = \left[\mathbb{E}[\pi_i; \alpha_i] - \max_{a_i \in A_i(h)} \mathbb{E}[\pi_i | (h, a_i); \alpha_i] \right]^+.$$

This implies that *there cannot be any "cooling off"* due to reference-point acclimatization. Formally, histories where nobody (not even chance) is active play no role and therefore can be deleted from the game form without affecting the analysis, unlike with slow play. For example, in the fast-play scenario, the ultimatum game form of Fig. D is equivalent to the one of Fig. B.

The fast-play frustration formula can be plugged into the SA decision utility function (7). As for the ABB decision utility, property (8) of B_{ij} extends to the multistage setting as follows:

$$B_{ij}(h; \alpha_i) = \begin{cases} 0, & \text{if } j \notin I(h') \text{ for all } h' \prec h, \\ F_i(a^1; \alpha_i), & \text{if } \{j\} = I(h') \text{ for all } h' \prec h. \end{cases} \quad (13)$$

In words, co-player j cannot be blamed if he was never active in the past, and he is fully blamed if instead he was the only active player. A relatively simple extension of could-have-been blame satisfies this property:

$$B_{ij}(h; \alpha_i) = \min \left\{ \left[\max_{h' \prec h, a'_j \in A_j(h')} \mathbb{E}[\pi_i | (h', a'_j); \alpha_i] - \mathbb{E}[\pi_i | h; \alpha_i] \right]^+, F_i(h; \alpha_i) \right\}. \quad (14)$$

We can follow a similar logic to extend ABI.

Proposition 5 *If B_{ij} is defined by (14), then it satisfies (13).*

We now illustrate our definition, elucidating a modeling choice:

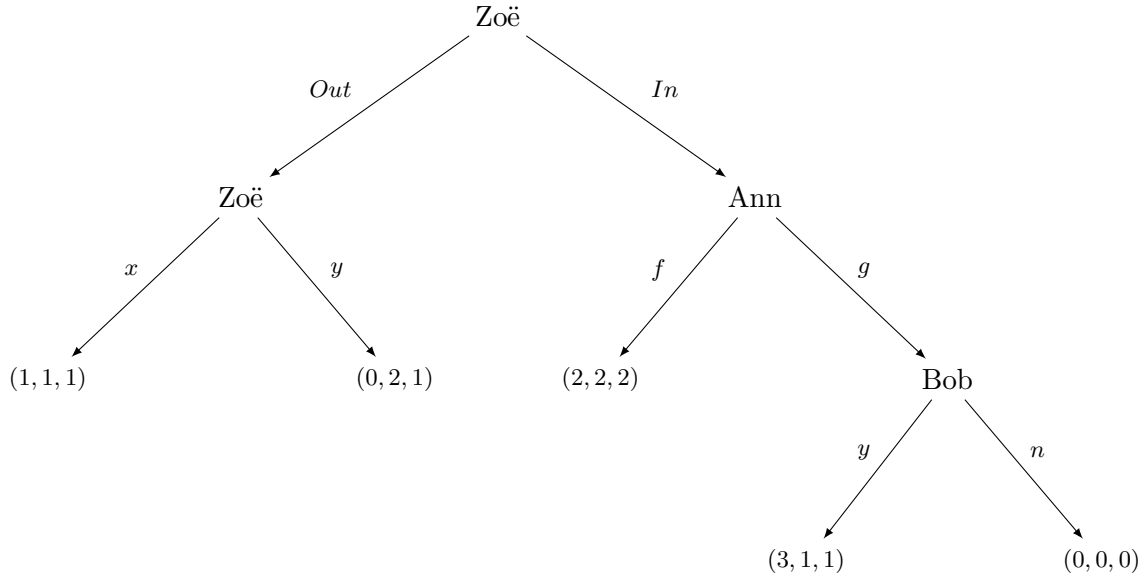


Figure E. Multistage Ultimatum featuring Zoë.

Example 13 Consider the game form in Fig. E (material payoffs are in alphabetical order). If Zoë chooses *In*, then Ann and Bob interact in an ultimatum minigame, but Zoë may instead exercise outside options and play (Out, x) or (Out, y) . Zoë's payoffs equal Bob's, except following (Out, y) where a payoff transfer from Ann to Bob occurs, relative to (Out, x) . Can strategy profile $(In-x, f, n)$ be an SE under ABB? Given equilibrium beliefs, this is the case if $0 - \theta_b \cdot 1 \cdot 0 \geq 1 - \theta_b \cdot 1 \cdot 3$, or $\theta_b \geq 1/3$. The calculation involves Bob blaming Ann, not Bob blaming Zoë, because if Zoë switched from *In* to *Out* (thus implementing (Out, x) instead of *In*) this would not improve Bob's payoff. This reflects a non-obvious modeling choice: Our definition assesses blame on the basis of single-agent deviations from the realized path, but if Bob alternatively assessed blame on the basis of multi-agent deviations, including off-realized-path deviations, he would consider that Zoë could have played (Out, y) . She would then have increased Bob's payoff from 1 to 2, preventing his frustration of 1. If Bob's blame of Zoë were thus 1, then $(In-x, f, n)$ would be an SE under ABB if $0 - \theta_b \cdot 1 \cdot 0 \geq 1 - \theta_b \cdot 1 \cdot 3 - \theta_b \cdot 1 \cdot 1$, or $\theta_b \geq 1/4 \neq 1/3$. (This also shows that SE under ABB is not invariant with respect to coalescing sequential moves.) Finally, note that also $(In-y, f, n)$ is an SE under ABB in the fast-play scenario for $\theta_b \geq 1/4$, because at (In, g) Zoë would be blamed for not switching to *Out* (implementing (Out, y)); but it is an SE under ABB in the slow-play scenario for larger parameter values, $\theta_b \geq 1/3$, because Bob would be frustrated only in the third period, after (In, g) , and Zoë—who played in the first—could not be blamed. ▲

The single- vs. multi-agent deviation issue illustrated here can arise also in two-stage games (with simultaneous moves), but the point is clearer, and perhaps more relevant, in games with more than two stages. We defend our chosen formulation thrice: It harmonizes well with how we define rational play, where players optimize only locally (although in equilibrium they predict correctly and choose as planned). The (hinted at) alternative definition would be formally convoluted. It is an open issue which formulation is empirically more relevant, so we stick with what is simpler.

6.3 Counterfactual anger and unique SE in hold-up

It is important to emphasize, worth a separate section, that anger (and in fact emotions more generally) can shape behavior without occurring. If anger is anticipated, this may steer behavior down alternative paths (cf. Proposition 1). We already saw examples, *e.g.*, (f, n) may be an SE in the Ultimatum Minigame, alongside (g, y) . Our next example highlights how there may be circumstances where the SE is *unique* and has that property. It also illustrates a difference between fast and slow play.

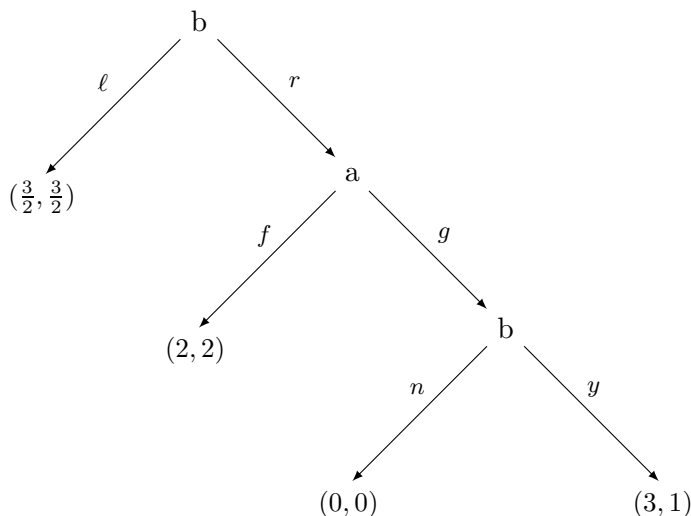


Figure F. Hold-up.

Example 14 Modify the Ultimatum Minigame by adding an initial move for Bob, as in Fig. F, to get an illustration of a hold-up problem (cf. Dufwenberg, Smith & Van Essen 2013).²⁷

²⁷Bob and Ann face a joint business opportunity worth $(2, 2)$ via path (r, f) ; however, r involves partnership-specific investment by Bob, which Ann can exploit choosing g (renegeing), etc. As always, we list payoffs by alphabetical order of players: (π_a, π_b) .

Under fast play, for each utility function seen so far,²⁸ if $\theta_b > 2/3$, there is a unique SE: Bob uses plan $(r-n)$, Ann plans for f . To verify this, the key step is to check that if Bob plans for (ℓ, y) and Ann for g this is *not* an SE; if Bob initially expects \$1.5, off-path at (r, g) , he would be frustrated and deviate to n . ▲

With slow play, by contrast, with $\theta_b > 2/3$, there are multiple SE, exactly as in the Ultimatum Minigame. In particular, both $(r-n, f)$ and $(\ell-y, g)$ are SE; in the latter, Bob’s updated expected payoff after (counterfactual) action r is only \$1, hence he cannot be frustrated by g .

7 Comparison with other models

In this section we compare and contrast the behavioral predictions resulting from frustration and anger with other models of strategic and social behavior. We first consider distributional preferences, which transform material payoffs at terminal histories but which otherwise retain the standard assumption that choices depend solely on their consequences in terms of material payoffs. We then discuss reciprocity as well as two other models of anger: that of Rotemberg (2005, 2008, 2011) and that of Akerlof (2016). Finally, we discuss connections to the reference-dependent utility model of Kőszegi and Rabin (2006) and its adaptation to study domestic violence by Card & Dahl (2011).

7.1 Distributional preferences

Like anger, models of distributional preferences such as inequality aversion (e.g. Fehr & Schmidt 1999, Bolton & Ockenfels 2000) predict costly punishment. However, a number of studies demonstrate that the decision to engage in costly punishment depends upon both payoffs reached at terminal histories as well as payoffs from unreached histories.²⁹ Our approach can capture this non-consequentialist aspect of behavior, while distributional preferences cannot. For example, in the ultimatum minigame of Fig. B, Bob’s decision to reject the greedy offer may depend upon not only the payoffs from accepting or rejecting the offer, but also upon the payoff that Ann could have given Bob had she chosen the fair offer. Holding payoffs and other beliefs constant, our models predict that Bob will be more likely to reject the greedy offer when either he assigns higher probability to receiving the fair one, or when the fair payoff is increased. In general, distributional preferences cannot capture behavioral patterns which depart from consequentialism, while our models can.

²⁸Except the blaming-unexpected-deviations version of ABB, which we did not define explicitly for fast play.

²⁹See e.g. Brandts & Solá (2001), Charness & Rabin (2002), Nelson (2002), Falk, Fehr, & Fischbacher (2003, 2008) and Sutter (2007).

The models in this paper assume that players care only about material payoffs and anger, disregarding distributional considerations. We do so to highlight the effects of frustration, blame, and anger on behavior in strategic interaction. However, real-world decision makers may have a mixture of material, distributional, and psychological motivations (e.g. Falk & Fischbacher 2006). Although we do not pursue the modeling of distributional concerns, doing so might involve straightforward modifications of our material payoff function. We leave this for future work.

7.2 Reciprocity

Negative reciprocity à la Rabin (1993), Dufwenberg & Kirchsteiger (2004), and Falk & Fischbacher (2006) joins anger as a motivation that can trigger hostility. Also like anger, negative reciprocity can result in non-consequential behavior, but anger and negative reciprocity differ in key ways. The following sketched comparison is with Dufwenberg & Kirchsteiger’s (2004) notion of sequential reciprocity equilibrium (SRE; refer to their article for formal definitions).

In the Hammering one’s thumb game (Fig. C), Andy may take it out on Bob if he is motivated by simple anger. Were he motivated by reciprocity, this could never happen: Bob’s kindness, since he is a dummy-player, equals 0, implying that Andy chooses as-if selfish. Reciprocity here captures intuitions similar to the ABI concept, but that analogy only carries so far, as we show next.

Reciprocity also allows for “miserable equilibria,” where a player reciprocates expected unkindness before it occurs. For example, in the Ultimatum Minigame of Fig. B, (g, n) may be a SRE. Ann makes offer g despite believing that Bob will reject; given her beliefs about Bob’s beliefs, Ann perceives Bob as seeing this coming, which makes him unkind, so she punishes by choosing g . Such self-fulfilling prophecies of destructive behavior have no counterpart under any of our anger notions. Since Ann moves at the root, Remarks ?? and 2 demonstrate that she cannot be frustrated, and hence chooses as-if selfish.³⁰ By Proposition 1, sacrificing material payoff to harm a co-player never occurs on the path of a pure-strategy SE with ABB, ABI, or (in two-player games) SA.

So under our definition, anger is “hot”: in pure-strategy SE’s, angry players do not expect to sacrifice to harm others. In addition, the cooling-off effects discussed below (Section 6) have no counterpart in reciprocity theory, which makes the same prediction in Figures B and D. Reciprocal players do not cool off. “*La vengeance est un plat qui se mange froid.*”

³⁰Another example is the hold-up game of Figure F. We gave conditions where $(r-n, f)$ was the unique SE. If Ann and Bob were reciprocal, $(\ell-n, g)$ and $(r-n, g)$ could be SRE, with miserable interaction, respectively, off and on the equilibrium path.

7.3 Other models of anger

Card & Dahl and Kőszegi & Rabin Card & Dahl (2011) show that reports of domestic abuse go up when football home teams favored to win lose. They argue that this is in line with Kőszegi & Rabin’s (2006, 2007, henceforth KR) theory of expectations-dependent reference points. KR model the loss felt when a player gets less than he expected, which one may think of as a form of disappointment with negative valence (cf. Bell 1985, Loomes & Sugden 1986). However, KR do not model other-regarding preferences directly: they focus on the consequences of their model for individual decisions. Our models study the social consequences of frustration: frustration results in lower weights on coplayer payoffs, and hence encourages costly punishment. Our simple anger model and the example of hammering-one’s thumb captures Card & Dahl’s result.

Another difference between this paper and KR is that in their work anticipation of the negative valence of future frustrations influences decision utility. Our decision makers are influenced by past frustrations, rather than future ones. Important modeling details then distinguish how we define frustration and how KR define loss (*e.g.*, how we cap frustration using the highest attainable payoff).

Rotemberg In a series of intriguing papers, Rotemberg explores how consumer anger shapes firms’ pricing (2005, 2011), as well as interaction in ultimatum games (2008). He proposes (versions of) a theory in which players are slightly altruistic, and consumers/responders also care about their co-players’ degrees of altruism. Namely, they abruptly become angry and punish a co-player whom they come to believe has an altruism parameter lower than some threshold. “One can thus think of individual i as acting as a classical statistician who has a null hypothesis that people’s altruism parameter is at least as large as some cutoff value. If a person acts so that i is able to reject this hypothesis, individual i gains ill-will towards this person” (Rotemberg 2008, p. 464).

As a literal statement of what upsets people, this assumption differs from our reading of the relevant psychology. Recall that frustration arises when individuals are unexpectedly denied things they care about. Matters like “own payoff” (our focus) and “fairness” or “quality of past decisions” (which we mentioned) come to mind; a co-player’s altruism being λ rather than $\lambda - \varepsilon$, where λ and ε are tiny numbers, hardly does. On the other hand, perhaps one may think of the approach as capturing a notion of scapegoating. Moreover, it is impressive how Rotemberg’s model captures the action in his data sets. It is natural to wonder whether our models could achieve that too. As regards behavior in ultimatum (and some other) games, there is already some existing evidence that is consistent with our approach; see the discussion below. Regarding pricing, we leave for empirical economists the task of exploring the topic.

Winter, Brams Winter (2014) and Winter *et al.* (2016) model anger and other emotions in games with a version of the indirect evolutionary approach:³¹ Like us, Winter *et al* assume that preferences over outcomes are “emotional” and endogenous, but we differ in the way we model emotions and make them endogenous. We assume that emotions depend on endogenous beliefs, while Winter *et al* model the rest points of an adaptation process of belief-independent preferences. Brams (2011) studies anger in sequential interactions by modeling players who take turns changing the state of a 2×2 payoff matrix and receive payoffs at the end of the game. However, like Winter’s, his model of anger is independent of beliefs, while we argue that beliefs are central to emotions.

Akerlof Akerlof (2016) models anger in a 2-person Bayesian game where the first mover decides whether or not to follow a rule, and the second mover decides whether or not to punish the first mover. In the model, Player 1’s compliance with the rule is motivated by a sense of duty. Player 2 may be sensitive to anger from noncompliance if she thinks that a “reasonable person,” modeled as a person with similar preferences to Player 2, would comply with the rule. Similarly to Rotemberg, Akerlof motivates costly punishment via preferences over others’ types. In contrast, we assume that anger and aggression arise from frustration and payoff expectations. Both Rotemberg’s and Akerlof’s approach begin with norms about behavior, and condition anger upon violation of those norms. In contrast, we develop models that capture the psychology of frustration and anger.

8 Discussion

Incorporating the effects of emotions in economic analysis is a balancing act. One wants to focus on sentiments that make empirical sense, but human psychology is multi-faceted and there is no unambiguous yardstick. Our chosen formulation provides a starting point for exploring how anger shapes interaction, and experimental or other evidence will help to assess empirical relevance and suggest revised formulas. We conclude by discussing sundry topics that may help gain perspective on, build on, or further develop our work.

Frustration Consider substituting $\mathbb{E}[\pi_i; \alpha_i] - \mathbb{E}[\pi_i|a^1; \alpha_i]$ for $F_i(a^1; \alpha_i)$ of Section 3. This alternative would measure *i*’s *actual* diminished expectations at a^1 , unlike $F_i(a^1; \alpha_i)$ which captures diminished expectations relative to what *i* believes is the most he can get (which we think of as the adequate way to capture goal-blockage). To appreciate how dramatically this change would impact behavior, consider a two-player common-interest game: Ann chooses *Out* or *In*; in the former case the game ends with payoffs (1, 1), in the latter case Bob chooses

³¹See, for example, Güth & Kliemt (1988).

between $(0, 0)$ and $(2, 2)$. *Mutatis mutandis*, for high enough θ_b , with the alternative, under SA and ABB, there is an SE where Ann chooses *Out* and Bob would go for $(0, 0)$. Following *In*, Bob would be frustrated because he (so-to-say) sees himself as locked-in with his stage-2 planned action. Our formulation of $F_i(a^1; \alpha_i)$ rules that out.

Consider a binary gamble where with probability $p > 0$ Ann wins $\$x > 0$, and otherwise gets $\$0$. Her frustration, using our definition, equals her initial expectation: $p \cdot x$. This embodies strong implications for how frustrations compare across contexts, *e.g.* the frustration of a highly expected failure to win the state lottery versus that of some unlikely small loss. We are agnostic as regards empirical relevance, but alert the reader to the issue.³²

The psychological evidence (cited in Section 1) says a player becomes frustrated when his goals are unexpectedly thwarted. We addressed but one aspect: own material rewards. Cases 1-3 indicate broad applied potential. Yet our focus is restrictive, as one may imagine other sources of frustration:

Case 4: In 2007 Apple launched its iPhone at $\$499$. Two months later they introduced a new version at $\$399$, re-priced the old model at $\$299$, and caused outrage among early adopters. Apple paid back the difference. Did this help long run profit?

Case 5: The 2008 TARP bank bail-out infuriated some US voters. Did this ignite Tea Party/Occupy-Wall Street movements?

In case 4, an early adopter is frustrated because he regrets he already bought, not because new information implies his expected rewards drop. In case 5, even an activist who is materially unaffected personally may be frustrated because of unexpected perceived unfairness. These examples are not exhaustive; further sources of frustration may *e.g.* involve shocks to self-esteem.³³ Techniques analogous to those we have developed may be applicable in these cases, but going in these directions is left for future research.

As regards the effects of frustration, we considered changes to a player's utility but neglected other plausible adjustments. Gneezy & Imas (2014) report data from an intriguing experiment involving two-player zero-sum material payoff games. In one game players gain if they are strong, in the other if they are smart. Before play starts, one subject may anger his opponent and force him to stay in the lab to do boring tasks. A thus frustrated player's performance is enhanced when strength is beneficial (possibly from increased adrenaline flow), but

³²The example involves one-player with a dummy-choice only to facilitate the frustration-calculation; interesting testable implications obviously arise more generally, *e.g.* in modified versions of the hammering one's thumb game.

³³See Baumeister, Smart & Boden (1996) for an interesting discussion linking (threatened) self-esteem and violence.

reduced when cool logic is called for (as if an angered player becomes cognitively impaired). Our model can capture the first consideration, but not the second. Specifically, we can let the consequences of actions depend also on beliefs, *e.g.*, because emotions affect strength or speed (cf. Rauh & Seccia 2006); this ultimately translates into belief-dependent utility (or cost) of actions. However, to capture the second effect, we would need a theory of endogenous cognitive abilities.

Valence and action-tendency Psychologists classify emotions in multiple ways. Two prominent aspects are **valence**, the intrinsic pleasantness or aversiveness of an emotion, and **action-tendency**, or how behavior is shaped as the emotion occurs. Both notions have bearing on anger. For example, most psychologists believe anger has negative valence (see, *e.g.*, Harmon-Jones & Sigelman 2001, p. 978). Perhaps such considerations steer people to avoid frustrations, say by not investing in the stock market. That said, the distinguishing feature of anger that psychologists stress concerns its action-tendency of aggression, not its valence. In developing our theory, we have exaggerated this, abstracting away from frustration avoidance, while emphasizing frustration-induced aggression. This is reflected in the decision utility functions, which are shaped by current frustration, but not by the anticipation of the negative valence of future frustrations.³⁴

Blame We explored various ways a player may blame others, but other notions are conceivable. For example, with anger from blaming behavior i 's blame of j depends on what i believes he would truly get at counterfactual histories, rather than the most he could get there. We view this modeling choice as reflecting local agency; i 's current agent views other agents of i as uncontrollable, and he has no direct care for their frustrations. Another example relates to how we model anger from blaming intentions: i 's blame of j depends on β_i , his second-order beliefs. Recall that the interpretation concerns beliefs about beliefs about material payoffs, not beliefs about beliefs about frustration, which would be third- rather than second-order beliefs. Battigalli & Dufwenberg (2007), in a context which concerned guilt rather than anger, worked with such a notion.

Our blame concepts one way or another assess the *marginal* impact of other players. For example, consider a game where i exits a building while all $j \in I \setminus \{i\}$, unexpectedly to i , simultaneously hurl buckets of water at i , who gets soaked. According to our approach, i cannot blame any j as long as there are at least two hurlers. One could imagine alternatives where

³⁴In previous work we modeled another emotion: guilt; see, *e.g.*, Battigalli & Dufwenberg (2007), Chang, Smith, Dufwenberg & Sanfey (2011). To gain perspective note that in that work our approach to anticipation of valence and action-tendency was reversed. Guilt has valence (negative!) as well as action-tendency (say to engage in "repair behavior"; see, *e.g.*, Silver 2007). In modeling guilt we highlighted the anticipation of its negative valence while neglecting action-tendencies.

i blames, say, all the hurlers on the grounds that *collectively* they could thwart i 's misery, or where i splits the blame among all the hurlers. Halpern (2016, Chapter 6) explores these issues and develops a model that assigns positive blame even when outcomes are overdetermined, as above.

People may also blame others in unfair ways, *e.g.* nominating scapegoats. Our notions of SA and ABB may embody related notions to some degree, but it has not been our intention to address such concerns systematically.

Several recent experiments explore interesting aspects of blame (Bartling & Fischbacher 2012, Gurdal *et al.* 2014, Celen *et al.* 2017). We emphasize that our focus on blame is restricted to its relation to frustration, not on reasons besides frustration that may lead people to blame each other.³⁵

Anger management People aware of their inclination to be angry may attempt to manage or contain their anger. Our players anticipate how frustrations shape behavior, and they may avoid or seek certain subgames because of that. However, there are interesting related phenomena we do not address: Can i somehow adjust θ_i say by taking an “anger management class?” If so, would rational individuals want to raise, or to lower, their θ_i ? How might that depend on the game forms they play? These are potentially relevant questions related to how we have modeled action-tendency. Further issues would arise if we were to consider aspects involving anticipated negative valence of future frustrations, or bursts of anger.

Experimental testing Our models tell stories of what happens when players prone to anger interact. It is natural to wonder about empirical relevance, and here experiments may be helpful.

A few experiments that measure beliefs, emotions, and behavior together provide support for the notion that anger and costly punishment result from outcomes which do not meet expectations. Pillutla & Murnighan (1996) find that reported anger predicted rejections better than perceived unfairness in ultimatum games. Fehr & Gächter (2002) elicit self-reports of the level of anger towards free riders in a public goods game, concluding that negative emotions including anger are the proximate cause of costly punishment.

Several other studies connect unmet expectations and costly punishment in ultimatum games. Falk, Fehr, & Fischbacher (2003) measure beliefs and behavior in ultimatum minigames. They report higher proportions of rejections of disadvantageous offers when responders' ex-

³⁵For example, Celen *et al.* (2014) present a model where i asks how he would have behaved had he been in j 's position and had j 's beliefs. Then i blames j if j appears to be less generous to i than i would have been, and may blame j even if i is not surprised/frustrated. Or imagine a model where players blame those considered unkind, as defined in reciprocity theory (cf. subsection below), independently of frustration.

pected payoffs are higher, consistent with our models.³⁶ Schotter & Sopher (2007) also measure second-mover expectations, concluding that unfulfilled expectations drive rejections of low offers. Similarly, Sanfey (2009) finds that psychology students who are told that a typical offer in the ultimatum game is \$4-\$5 reject low offers more frequently than students who are told that a typical offer is \$1-\$2.

A series of papers by Frans van Winden and coauthors records both emotions and expectations in the power-to-take game (which resembles ultimatum games, but allows for partial rejections).³⁷ Second-mover expectations about first-mover “take rates” are a key factor in the decision to destroy income, and anger-like emotions are triggered by the difference between expected and actual take rates. The difference between the actual and reported “fair” take rate is not significant in determining anger-like emotions, suggesting that deviations from expectations, rather than from fairness benchmarks, drive both anger and the destruction of endowments in the games.

Apropos the cooling off effects discussed in Section 6, Grimm & Mengel (2011) run ultimatum games that force some responders to wait ten minutes before making their choice. Without delay, less than 20% of low offers were accepted while 60–80% were accepted if the acceptance decision were delayed.

A literature in neuroscience connects expectations with social norms to study the neural underpinnings of emotional behavior. In Xiang, Lohrenz & Montague (2013), subjects respond to a sequence of ultimatum game offers whilst undergoing fMRI imaging. Unbeknownst to subjects, the experimenter controls the distribution of offers in order to manipulate beliefs. Rejections occur more often when subjects expect high rather than low offers. The authors connect norm violations (i.e., lower than expected offers) with reward prediction errors from reinforcement learning, which are known to be the computations instantiated by the dopaminergic reward system. Xiang *et al.* note that “when the expectation (norm) is violated, these error signals serve as control signals to guide choices. They may also serve as the progenitor of subjective feelings.”

It would be useful to develop tests specifically designed to target key features of our theory. For example, which version—SA, ABB, ABI—seems more empirically relevant, and how does the answer depend on context (*e.g.*, is SA more relevant for tired subjects)? Some insights may again be gleaned from existing studies. For example, Gurdal *et al.* (2014) study games where an agent invests on behalf of a principal, choosing between a safe outside option and a risky alternative. If the latter is chosen, then it turns out that many principals punish the agent if and only if by chance a poor outcome is realized. This seems to indicate some relevance of our ABB solution (relative to ABI). That said, Gurdal *et al.*’s intriguing design

³⁶See the data in Figure 2 and Table 1 of their paper. Brandts & Solá (2001) find similar behavioral results (see Table 1). Compare also with our discussion of Example 3 in Section 4.

³⁷Bosman & van Winden (2002), Bosman, Sutter & van Winden (2005), Reuben & van Winden (2008).

is not tailored to specifically test our theory (and beliefs and frustrations are not measured), so more work seems needed to draw clearer conclusions.

One study, Persson (2017), is directly motivated as a test of simple anger. Persson looks at the Hammering one’s thumb game, and documents that frustrations occur much as predicted and yet no punishments occur. His results thus favor ABB or ABI over SA in that context.

Our models are abstractions. We theorize about the consequences of anger while neglecting myriad other obviously important aspects of human motivation (say altruism, warm glow, inequity aversion, reciprocity, social status, or emotions like guilt, disappointment, regret, or anxiety). Our models are not intended to explain every data pattern, but rather to highlight the would-be consequences of anger, if anger were the only form of motivation at play (in addition to concern for material payoffs). This statement may seem trivially obvious, but it has subtle implications for how to evaluate experimental work. Experimental data may find that one of the forms of motivation that our theory abstracts away from affects subjects’ choices. However, to reject our theory, it may be more relevant to ask if behaviors that are in fact driven by anger (as measured by, *e.g.*, emotion self-reports, physiological activity, or both, as in Chang *et al.* 2011) diverge from our predictions. If they were, that might indicate that our theory could benefit from revision.

Applications Formulating, motivating, and elucidating the key definitions of our models is more than a mouthful, so we have not taken this paper in the direction of doing applied economics. Make no mistake about it though, the hope that our models will prove useful for such work has been a primary driving force. Our psychologically grounded models of frustration, anger, and blame may shed light on many of the themes (e.g. pricing, violence, politics, recessions, haggling, terror, and traffic) that we listed at the start of this paper. We hope to do some work in these directions ourselves.

A Appendix

This appendix contains proofs of the results stated in the main text.

A.1 Preliminaries

To ease exposition, some of the key definitions and equations contained in the main text are repeated below.

For each topological space X , we let $\Delta(X)$ denote the space of Borel probability measures on X endowed with the topology of weak convergence of measures. Every Cartesian product of topological spaces is endowed with the product topology. A topological space X is metrizable

if there is a metric that induces its topology. A Cartesian product of a countable (finite, or denumerable) collection of metrizable spaces is metrizable.

$\Delta_i^1 \subseteq \times_{h_i \in H_i} \Delta(Z(h_i))$ is the set of first-order beliefs, that is, the set of $\alpha_i = (\alpha_i(\cdot|Z(h_i)))_{h_i \in H_i}$ such that:

- for all $h_i, h'_i \in H_i$, if $h_i \prec h'_i$ then for every $Y \subseteq Z(h'_i)$

$$\alpha_i(Z(h'_i)|Z(h_i)) > 0 \Rightarrow \alpha_i(Y|Z(h'_i)) = \frac{\alpha_i(Y|Z(h_i))}{\alpha_i(Z(h'_i)|Z(h_i))}; \quad (15)$$

- for all $h \in H$, $a_i \in A_i(h)$, $a_{-i} \in A_{-i}(h)$ (using obvious abbreviations)

$$\alpha_{i,-i}(a_{-i}|h) = \alpha_{i,-i}(a_{-i}|h, a_i). \quad (16)$$

$\Delta_i^2 \subseteq \times_{h_i \in H_i} \Delta(Z(h_i) \times \Delta_{-i}^1)$ —where $\Delta_{-i}^1 = \times_{j \neq i} \Delta_j^1$ — is the set of second-order beliefs, that is, the set of $\beta_i = (\beta_i(\cdot|h_i))_{h_i \in H_i}$ such that:

- if $h_i \prec h'_i$ then

$$\beta_i(h'_i|h_i) > 0 \Rightarrow \beta_i(E|h'_i) = \frac{\beta_i(E|h_i)}{\beta_i(h'_i|h_i)} \quad (17)$$

for all $h_i, h'_i \in H_i$ and every event $E \subseteq Z(h'_i) \times \Delta_{-i}^1$;

- i 's beliefs satisfy an own-action independence property:

$$\beta_i(Z(h, (a_i, a_{-i})) \times E_\Delta | (h, a_i)) = \beta_i(Z(h, (a'_i, a_{-i})) \times E_\Delta | (h, a'_i)), \quad (18)$$

for every $h \in H$, $a_i, a'_i \in A_i(h)$, $a_{-i} \in A_{-i}(h)$, and (measurable) $E_\Delta \subseteq \Delta_{-i}^1$. The space of second-order beliefs of i is denoted Δ_{-i}^2 .

Note that (16) and (18) are given by equalities between marginal measures (on $A_{-i}(h)$ and $A_{-i}(h) \times \Delta_{-i}^1$ respectively).

Lemma 1 *For each player $i \in I$, Δ_i^2 is a compact metrizable space.*

Proof Let Θ be a non-empty, compact metrizable space. Lemma 1 in Battigalli & Siniscalchi (1999) (B&S) establishes that the set of arrays of probability measures $(\mu(\cdot|h_i))_{h_i \in H_i} \in \times_{h_i \in H_i} \Delta(Z(h_i) \times \Theta)$ such that

$$h_i \prec h'_i \wedge \mu(h'_i|h_i) > 0 \Rightarrow \mu(E|h'_i) = \frac{\mu(E|h_i)}{\mu(h'_i|h_i)}$$

is closed. Note that, in the special case where Θ is a singleton, each $\Delta(Z(h_i) \times \Theta)$ is isomorphic to $\Delta(Z(h_i))$; hence, the set of first-order beliefs satisfying (15) is closed. Letting $\Theta = \Delta_{-i}^1$, we obtain that the set of second-order beliefs satisfying (17) is closed.

Since $\times_{h_i \in H_i} \Delta(Z(h_i))$ is a compact subset of a Euclidean space and eq. (16) is a closed condition (equalities between marginal measures are preserved in the limit), Lemma 1 in B&S implies that Δ_i^1 is a closed subset of a compact metrizable space. Hence, Δ_i^1 is a compact metrizable space.

It is well known that if X_1, \dots, X_K are compact metrizable, so is $\times_{k=1}^K \Delta(X_k)$ (see Aliprantis & Border 2006, Theorem 15.11). Hence, by Lemma 1 in B&S, the set of second-order beliefs satisfying (17) is a closed subset of a compact metrizable space. Since eq. (18) is a closed condition (equalities between marginal measures are preserved in the limit), this implies that Δ_i^2 is compact metrizable. ■

Lemma 2 *For each profile of behavioral strategies $\sigma = (\sigma_i)_{i \in I}$ there is a unique profile of second-order beliefs $\beta^\sigma = (\beta_i^\sigma)_{i \in I}$ such that (σ, β^σ) is a consistent assessment. The map $\sigma \mapsto \beta^\sigma$ is continuous.*

Proof Write $\mathbb{P}^\sigma(h'|h)$ for the probability of reaching h' from h , e.g.,

$$\mathbb{P}^\sigma(a^1, a^2 | \emptyset) = \left(\prod_{j \in I} \sigma_j(a_j^1 | \emptyset) \right) \left(\prod_{j \in I} \sigma_j(a_j^2 | a^1) \right).$$

Define α_i^σ as $\alpha_i^\sigma(z|h) = \mathbb{P}^\sigma(z|h)$ for all $i \in I$, $h \in H$, and $z \in Z$. Define β^σ as $\beta_i^\sigma(\cdot|h) = \alpha_i^\sigma(\cdot|h) \times \delta_{\alpha_{-i}^\sigma}$ for all $i \in I$, $h \in H$. It can be checked that (1) $\beta^\sigma \in \Delta_i^2$ for each $i \in I$, (2) (σ, β^σ) is a consistent assessment, and (3) if $\beta \neq \beta^\sigma$, then either (a) or (b) of the definition of consistency is violated. It is also apparent from the construction that the map $\sigma \mapsto \beta^\sigma$ is continuous, because $\sigma \mapsto \alpha^\sigma$ is obviously continuous, and the Dirac-measure map $\alpha_{-i} \mapsto \delta_{\alpha_{-i}}$ is continuous. ■

Lemma 3 *The set of consistent assessments is compact.*

Proof Lemma 1 implies that $\times_{i \in I} (\Sigma_i \times \Delta_i^2)$ is a compact metrizable space that contains the set of consistent assessments. Therefore, it is enough to show that the latter is closed. Let $(\sigma^n, \beta^n)_{n \in \mathbb{N}}$ be a converging sequence of consistent assessments with limit $(\sigma^\infty, \beta^\infty)$. For each $i \in I$, let α_i^n be the first-order belief derived from β_i^n ($n \in \mathbb{N} \cup \{\infty\}$), that is,

$$\alpha_i^n(Y|h) = \beta_i^n(Y \times \Delta_{-i}^1 | h)$$

for all $h \in H$ and $Y \subseteq Z(h)$. By consistency, for all $n \in \mathbb{N}$, $i \in I$, $h \in H$, $a \in A(h)$, it holds that

- (a.n) $\alpha_i^n(a|h) = \beta_i^n(Z(h, a) \times \Delta_{-i}^1|h) = \prod_{j \in I} \sigma_j^n(a_j|h)$,
- (b.n) $\text{marg}_{\Delta_{-i}^1} \beta_i^n(\cdot|h) = \delta_{\alpha_{-i}^n}$, where each α_j^n is determined as in (a.n).

Then,

$$\alpha_i^\infty(a|h) = \beta_i^\infty(Z(h, a) \times \Delta_{-i}^1|h) = \prod_{j \in I} \sigma_j^\infty(a_j|h)$$

for all $i \in I$, $h \in H$, $a \in A(h)$. Furthermore, $\text{marg}_{\Delta_{-i}^1} \beta_i^\infty(\cdot|h) = \delta_{\alpha_{-i}^\infty}$ for all $i \in I$ and $h \in H$, because $\alpha_{-i}^n \rightarrow \alpha_{-i}^\infty$ and the marginalization and Dirac maps $\beta_i \mapsto \text{marg}_{\Delta_{-i}^1} \beta_i$ and $\alpha_{-i} \mapsto \delta_{\alpha_{-i}}$ are continuous. ■

A.2 Proof of Proposition 1

Fix $i \in I$ arbitrarily. First-order belief α_i is derived from β_i and, by consistency, gives the behavioral strategy profile σ . Therefore, by assumption each $h' \preceq h$ has probability one under α_i , which implies that $\mathbb{E}[\pi_i|h'; \alpha_i] = \mathbb{E}[\pi_i; \alpha_i]$, hence $F_i(h'; \alpha_i) = 0$. Since blame is capped by frustration, $u_i(h', a'_i; \beta_i) = \mathbb{E}[\pi_i|h'; \alpha_i]$. Therefore, sequential rationality of the equilibrium assessment implies that $\text{Supp} \sigma_i(\cdot|h') \subseteq \arg \max_{a'_i \in A_i(h')} \mathbb{E}[\pi_i|h'; \alpha_i]$. If there is randomization only in the last stage (or none at all), then players maximize locally their expected material payoff on the equilibrium path. Hence, the second claim follows by inspection of the definitions of agent form of the material-payoff game and Nash equilibrium. ■

A.3 Proof of Proposition 2

Let $(\bar{\sigma}, \bar{\beta}) = (\bar{\sigma}_i, \bar{\beta}_i)_{i \in I}$ be the SE of the material payoff game, which is in pure strategies by the perfect information assumption. Fix decision utility functions $u_i(h, a_i; \cdot)$ of the ABI, or ABB kind, and a sequence of real numbers $(\varepsilon_n)_{n \in \mathbb{N}}$, with $\varepsilon_n \rightarrow 0$ and $0 < \varepsilon_n < \frac{1}{\max_{i \in I, h \in H} |A_i(h)|}$ for all $n \in \mathbb{N}$. Consider the constrained psychological game where players can choose mixed actions in the following sets:

$$\Sigma_i^n(h) = \{\sigma_i(\cdot|h) \in \Delta(A_i(h)) : \|\sigma_i(\cdot|h) - \bar{\sigma}_i(\cdot|h)\| \leq \varepsilon_n\}$$

if h is on the $\bar{\sigma}$ -path, and

$$\Sigma_i^n(h) = \{\sigma_i(\cdot|h) \in \Delta(A_i(h)) : \forall a_i \in A_i(h), \sigma_i(a_i|h) \geq \varepsilon_n\}$$

if h is off the $\bar{\sigma}$ -path. By construction, these sets are non-empty, convex, and compact. Since the decision utility functions are continuous, and the consistent assessment map $\sigma \mapsto \beta^\sigma$ is continuous (Lemma 2), correspondence

$$\sigma \mapsto \times_{h \in H} \times_{i \in I} \arg \max_{\sigma'_i(\cdot|h) \in \Sigma_i^n(h)} \sum_{a_i \in A_i(h)} \sigma'_i(a_i|h) u_i(h, a_i; \beta_i^\sigma)$$

is upper-hemicontinuous, non-empty, convex, and compact valued; therefore (by Kakutani's theorem), it has a fixed point σ^n . By Lemma 3, the sequence of consistent assessments $(\sigma^n, \beta^{\sigma^n})_{n=1}^\infty$ has a limit point (σ^*, β^*) , which is consistent too. By construction, $\bar{\sigma}(\cdot|h) = \sigma^*(\cdot|h)$ for h on the $\bar{\sigma}$ -path, therefore $(\bar{\sigma}, \bar{\beta})$ and (σ^*, β^*) are realization-equivalent. We let $\bar{\alpha}_i$ (respectively, α_i^*) denote the first-order beliefs of i implied by $(\bar{\sigma}, \bar{\beta})$ (respectively, (σ^*, β^*)).

We claim that the consistent assessment (σ^*, β^*) is an SE of the psychological game with decision utility functions $u_i(h, a_i; \cdot)$. We must show that (σ^*, β^*) satisfies sequential rationality. If h is off the $\bar{\sigma}$ -path, sequential rationality is satisfied by construction. Since $\bar{\sigma}$ is deterministic and there are no chance moves, if h is on the $\bar{\sigma}$ -path (*i.e.*, on the σ^* -path) it must have unconditional probability 1 according to each player's beliefs and there cannot be any frustration; hence, $u_i(h, a_i; \beta_i^*) = \mathbb{E}[\pi_i|h, a_i; \alpha_i^*]$ ($i \in I$) where α_i^* is determined by σ^* . If, furthermore, it is the second stage ($h = \bar{a}^1$, with $\bar{\sigma}(\bar{a}^1|\emptyset) = 1$), then —by construction— $\mathbb{E}[\pi_i|h, a_i; \alpha_i^*] = \mathbb{E}[\pi_i|h, a_i; \bar{\alpha}_i]$, where $\bar{\alpha}_i$ is determined by $\bar{\sigma}$. Since $\bar{\sigma}$ is an SE of the material-payoff game, sequential rationality is satisfied at h . Finally, we claim that (σ^*, β^*) satisfies sequential rationality also at the root $h = \emptyset$. Let $\iota(h)$ denote the active player at h . Since $\iota(\emptyset)$ cannot be frustrated at \emptyset , we must show that action \bar{a}^1 with $\bar{\sigma}(\bar{a}^1|\emptyset) = 1$ maximizes his expected material payoff given belief $\alpha_{\iota(\emptyset)}$. According to ABB and ABI, player $\iota(a^1)$ can only blame the first mover $\iota(\emptyset)$ and possibly hurt him, if he is frustrated. Therefore, in assessment (σ^*, β^*) at node a^1 , either $\iota(a^1)$ plans to choose his (unique) payoff maximizing action, or he blames $\iota(\emptyset)$ strongly enough to give up some material payoff in order to bring down the payoff of $\iota(\emptyset)$. Hence, $\mathbb{E}[\pi_{\iota(\emptyset)}|a^1; \alpha_{\iota(a^1)}^*] \leq \mathbb{E}[\pi_{\iota(\emptyset)}|a^1; \bar{\alpha}_{\iota(a^1)}]$ (anger). By consistency of (σ^*, β^*) and $(\bar{\sigma}, \bar{\beta})$, $\alpha_{\iota(a^1)}^* = \alpha_{\iota(\emptyset)}^*$ and $\bar{\alpha}_{\iota(a^1)} = \bar{\alpha}_{\iota(\emptyset)}$ (cons.). Since (σ^*, β^*) is realization-equivalent to $(\bar{\sigma}, \bar{\beta})$ (r.e.), which is the material-payoff equilibrium (m.eq.), for each $a^1 \in A(\emptyset)$,

$$\begin{aligned} \mathbb{E}[\pi_{\iota(\emptyset)}|\bar{a}^1; \alpha_{\iota(\emptyset)}^*] &\stackrel{\text{(r.e.)}}{=} \mathbb{E}[\pi_{\iota(\emptyset)}|\bar{a}^1; \bar{\alpha}_{\iota(\emptyset)}] \stackrel{\text{(m.eq.)}}{\geq} \\ \mathbb{E}[\pi_{\iota(\emptyset)}|a^1; \bar{\alpha}_{\iota(\emptyset)}] &\stackrel{\text{(cons.)}}{=} \mathbb{E}[\pi_{\iota(\emptyset)}|a^1; \bar{\alpha}_{\iota(a^1)}] \stackrel{\text{(anger)}}{\geq} \\ \mathbb{E}[\pi_{\iota(\emptyset)}|a^1; \alpha_{\iota(a^1)}^*] &\stackrel{\text{(cons.)}}{=} \mathbb{E}[\pi_{\iota(\emptyset)}|a^1; \alpha_{\iota(\emptyset)}^*]. \end{aligned}$$

This completes the proof for the ABB and ABI cases. If there are only two players, then we have a leader-follower game and SA is equivalent to ABB (Remark 1 of “Frustration and Anger in Games”), so (σ^*, β^*) is an SE in this case too. ■

A.4 Proof of Proposition 3

Let ℓ and f respectively denote the leader and the follower. To ease notation, let us assume that the leader has no terminating action, that is, each terminal history has length 2 (this is without loss of generality because $A_f(a_\ell)$ may be a singleton for some a_ℓ). Since a two-person

leader-follower game form has perfect information, its unique material-payoff SE, viz. $(\bar{\sigma}_\ell, \bar{\sigma}_f)$, is also its unique pure-strategy subgame perfect equilibrium: for each $a_\ell \in A_\ell(\emptyset)$, let

$$\bar{\mathbf{a}}_f(a_\ell) = \arg \max_{a_f \in A(a_f)} \pi_f(a_\ell, a_f)$$

denote the material best reply of f to a_ℓ (unique by assumption); then

$$\begin{aligned} \forall a_\ell \in A_\ell(\emptyset), \bar{\sigma}_f(\bar{\mathbf{a}}_f(a_\ell) | a_\ell) &= 1, \\ \bar{\sigma}_\ell \left(\arg \max_{a_\ell \in A_\ell(\emptyset)} \pi_\ell(a_\ell, \bar{\mathbf{a}}_f(a_\ell)) | \emptyset \right) &= 1 \end{aligned} \quad (19)$$

(the argmax is a singleton by assumption). Let $(\bar{a}_\ell, \bar{\mathbf{a}}_f)$ denote this pure-strategy equilibrium. We must prove that $\mathbb{E}[\pi_f; \beta] \geq \pi_f(\bar{a}_\ell, \bar{\mathbf{a}}_f(\bar{a}_\ell))$ for every SE assessment (σ, β) .

Now, fix arbitrarily an SE (σ, β) , thus, by consistency, σ is derived from the first-order beliefs implied by β . Observe that

$$\forall a_\ell \in A_\ell(\emptyset) \setminus \{\bar{a}_\ell\}, \mathbb{E}[\pi_\ell | a_\ell; \beta] \leq \pi_\ell(a_\ell, \bar{\mathbf{a}}_f(a_\ell)) < \pi_\ell(\bar{a}_\ell, \bar{\mathbf{a}}_f(\bar{a}_\ell)). \quad (20)$$

The first inequality holds because either f —upon observing a_ℓ —is frustrated enough to deviate from his material-payoff maximizing action $\bar{\mathbf{a}}_f(a_\ell)$ and punish f , or he replies with $\bar{\mathbf{a}}_f(a_\ell)$; the second inequality holds because $(\bar{a}_\ell, \bar{\mathbf{a}}_f)$ is the unique material-payoff equilibrium. Since in every equilibrium the leader, who cannot be frustrated, maximizes his expected material payoff, it must be the case that either (i) $\sigma_\ell(\bar{a}_\ell | \emptyset) = 1$, or (ii) $\mathbb{E}[\pi_\ell | \bar{a}_\ell; \beta] < \pi_\ell(\bar{a}_\ell, \bar{\mathbf{a}}_f(\bar{a}_\ell))$, otherwise eq. (20) implies that he would choose \bar{a}_ℓ with probability 1. In case (i), f cannot be frustrated after \bar{a}_ℓ ; hence, $\sigma_f(\bar{\mathbf{a}}_f(\bar{a}_\ell) | \bar{a}_\ell) = 1$, $\sigma_\ell(\bar{a}_\ell | \emptyset) = 1$, and the equilibrium payoff of f is $\pi_f(\bar{a}_\ell, \bar{\mathbf{a}}_f(\bar{a}_\ell))$, the same as in the material-payoff equilibrium. In case (ii), $\sigma_f(\bar{\mathbf{a}}_f(\bar{a}_\ell) | \bar{a}_\ell) < 1$, that is, f is not choosing his material-payoff maximizing action $\bar{\mathbf{a}}_f(\bar{a}_\ell)$ because he is frustrated; therefore,

$$\mathbb{E}[\pi_f; \beta] > \mathbb{E}[\pi_f | \bar{a}_\ell; \beta] > \pi_f(\bar{a}_\ell, \bar{\mathbf{a}}_f(\bar{a}_\ell)).$$

where the first inequality is implied by frustration upon observing \bar{a}_ℓ , and the second inequality follows from consistency of σ with β and $\sigma_f(\bar{\mathbf{a}}_f(\bar{a}_\ell) | \bar{a}_\ell) < 1$. Thus, in each case $\mathbb{E}[\pi_f; \beta] \geq \pi_f(\bar{a}_\ell, \bar{\mathbf{a}}_f(\bar{a}_\ell))$. ■

A.5 Proof of Proposition 4

We denote the leader by $\iota(\emptyset)$. Let $(\sigma_i, \beta_i)_{i \in I}$ be an SE under ABB/SA with parameter profile $(\theta_i)_{i \in I}$, and suppose that the leader's strategy has full support: $\text{Supp} \sigma_{\iota(\emptyset)}(\cdot | \emptyset) = A_{\iota(\emptyset)}(\emptyset)$. Construct a polymorphic consistent assessment $\bar{\lambda}$ as follows: For each follower i , $T_i(\bar{\lambda}_i) = \{t_i\}$

(a singleton) and $(\bar{\sigma}_{t_i}, \bar{\beta}_{t_i}) = (\sigma_i, \beta_i)$. For the leader $\iota(\emptyset)$, $T_{\iota(\emptyset)}(\bar{\lambda}_{\iota(\emptyset)}) = A_{\iota(\emptyset)}(\emptyset)$, and, for each type $a_{\iota(\emptyset)}$, $\bar{\sigma}_{a_{\iota(\emptyset)}}(a_{\iota(\emptyset)}|\emptyset) = 1$ and $\bar{\alpha}_{a_{\iota(\emptyset)}}(\cdot|a^1) = \prod_{i \in I} \sigma_i(\cdot|a^1)$ for all non-terminal a^1 , where $\bar{\alpha}_{a_{\iota(\emptyset)}}$ is the first-order belief derived from $\bar{\beta}_{a_{\iota(\emptyset)}}$. By construction, each type of leader is indifferent, because the leader (who acts as-if selfish, cf. Remark 2) is indifferent in the original assessment $(\sigma_i, \beta_i)_{i \in I}$. As for the followers, they have the same first-order beliefs, hence the same second-stage frustrations as in $(\sigma_i, \beta_i)_{i \in I}$. Under ABB/SA, blame always equals frustration in leader-followers games. As for ABI, Bayes' rule implies that, after observing $a^1 = a_{\iota(\emptyset)}$, each follower becomes certain that the leader indeed planned to choose $a_{\iota(\emptyset)}$ with probability one, and blame equals frustration in this case too. Therefore, the incentive conditions of the followers hold in $\bar{\lambda}$ as in $(\sigma_i, \beta_i)_{i \in I}$ for all kinds of decision utility (ABI, ABB, SA) under the same parameter profile $(\theta_i)_{i \in I}$. ■

A.6 Proof of Proposition 5

Fix $h \in H$. We consider the following simple extension of could-have-been blame in multistage games under fast play:

$$B_{ij}(h; \alpha_i) = \min \left\{ \left[\max_{h' \prec h, a'_j \in A_j(h')} \mathbb{E} [\pi_i(h', a'_j); \alpha_i] - \mathbb{E}[\pi_i|h; \alpha_i] \right]^+, F_i(h; \alpha_i) \right\}. \quad (21)$$

We must show that $B_{ij}(h; \alpha_i) = 0$ if j is not active at any $h' \prec h$, and $B_{ij}(h; \alpha_i) = F_i(h; \alpha_i)$ if j is the only active player at each $h' \prec h$.

First note that if j was never active before, then $A_j(h')$ is a singleton for each $h' \prec h$, hence the term in brackets of (14) is zero. Next suppose that i is frustrated at h and j was the only active player in the past. Then there must be some $\bar{h} \prec h$ such that j deviated from i 's expectations $\alpha_i(\cdot|\bar{h})$ for the first time, that is, \bar{h} is the shortest predecessor $h' \prec h$ such that $\alpha_j(a'_j|h') < 1$ for $(h', a'_j) \preceq h$. Such \bar{h} must have probability one according to the initial belief $\alpha_i(\cdot|\emptyset)$, thus $\mathbb{E}[\pi_i|\bar{h}; \alpha_i] = \mathbb{E}[\pi_i; \alpha_i]$. Since $\max_{a'_j \in A_j(\bar{h})} \mathbb{E} [\pi_i(\bar{h}, a'_j); \alpha_i] \geq \mathbb{E}[\pi_i|\bar{h}; \alpha_i]$, we have $\max_{a'_j \in A_j(\bar{h})} \mathbb{E} [\pi_i(\bar{h}, a'_j); \alpha_i] \geq \mathbb{E}[\pi_i; \alpha_i]$. Therefore

$$\begin{aligned} & \max_{h' \prec h, a'_j \in A_j(h')} \mathbb{E} [\pi_i(h', a'_j); \alpha_i] - \mathbb{E}[\pi_i|h; \alpha_i] \\ & \geq \max_{a'_j \in A_j(\bar{h})} \mathbb{E} [\pi_i(\bar{h}, a'_j); \alpha_i] - \mathbb{E}[\pi_i|h; \alpha_i] \\ & \geq \mathbb{E}[\pi_i; \alpha_i] - \mathbb{E}[\pi_i|h; \alpha_i] \\ & \geq \mathbb{E}[\pi_i; \alpha_i] - \max_{a_i \in A_i(h)} \mathbb{E}[\pi_i|(h, a_i); \alpha_i] = F_i(h; \alpha_i), \end{aligned}$$

which implies $B_{ij}(h; \alpha_i) = F_i(h; \alpha_i)$ according to (21). ■

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