Agreements with Reciprocity: Co-financing and MOUs*

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Abstract

Institutions for co-financing agreements often exist to encourage public good investment. Can such frameworks deliver maximal investment when agents are motivated by reciprocity? We demonstrate that indeed they can, but not how one might expect. If maximal investment is impossible in the absence of the institution and public good returns are high, then an agreement signed by all parties cannot lead to full investment. However, if all parties reject the agreement, then full investment is attainable via a gentlemen's agreement or memorandum of understanding (MOU). Agreement institutions may thus do more than just facilitate the signing of binding agreements; they may play a critical role in igniting informal cooperation underpinned by reciprocity.

Keywords: Co-financing agreements, gentlemen's agreements, MOUs, public goods, reciprocity.

JEL codes: C72, D03, F53, H41

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1 Introduction

Institutions play an important role in creating the conditions for investment in public goods. Among other things, they facilitate the negotiation and enforcement of binding agreements. One common type of agreement is a co-financing, or cost-sharing, agreement; signatories make a binding commitment to co-finance each other's future public good investments. The agreement does not commit a signatory to invest in public goods per se. However, should any signatory initiate a public good investment, its co-signatories are committed to share the cost.[1] Such agreements have been used to finance critical investment in public goods, ranging from disease eradication to climate change mitigation.[2]

Theoretically, co-financing agreements can increase public good investment (cf. Varian 1994).[3] This is because a signatory can be pivotal in inducing other signatories to invest in public goods, as only with its participation would the private cost of a public good be less than the private benefit. However, full investment remains impossible as if there are many signatories, an individual signatory is no longer pivotal, thus it deviates to not signing and not investing.

These insights rely on the assumption that agents care only about their material payoffs. Yet behaviour in public good contexts often exhibits conditional cooperation (e.g. Fischbacher et al. 2001), cooperating only if others do. Such behaviour can be rationalised using reciprocity theory (Rabin 1993, Dufwenberg and Kirchsteiger 2004 (D&K), Falk and Fischbacher 2006). It describes agents as having a desire to be kind to those who are kind to them, and unkind to those who are unkind to them. For example, if agent A invests, agent B may view A as kind and invest himself.

[1] These agreements are often politically more feasible than binding commitments to actually invest in public goods.

[2] In April 2016 The World Bank and The Asian Infrastructure Investment Bank signed a co-financing agreement focusing on water, transport and energy. Each party contributed $216 million to the first project, upgrading slums in Indonesia: www.brettonwoodsproject.org/2016/06/world-bank-and-aiib-signs-joint-co-financing-agreement. Such agreements are also signed by private companies. The Asian Development Bank, for instance, has an agreement with Chevron to invest in IT, construction and engineering education: www.adb.org/site/cofinancing/partners. One area where cost-sharing agreements are extensively used is in R&D investments (Katz 1986).

[3] Indeed higher investment is observed in related experimental games (Andreoni and Varian 1999, Falkinger et al. 2000 and Charness et al. 2007.)

The implications of reciprocity for public good provision are both well established (e.g. Sugden 1984) and straightforward. If agents care enough about reciprocity, maximal investment is possible, otherwise it is not. Such investment is not due to a formal agreement to invest, but rather an informal one (referred to as a gentlemen's agreement, a tacit agreement or a memorandum of understanding (MOU), for example).[4] By contrast, little is known about the implications of reciprocity for formal agreements over public goods.

An obvious question follows: How do co-financing agreements perform under reciprocity? More specifically, one may wonder: Can a co-financing mechanism deliver full investment? Does such investment follow if all players sign the agreement? Is it impossible if all players reject the agreement? To answer these questions, we apply D&K's model of reciprocity to an agreements game where players choose whether or not to sign a cost-sharing agreement, then play a public good game. We find that if in the absence of the mechanism, full investment via a MOU is impossible and the public good return is high, then such investment remains impossible if all players sign. However, if all players reject the agreement, then full investment becomes attainable via a MOU.

For some intuition, consider the interaction of kindness and co-financing agreements. Roughly, D&K say that agent i is kind to agent j if i could have given j a much lower payoff by changing his behaviour. Agent i deviating from a situation where all players sign and invest does reduce j's payoff, but not by much, as the cost-sharing agreement still has many signatories thus provides large investment incentives. By contrast, if i deviates from a situation where no-one signs and all invest, j's payoff is reduced considerably as there is no such cost-sharing agreement. Kindness and hence reciprocity incentives to invest in public goods are thus larger when there are no signatories than when there are many.

Our results provide several important insights. First, the existence of an institution for making binding agreements is potentially critical for triggering informal cooperation via MOUs and other informal agreements. Second, since our main result exemplifies a more general point that "high investment is possible with few signatories and impossible with many", formal agreements with few signatories may achieve better outcomes than those with many. Third, and pointing to the more general feature underlying the pre-

[4]We shall refer to informal agreements to invest as MOUs throughout.

vious insights, prior stages in games (here, an agreement stage) can increase a player's influence over others' payoffs, since others may condition their actions on his early choice. This increase in payoff influence, can "amplify" psychological payoffs (in our case kindness) and make otherwise impossible outcomes attainable.

We add to an active literature studying agreements (e.g. Barrett 2003, Harstad 2012, 2015, Battaglini and Harstad 2016, Martimort and Sand-Zantman 2016) and an emerging literature on mechanism design where players have reciprocity preferences (Netzer and Volk 2014, Bierbrauer and Netzer 2016, Bierbrauer et al. 2017, Dufwenberg and Patel 2017).

Our particular mechanism, cost-sharing agreements, falls into a class of mechanisms where commitments on strategy-conditional side-payments are made before a game is played (Jackson and Wilkie 2005, Ellingsen and Paltseva 2016). Cost-sharing is an important case of models where agents make commitments to match others' public good investments (Guttman 1978, 1987, Boadway et al. 2007) or to compensate others for their investment (Varian 1994). Our game may also be relevant for agreements on R&D investment (Katz 1986) and International Environmental Agreements (IEAs) (Barrett 1994), if they involve binding co-financing.

Understanding the role of reciprocity in IEAs is important for environmental economists. Nyborg (2015) concurrently developed a model that extends D&K to cooperative games in order to apply it to Barrett's IEAs model. She finds that reciprocity can create weakly larger stable coalitions that exhibit higher abatement. Less closely related are Hadjiyiannis et al. (2012) and Kolstad (2014). The former studies the effect of a different notion of reciprocity on abatement in a two-player game with no possibility to sign an agreement. The latter examines the effect of equity- and efficiency-concerns (Charness and Rabin 2000) in an IEAs game.

We structure the paper as follows. Section 2 presents a set of preliminaries needed for our main result. Section 3 states and explains our main result on how full investment is impossible if all players sign, but is possible if no-one signs. Section 4 argues that our result illustrates a more general principle that high investment is possible with few signatories but not with many. Section 5 states further results on games with very few players and where reciprocity concerns are very high or low. Section 6 offers reflections on alternative definitions of reciprocity and other game forms. Section 7 concludes.

2 Preliminaries

We introduce a public good game (2.1), an agreements game (2.2), and D&K's reciprocity model (2.3) which we apply to the public good game (2.4).

2.1 The public good game (Γ_P)

Let $N = \{1, \ldots, n\}$ be the set of players where $n \geq 4$.[5] Each $i \in N$ simultaneously chooses $a_i \in \{0, 1\}$, where 1 corresponds to investing in a public good and 0 to not doing so. Let $a = (a_i)_{i \in N}$. Player i's material payoff is

$$\pi_i(a) = \beta \sum_{j \in N} a_j - \gamma a_i,$$

where β is the public good benefit and γ the cost. Assume that $n\beta > \gamma > \beta > 0$ so that the individual cost exceeds the benefit and all players investing maximises total payoffs. Γ_P has a unique NE where for all $i \in N$, $a_i = 0$.

While there are no formal agreements in this game form, a legitimate interpretation of the strategy profile where all players invest is that players have a MOU (or gentleman's agreement) to invest. Clearly the MOU profile is not a NE of Γ_P.

2.2 The agreements game (Γ_A)

The agreements game appends a prior stage to the public good game where each player decides whether or not to sign a co-financing agreement: a binding commitment to share public good investment costs. In addition to capturing this formal agreement, the game form still nests the previous informal agreement, a MOU. If all players refuse to sign the co-financing agreement, but still invest in the public good, they can be said to have an informal agreement to invest, a MOU. Rejecting a formal agreement does not necessarily imply a MOU as players may not plan to invest. We now describe the game more precisely.

As before, $N = \{1, \ldots, n\}$, $n \geq 4$. In stage 1, the "sign-up stage", each $i \in N$ simultaneously chooses $a_i^1 \in \{0, 1\}$; 1 means signing the agreement, 0 not doing so. In stage 2, the "investment stage", each $i \in N$ simultaneously

[5] We discuss $n < 4$ in Section 5.

chooses $a_i^2 \in \{0,1\}$; 1 means investing in the public good, 0 not doing so. Let $a^1 = (a_i^1)_{i\in N}$, $a^2 = (a_i^2)_{i\in N}$ and $a = (a^1, a^2)$.

Let $m(a^1) = \sum_{i\in N} a_i^1$ be the number of signatories, $x(a^2) = \sum_{i\in N} a_i^2$ the number of players who invest, $x^m(a) = \sum_{i\in N} a_i^1 a_i^2$ the number of signatories who invest. Player i's material payoff is

$$\pi_i(a) = \begin{cases} \beta x(a^2) - \gamma a_i^2 & \text{if } a_i^1 = 0 \\ \beta x(a^2) - \gamma \frac{x^m(a)}{m(a^1)} & \text{if } a_i^1 = 1 \end{cases},$$

where β is the public good benefit and γ the cost. The investment stage involves the same decision as in Γ_P, but with different payoff consequences for signatories. Signatories have made a binding commitment to share their investment costs. As before, $n\beta > \gamma > \beta > 0$ and to avoid knife-edge cases assume $\frac{\gamma}{\beta}$ is non-integer and $n > \lceil \frac{\gamma}{\beta} \rceil$.[6] A strategy for i, $s_i \in S_i$, specifies an initial choice and a choice for each stage 1 history. We focus on pure strategies throughout. Finally, let $S = \times_{i\in N} S_i$ and $s \in S$.

As previously explained, we interpret the strategy profile where zero players choose to sign but all invest, as players having a MOU to invest.

The following observation highlights important properties of SPE of Γ_A.

Observation 1 (Agreements game SPE)

(a) *There does not exist a full investment SPE in* Γ_A.

(b) *There exist zero investment SPE in* Γ_A *with* $[0, \lceil \frac{\gamma}{\beta} \rceil - 1)$ *signatories.*

(c) *There exist positive investment SPE in* Γ_A *with* $\lceil \frac{\gamma}{\beta} \rceil$ *signatories.*

Our proofs are rather tedious. We provide them in the appendix. In the main text we highlight key intuitions. Two aspects of Observation 1 are particularly noteworthy: a MOU is not a SPE and SPE exhibit less than full investment. To understand why, solve backwards. Non-signatories never invest as they bear their full investment cost (thus a MOU to invest is not a SPE). A signatory only invests if his costs are shared with sufficiently many co-signatories, namely if $m(a^1) \geq \lceil \frac{\gamma}{\beta} \rceil$. Given this, consider the sign-up stage. There exist SPE with very few signatories, (b), as stage 1 deviation does not induce investment. There also exist SPE with $\lceil \frac{\gamma}{\beta} \rceil$ signatories, (c), as each

[6] $\lceil \frac{\gamma}{\beta} \rceil$ is the lowest integer strictly greater than $\frac{\gamma}{\beta}$.

signatory is pivotal in inducing all signatories to invest. If there are greater than $\left\lceil \frac{\gamma}{\beta} \right\rceil$ signatories, a signatory can increase his payoff by deviating to not signing as other signatories continue to invest, hence full investment is not SPE, (a).

Observation 1 echos the results of Varian (1994) and Barrett (1994).[7]

2.3 Modelling reciprocity

We now incorporate reciprocity following D&K.[8] Their approach uses "kindness functions". To determine whether i is kind to j one needs a reference point, the equitable payoff. In defining this reference point, D&K argue that only the set of efficient strategies, E_i, those not involving "wasteful" play, are relevant.[9] For Γ_P, $E_i = S_i$; for Γ_A, $s_i \notin E_i$ iff s_i prescribes not investing after a history of i signing and $m(a^1) \geq \left\lceil \frac{\gamma}{\beta} \right\rceil$.

Let $b_{ik} \in S_k$ denote i's (point) belief of k's strategy, then given $(b_{ik})_{k \neq i}$ the *equitable payoff* for j is the average of the most i believes he can "give" to j given his strategy set and the least he believes he can "give" given his efficient strategies:

$$\pi_j^{e_i}\left((b_{ik})_{k\neq i}\right) = \tfrac{1}{2}\left[\max\{\pi_j\left(s_i,(b_{ik})_{k\neq i}\right)|s_i \in S_i\} + \min\{\pi_j\left(s_i,(b_{ik})_{k\neq i}\right)|s_i \in E_i\}\right].$$

To define i's kindness to j, let $s_i(h)$ be i's (updated) strategy which is identical to s_i except that a_i^1 must be consistent with reaching h. Let $b_{ik}(h)$ be i's (updated) belief of k's strategy. Given $s_i(h)$ and $(b_{ik}(h))_{k\neq i}$, i's *kindness* to j at h is

$$\kappa_{ij}\left(s_i(h),(b_{ik}(h))_{k\neq i}\right) = \pi_j\left(s_i(h),(b_{ik}(h))_{k\neq i}\right) - \pi_j^{e_i}\left((b_{ik}(h))_{k\neq i}\right).$$

The material payoff i believes j receives (first term on RHS) is compared to the equitable payoff (second term on RHS). If $\kappa_{ij}(.) > 0$, i is kind to j. If $\kappa_{ij}(.) < 0$, i is unkind to j. If $\kappa_{ij}(.) = 0$, i has zero kindness toward j.

[7] Both authors suggest mechanisms that give some SPE with partial investment and some with zero investment.

[8] Our presentation is tailored to Γ_P and Γ_A; see D&K for general games.

[9] A strategy is efficient if there does not exist another strategy which for all histories and others' strategies gives no player a lower payoff, and for some history and others' strategies gives at least one player a strictly higher payoff. See D&K pp. 275-6 for more on motivation, and the precise definition for general games. See Section 6.1 of the current paper for the implications of an alternative definition of efficient strategies.

To capture reciprocity incentives D&K need a function reflecting how kind i perceives j as being. Let $c_{ijk}(h)$ denote i's updated (point) belief about j's (point) belief about k's strategy. Given $b_{ij}(h)$ and $(c_{ijk}(h))_{k\neq j}$, i's *perceived kindness* of j towards i at history h is

$$\lambda_{iji}\left(b_{ij}(h),(c_{ijk}(h))_{k\neq j}\right)=\pi_i\left(b_{ij}(h),(c_{ijk}(h))_{k\neq j}\right)-\pi_j^{e_j}\left((c_{ijk}(h))_{k\neq j}\right).$$

The material payoff that i believes j believes i receives (first term on RHS) is compared to the equitable payoff (second term on RHS). If $\lambda_{iji}(.)>0$ then i perceives j as kind to him, etc.

Player i 's utility at history h sums material and reciprocity payoffs

$$\begin{aligned}&U_i\left(s_i(h),\left(b_{ik}(h),(c_{ikl}(h))_{l\neq k}\right)_{k\neq i}\right)\\&=\underbrace{\pi_i\left(s_i(h),(b_{ik}(h))_{k\neq i}\right)}_{\text{material payoff}}\\&\underbrace{+Y\sum_{j\in N\setminus\{i\}}\left(\kappa_{ij}\left(s_i(h),(b_{ik}(h))_{k\neq i}\right)\cdot\lambda_{iji}\left(b_{ij}(h),(c_{ijl}(h))_{l\neq j}\right)\right),}_{\text{reciprocity payoff}}\end{aligned}\tag{1}$$

where $Y\geq 0$ is the sensitivity to reciprocity payoffs. If $Y>0$ then i's preference for reciprocation toward j is captured by i's utility increasing when $\kappa_{ij}(.)$ and $\lambda_{iji}(.)$ are non-zero with matching signs, reflecting mutual kindness or unkindness.

Let $\widehat{\Gamma}_P$ and $\widehat{\Gamma}_A$ denote the public good game and agreements game where players' utilities are (1).[10] To define an appropriate solution concept, let $s_i'(h)$ be the strategy identical to $s_i(h)$ at all histories except at h, where it differs (given binary action choices there is only one such strategy).

[10] $\widehat{\Gamma}_P$ and $\widehat{\Gamma}_A$ are psychological games (Geanakoplos et al. 1989, Battigalli and Dufwenberg 2009).

Definition 1 *$s \in S$ is a sequential reciprocity equilibrium (SRE) if for all i and at each history h it holds that*

(i) $U_i\left(s_i(h), \left(b_{ik}(h), (c_{ikl}(h))_{l\neq k}\right)_{k\neq i}\right) \geq U_i\left(s_i'(h), \left(b_{ik}(h), (c_{ikl}(h))_{l\neq k}\right)_{k\neq i}\right)$,

(ii) $c_{ijk} = b_{jk} = s_k$ *for all* $j \neq i$ *and* $k \neq j$.

Condition (i) implies i best-responds at each history given his beliefs. Condition (ii) requires beliefs to be correct. If $Y = 0$ then Definition 1 describes a SPE (+ correct beliefs) in a game where utility equals material payoffs.

2.4 Reciprocity in the public good game

Full investment via a MOU was impossible in Γ_P (2.1). Can it be a SRE in $\widehat{\Gamma}_P$? If so, given equilibrium beliefs, for all $i, j \in N$, $\kappa_{ij}(1,.) = \lambda_{iji}(1,.) = \frac{\beta}{2}$ and $\kappa_{ij}(0,.) = -\frac{\beta}{2}$. Therefore i would not deviate to not investing if $n\beta - \gamma + (n-1)Y(\frac{\beta}{2})^2 \geq (n-1)\beta + (n-1)Y(-\frac{\beta}{2})(\frac{\beta}{2})$. Put differently, a MOU for full investment is a SRE of $\widehat{\Gamma}_P$ if $Y \geq Y^*$, where

$$Y^* = \frac{2(\gamma - \beta)}{\beta^2(n-1)}.$$

Intuitively, a sufficiently high reciprocity sensitivity ensures that the reciprocity cost of deviating to not investing (being unkind to co-players who are kind to you) outweighs the material gain. Our main result, in the next section, concerns whether the agreements mechanism ($\widehat{\Gamma}_A$) can deliver full investment when a MOU for full investment is impossible in $\widehat{\Gamma}_P$, i.e. $Y < Y^*$.

3 Main result

Can the agreements game deliver full investment if this is impossible without the agreements mechanism ($Y < Y^*$) and the stakes are high (large β)? Indeed it can, but not how one might expect. If all players sign the co-financing agreement, full investment remains out of reach. However, full investment is possible if all players refuse to sign and have a MOU to invest instead. We first state the result formally (3.1), then provide intuition via examples (3.2-3).

3.1 Formal statement

Theorem 1 *For all $\gamma > 0$ and $n \geq 4$, there exists $\beta' \in (\gamma/n, \gamma)$ and $Y' \in (0, Y^*)$ such that,*

- ***(a)*** *if $\beta \geq \beta'$ and $Y < Y^*$, there does not exist a full investment SRE in $\widehat{\Gamma}_A$ with n signatories,*
- ***(b)*** *if $\beta \geq \beta'$ and $Y \in [Y', Y^*)$, there exists a full investment SRE in $\widehat{\Gamma}_A$ with 0 signatories. The SRE is described by 0 signing, then non-signatories investing iff there are 0 signatories and signatories investing if there are at least $\left\lceil \frac{\gamma}{\beta} \right\rceil$ signatories.*

The theorem suggests that for high return public goods where full investment is impossible, the introduction of an agreements mechanism makes full investment possible. Encouraging all players to actually sign however is counterproductive. Instead, it is potentially a good thing if all parties reject the co-financing agreement as a MOU to invest is then incentive compatible.

The strategies that support the full investment result – part (b) of the theorem – may seem to require a deep understanding of the game by the players. If a player deviates in stage 1 by signing, then no players invest in stage 2. So a deviation in stage 1 would result in a large decrease in material payoff for all players, and (as we show below) this is central for incentive-compatibility. This requires that all players understand the full consequences of varying degrees of take-up of the agreement, which may seem a tall order. However, under our MOU interpretation where coordination is achieved through discussions, such consequences would soon become common knowledge as negotiators reveal the implications of deviation.

3.2 Highlighting the intuition: part (b)

We begin with part (b) of Theorem 1, the SRE involving a MOU to invest, as it is more straightforward. Example 1 takes the stated profile, where no-one signs and all invest on path, and shows that it is a SRE for an interval of Y less than Y^*.

Example 1 (No-one signs): Take $\widehat{\Gamma}_A$ and let $n = 4$, $\gamma = 10$, $\beta = 9$ and $Y \in [\frac{1}{351}, Y^*)$, where $Y^* = \frac{2(10-9)}{9^2(4-1)} = \frac{2}{243}$. Consider the profile where,

- no-one signs,
- non-signatories invest iff there are 0 signatories,
- signatories invest iff there are at least 2 signatories.

Reason as follows to see that the profile is a SRE. Following a history of 0 signatories, i does not deviate to not investing if

$$4 \cdot 9 - 10 + 3Y \cdot \left(26 - \tfrac{1}{2}(26+0)\right) \cdot \underbrace{\left(26 - \tfrac{1}{2}(26+0)\right)}_{\lambda_{iji}(.)}$$

$$\geq 3 \cdot 9 + 3Y \cdot \left(17 - \tfrac{1}{2}(26+0)\right) \cdot \underbrace{\left(26 - \frac{1}{2}(26+0)\right)}_{\lambda_{iji}(.)}. \tag{2}$$

Solving gives $Y \geq \frac{1}{351}$, note that $\frac{1}{351} < Y^* = \frac{2}{243}$.

Following a history of 1 signatory, i does not deviate to investing for any Y. Following a history of 2 signatories, signatory i does not deviate to not investing if $Y \leq \frac{4}{157}$; note that $Y^* = \frac{2}{243} < \frac{4}{157}$. Following histories of 2 or 3 signatories, non-signatory i does not deviate to investing if $Y \leq Y^*$. Following a history of 3 or 4 signatories, signatory i does not deviate to not investing for any Y. Player i does not deviate to signing in the sign-up stage for any Y. The profile is thus a SRE for $Y \in [\frac{1}{351}, Y^*)$. ▲

Kindness amplification

Why is a MOU to invest incentive compatible in Example 1, when it would not be if there had been no option to sign a co-financing agreement ($\widehat{\Gamma}_P$ with $Y < Y^*$)? The explanation centers on what we call *kindness amplification*.

To see this, contrast the 0 signatory subgame in Example 1 with the full investment profile in $\widehat{\Gamma}_P$. In both cases, i has the same material incentive to deviate to not investing: a material payoff increase of $3 \cdot 9 - (4 \cdot 9 - 10) = 1$. His reciprocity incentive however differs as kindness is higher (or amplified) in Example 1 (e.g. $\lambda_{iji}(.) = \frac{9}{2}$ in $\widehat{\Gamma}_P$, see section 2.4, but $\lambda_{iji}(.) = 13$ in Example 1, see (2)). The amplified kindness implies that for a given Y, the decrease in i's reciprocity payoff from deviating to not investing is larger in Example 1

$(3Y \cdot 13 \cdot 13 - 3Y \cdot 4 \cdot 13 = 351Y)$ than in $\widehat{\Gamma}_P$ $(3Y(\frac{9}{2})(\frac{9}{2}) - 3Y(-\frac{9}{2})(\frac{9}{2}) = \frac{243}{2}Y)$. A lower Y is thus sufficient to prevent i deviating in Example 1 than in $\widehat{\Gamma}_P$, hence our result.

The fundamental reason as to why the agreements mechanism can amplify kindness is that it can increase a player's ability to influence others' behaviour. For instance, if j signs in Example 1, *no-one* invests, implying i's material payoff falls by $4 \cdot 9 - 10 = 26$. If j deviates to not investing in $\widehat{\Gamma}_P$, others' actions are unchanged so i 's material payoff falls by only $9 < 26$. Thus i perceives j as kinder in Example 1 than in $\widehat{\Gamma}_P$.

3.3 Highlighting the intuition: part (a)

We now illustrate part (a) of Theorem 1. Example 2 has three parts, it takes the parameters of Example 1 and demonstrates that for $Y < Y^*$, no profile where all players sign and invest on path is a SRE. The key intuition is that kindness cannot be amplified as effectively as when there are zero signatories on path (3.2). Part (i) demonstrates that a particular off-path investment-stage behaviour implies sign-up stage deviation. Parts (ii) and (iii) show that while alternative off-path investment-stage behaviours may avoid sign-up stage deviation, they necessarily involve investment-stage deviation.

Example 2(i) (Sign-up stage deviation): Take $\widehat{\Gamma}_A$ and let $n = 4$, $\gamma = 10$, $\beta = 9$ and $Y < Y^* = \frac{2}{243}$. Consider any profile where

- all players sign & invest on path,
- only signatories invest if there are 3 signatories.

Player i's sign-up stage incentives are identical to those for the full investment profile in $\widehat{\Gamma}_P$, i thus deviates to not signing as $Y < Y^*$. ▲

It follows from Example 2(i) that all remaining candidate SRE (where all players sign and invest on path) must involve: (a) a signatory who does not invest when there are 3 signatories, or, (b) a non-signatory who invests when there are 3 signatories. Examples 2(ii) and 2(iii) rule out candidate SRE with properties (a) and (b) respectively by demonstrating that players deviate in the investment stage.

Example 2(ii) (Signatory not investing with 3 signatories): Take $\widehat{\Gamma}_A$ and let $n = 4$, $\gamma = 10$, $\beta = 9$ and $Y < Y^* = \frac{2}{243}$. Consider a profile where

- all players sign & invest on path,
- at history h where there are 3 signatories, some signatory i does not invest.

By deviating to investing at h, signatory i can increase his material payoff by $9 - \frac{10}{3} = \frac{17}{3}$. Can i's reciprocity incentives prevent this deviation? Since i not investing is less kind than investing, to maximise the reciprocity cost of i's deviation examine the profile where

- all players sign & invest on path,
- no-one invests if there are 3 signatories,
- all players invest if there are 2 signatories.

Player i's reduction in reciprocity payoff from deviation at h is then $\frac{1144}{3}Y < \frac{2288}{729} < \frac{17}{3}$ (first inequality from $Y < Y^* = \frac{2}{243}$), thus i deviates. ▲

Weak kindness amplification
Why can kindness be sufficiently amplified to prevent deviation in Example 1 but not in Example 2(ii)? The answer concerns the size of the material loss that the reciprocity payoff need compensate in the relevant subgames. In Example 1, following a history of zero signatories, the material loss of investing was 1. In Example 2(ii), following a history of 3 signatories, the material loss to a signatory of not investing was $\frac{17}{3} > 1$. More generally, for high β, the size of the material loss from not deviating in the relevant subgame is lower for the case when there are zero signatories than when there are many. Lower kindness is thus sufficient for non-deviation when there are few signatories than when there are many.

To complete Example 2, part (iii) examines the remaining candidate SRE (where all sign and invest on path) which involve all players investing when there are 3 signatories (by Examples 2(i)-(ii)). It shows that for a non-signatory to invest when there are 3 signatories, a signatory must not invest when there are 2 signatories. However, since this signatory has an incentive to deviate to investing, these profiles are not SRE either.

Example 2(iii) (Non-signatory investing with 3 signatories): Take $\widehat{\Gamma}_A$ and let $n = 4$, $\gamma = 10$, $\beta = 9$ and $Y < Y^* = \frac{2}{243}$. Consider a profile where

- all players sign & invest on path,
- all players invest at h where all but i have signed

By deviating to not investing at h, non-signatory i increases his material payoff by $10 - 9 = 1$. Whether i's reciprocity incentives prevent deviation depends on behaviour following histories with 2 signatories. If all invest when there are 2 signatories, i's incentives are identical to the full investment profile in $\widehat{\Gamma}_P$, thus i deviates at h ($Y < Y^*$). If only the signatories invest when there are 2 signatories, then i deviates at h if $Y < \frac{1}{108}$, which is true ($Y < Y^* = \frac{2}{243} < \frac{1}{108}$). For i not to deviate at h, it must be that at least 3 players do not invest when there are 2 signatories.

Suppose then that some signatory j does not invest when there are 2 signatories. Signatory j has a material incentive to deviate to investing. To maximise the reciprocity cost of this deviation, suppose no-one invests if there are 2 signatories and all invest if there is 1 signatory. Signatory j deviates to investing when there are 2 signatories if $Y < \frac{2}{143}$, which is true ($Y < Y^* = \frac{2}{243} < \frac{2}{143}$). ▲

For the non-signatory to invest when there are 3 signatories his perceptions of others' kindness must be sufficiently amplified, this requires signatories to not invest when there are 2 signatories. However, reciprocity incentives are not sufficiently large to prevent a signatory deviating to investing for reasons analogous to those discussed following Example 2(ii). Thus there does not exist a full investment SRE where all sign on path.

To summarise, the fundamental intuition behind part (a) of Theorem 1 is the difficulty of amplifying kindness when many players sign. In order to prevent sign-up stage deviation, kindness must be amplified, this requires low investment in subgames where there are $n-1$ or $n-2$ (3 or 2 in Example 2) signatories. However, signatories have relatively large material incentives to invest in such subgames (high β) and reciprocity payoffs are relatively low ($Y < Y^*$), hence the impossibility.

4 Robustness

Theorem 1 examined whether particular profiles (those with either zero or n signatories, and full investment) are SRE. While these are clearly interesting profiles, one may nonetheless wonder whether Theorem 1 exemplifies the more general finding that "high investment is possible with few signatories and impossible with many signatories" or whether the equilibria in focus are just a special case of what may happen? This subsection presents insights that suggest that our result is more than just a special case.

First consider the number of signatories. We defined a MOU to invest as the profile where zero players sign a co-financing agreement, then all players invest. It seems equally reasonable to interpret a profile where very few players sign the agreement, then all players invest, as a MOU to invest (at least for the many non-signatories). Proposition 1(b) demonstrates that such a MOU is also a SRE (cf. Theorem 1(b)). Theorem 1(a) identified the impossibility of full investment if there are n signatories. Proposition 1(a) shows that this impossibility remains if there are $n-1$ signatories.

Proposition 1 *For all $\gamma > 0$ and $n \geq 4$, there exists $\beta''' \in (\gamma/n, \gamma)$ and $Y''' \in (0, Y^*)$ such that,*

- ***(a)*** *if $\beta \geq \beta'''$ and $Y < Y^*$, there does not exist a full investment SRE in $\widehat{\Gamma}_A$ with $n-1$ signatories,*
- ***(b)*** *if $\beta \geq \beta'''$ and $Y \in [Y''', Y^*)$, there exists a full investment SRE in $\widehat{\Gamma}_A$ with one signatory. The SRE is described by one player signing, then non-signatories investing iff there is exactly one signatory, and signatories investing always.*[11]

The intuition behind the result is analogous to that driving Theorem 1 (see section 3.2-3), kindness can be sufficiently amplified when there is only one signatory but not when there are $n-1$.

Proposition 1 is a reassuring robustness check, but like our main result it focuses on specific equilibria (those involving full investment and a specific number of signatories). How representative of the set of equilibria are these equilibria? A complete characterisation of the set of SRE is too complex in

[11] Proof available on request.

general, we can however study an example to get a sense of how representative our main result is.

Example 3 (Symmetric SRE): Take $\widehat{\Gamma}_A$ and let $n = 4$, β be sufficiently high, and Y be sufficiently high, but less than Y^*. Grouping equilibria by behaviour on the path of play, we note that there exist only five types of symmetric SRE (symmetric in the sense that signatories make the same investment choice as each other and non-signatories make the same investment choice as each other):[12]

Type I: Zero players signing, then all investing on path.

Type II: One player signing, then all investing on path.

Type III: Two players signing, then all investing on path (only exists if $Y < \frac{2(\gamma-\beta)}{3\beta\gamma}$).

Type IV: Two players signing, then only signatories investing on path.

Type V: Zero players signing, then zero investing on path.

Types I-II correspond to the MOU SRE referred to in Theorem 1(b) and Proposition 1(b) respectively. Type III could be interpreted as a MOU to invest (only two signatories, but all players invest). Type IV is the SPE profile where only signatories invest (no MOU to invest). Type V is qualitatively very different from our main result in that there are no signatories and no investment (no MOU to invest). Despite the existence of SRE with no MOU to invest, it is worth noting that the SRE that can be thought of as involving a MOU to invest are far more numerous than the SREs of Type IV-V. A final feature qualitiatively similar to our main result is the non-existence of SRE where many players sign and there is positive investment.

Taken together, Example 3 and Proposition 1 suggest that our main result exemplifies a more general insight that high investment is feasible via a MOU where very few players sign a co-financing agreement, but high investment is impossible if there are many signatories.

5 Further results

This section addresses three further questions. Does the possibility of full investment via a MOU and the impossibility of full investment with n signatories arise in 2- and 3-player games? Can the agreements mechanism

[12] Full details available on request.

give full investment with n signatories when a MOU to invest is a SRE even without a mechanism ($Y \geq Y^*$)? When reciprocity is low ($Y \in (0, Y^*)$) can the mechanism deliver at least the investment levels possible under material preferences? We show that the answers are no, yes and yes.

Three- and two-player games
In contrast to Theorem 1(a), for 3-player games, the agreements mechanism can give full investment with n signatories for an interval of Y less than Y^*.[13]

Proposition 2 (3-players) *For $n = 3$, all β and γ, there exists $Y'' \in (0, Y^*)$ such that,*

- ***(a)*** *if $Y \in [Y'', Y^*)$ there exists a full investment SRE in $\widehat{\Gamma}_A$ with 3 signatories. The SRE is described by 3 signing, then i does not invest iff there is only 1 signatory.*
- ***(b)*** *if $Y \in [Y'', Y^*)$ there exists a full investment SRE in $\widehat{\Gamma}_A$ with 0 signatories. The SRE is described by 0 signing, then i does not invest iff there is only 1 signatory.*

The impossibility identified in Section 3 does not arise here as kindness can be amplified more easily when $n = 3$. From Section 3, recall that a candidate SRE where n sign and invest on path, required low investment in subgames where there were $n - 1$ or $n - 2$ signatories (see Example 2(ii)-(iii)). Large material incentives to invest in such subgames prevented low investment for $n > 3$. By contrast, for $n = 3$, we get $n - 2 = 1$ signatory and players have no material incentive to invest if there is only one signatory. Kindness can thus be sufficiently amplified and all players signing and investing on path is a SRE (Proposition 2(a)).

Now consider 2-player games. For such games, drop the assumptions that $\frac{\gamma}{\beta}$ is non-integer and $n > \lceil \frac{\gamma}{\beta} \rceil$ as they would contradict that $n\beta > \gamma > \beta > 0$.[14]

[13] From now on, allow Γ_A and Γ_P to be 2- or 3-player games.

[14] Note that there exists a full investment SPE in the 2-player Γ_A where both sign and invest on path.

Proposition 3 (2-players) *For $n = 2$, all γ and $Y \in (0, Y^*)$,*

- ***(a)*** *if $\beta \geq \frac{2}{3}\gamma$, then there exists a full investment SRE in $\widehat{\Gamma}_A$ with 2 signatories. The SRE is described by 2 signing, then i invests iff there are 2 signatories.*
- ***(b)*** *for all β, there does not exist a full investment SRE in $\widehat{\Gamma}_A$ with 0 signatories.*

In the 2-player agreements game, there exists a full investment SPE with 2 signatories. Kindness amplification is not needed to compensate players for a material loss, thus impossibility does not occur (part (a)). By contrast, full investment via a MOU does require kindness amplification. As part (b) suggests, kindness cannot be sufficiently amplified in the 2-player game, this is because there is no other player, k, such that i can influence k's strategy and thereby amplify i's influence over j's material payoff.

High reciprocity ($Y \geq Y^*$)
Our main result examined cases where full investment was impossible with no mechanism ($Y \in (0, Y^*)$). We now consider whether the mechanism precludes full investment when it was possible with no mechanism ($Y \geq Y^*$).

Proposition 4 (High reciprocity) *For all $n \geq 2$, γ, β and $Y \geq Y^*$ there exists a full investment SRE in $\widehat{\Gamma}_A$. The SRE is described by n signing, then i does not invest iff there are $\lceil \frac{\gamma}{\beta} \rceil - q$ signatories where $q > 0$ and odd.*

If full investment were possible without the mechanism, it remains possible with it. As the reciprocity sensitivity is high ($Y > Y^*$), when there are many signatories ($\geq \lceil \frac{\gamma}{\beta} \rceil$), reciprocity payoffs compensate material costs of investment with no need for kindness amplification. When there are few signatories ($\leq \lceil \frac{\gamma}{\beta} \rceil$), the alternating between all investing and zero investing ensures kindness is sufficiently amplified and that there are no deviations.

Low reciprocity ($Y \in (0, Y^*)$)
Observation 1(a) stated that full investment was impossible with material preferences. Our results have thus far considered whether it is attainable with reciprocity. We found that full investment is indeed attainable if the reciprocity sensitivity is high enough (e.g. Theorem 1, Proposition 4). What about when the reciprocity sensitivity is not sufficiently high? We know from

Observation 1(c) that even with material preferences (i.e. $Y = 0$), equilibria exist with $\lceil \frac{\gamma}{\beta} \rceil$ players investing. Do equilibria exist with at least $\lceil \frac{\gamma}{\beta} \rceil$ players investing for small, non-zero reciprocity sensitivities? The answer is yes.

Proposition 5 (Low reciprocity) *For all $n \geq 4$ and γ, there exists $Y''' \in (0, Y^*)$, $\beta'' \in (\gamma/n, \gamma)$ and $m'(Y) \in \left[\lceil \frac{\gamma}{\beta} \rceil, \lfloor \frac{\sqrt{8n-7}-1}{2} \rfloor \right]$ such that*

- ***(a)*** *if $\beta \geq \beta''$ and $Y \leq Y'''$ there exists a SRE where $m'(Y)$ players sign and invest on path. The SRE is described by $m'(Y)$ players signing, then i invests iff i signed and there are at least $m'(Y)$ signatories.*
- ***(b)*** *$m'(Y)$ is non-decreasing in Y.*

When the reciprocity sensitivity is sufficiently low at least the maximal SPE investment, $\lceil \frac{\gamma}{\beta} \rceil$, is attainable as a SRE. As Y increases, investment greater than $\lceil \frac{\gamma}{\beta} \rceil$ is possible in SRE, since reciprocity payoffs increase due to mutual kindness among signatories and kindness amplification.

6 Two reflections

6.1 The role of efficient strategies a la D&K

Our main result depends critically on using D&K's notion of "efficient strategies," rather than the corresponding notion in Rabin (1993). In this section we present the issue and defend the view that D&K's definition is adequate.

The key issue concerns how to calculate the zero-kindness threshold (i.e. the equitable payoff, $\pi_j^{e_i}$). In D&K's theory this is done with respect to the average of the lowest and highest payoff that i can achieve for j, if i uses an efficient strategy, a notion D&K define independently of player i's beliefs (see p. 276 in D&K; p. 6 in this paper). Rabin, by contrast (see his p. 1286) calculates the zero kindness threshold with respect to those strategies that produce payoffs on the Pareto frontier, *given* i's beliefs. With such a definition the equilibrium highlighted in our main theorem would collapse: Under Rabin's definition a player who deviated at the sign-up stage would do something inefficient, hence irrelevant for calculating the zero-kindness threshold. In effect, the dramatic kindness amplification discussed in section 3.2. would be lost.

While we note this fact, we propose that it does not undermine the force of our result as Rabin's definition imposes psychologically non-obvious restrictions. In particular, according to Rabin, the kindness associated with a deviation from a purported equilibrium is calculated as if the player in question had equilibrium beliefs. This assumption is hard to justify, because, after all, a deviating player is demonstrably *not* playing along with the equilibrium! An intelligent player, who tries to assess the kindness of a deviating co-player, should thus ask what payoff consequences the deviator has in mind, and in particular whether he is inefficiently attempting to lower the payoffs of all players. D&K's key definition identifies the set of strategies (the inefficient ones) for which no story can be told that explains the behaviour unless that story involves "waste" at some history. This is a set with a meaningful interpretation, which helps one think about and interpret the zero kindness threshold.

For more critical discussion, comparing D&K and Rabin's efficiency definitions, we refer the reader to section 5 of D&K. It is shown that Rabin's definition makes kindness change discontinuously with beliefs, and that equilibrium existence is not guaranteed. We find the first aspect psychologically counterintuitive. The second aspect is a consequence of the first. We believe these aspects speak in favour of D&K's efficiency definition as well.

6.2 The phenomenon of kindness amplification

The main motivation of our work is to explore the impact of reciprocity in our particular game forms, given our particular co-financing agreements and MOUs interpretation. Our results uncovered a phenomenon that we labelled kindness amplification. Analogous effects can be found in related game forms with alternative economic interpretations. While it is beyond the scope of this paper to explore the topic at length we now present the simplest example which demonstrates that the phenomenon can imply similar but not identical implications for public good provision in other game forms.

Example 4 (Cheap-talk first stage): Define Γ_C as identical to Γ_A except that i's material payoff is $\pi_i(a) = \beta x(a^2) - \gamma a_i^2$. First-stage actions in Γ_C thus have no material payoff consequences and can be interpreted as cheap-talk messages sent prior to a public good game. For all γ and $n \geq 4$, there exists $\beta_C \in (\gamma/n, \gamma)$ and $Y_C \in (0, Y^*)$ such that

(i) if $\beta \geq \beta_C$ and $Y \in [Y_C, Y^*)$, there exists a full investment SRE where all players choose 0 in stage 1. The SRE is described by all choosing 0, then players investing iff an even number of players chose 0 in the first stage.

(ii) if $\beta \geq \beta_C$ and $Y \in [Y_C, Y^*)$, there exists a full investment SRE where all players choose 1 in stage 1. The SRE is described by all choosing 1, then players investing iff an odd number of players chose 0 in the first stage.[15]

The example illustrates that the existence of any material-payoff-irrelevant first-stage action choice can amplify kindness and thus lead to full investment (the profile may be interpreted as a MOU to invest). The intuition is identical to that behind the Theorem 1(b) where no-one signed (i.e. took a material-payoff-irrelevant-action) and then all players invested. In this cheap-talk game however, since there are no material payoff consequences of sending a particular message (unlike where at least two players choose to sign a co-financing agreement), there is no analog to Theorem 1(a) i.e. full investment is not impossible if all players take a particular first-stage action. The asymmetric effects of all signing versus no-one signing in our main result cannot be generated by a cheap-talk first stage.

7 Conclusion

In this paper we studied the ability of a co-financing agreements mechanism to raise public good investment given reciprocity motivations. Our main result is that when full investment is impossible in the absence of an agreements mechanism and the return to the public good is high, then full investment remains impossible if all players sign the agreement, but is possible (via a MOU) if all players reject the agreement. Critically, the informal agreement to invest (the MOU) would not be possible if players had not met, tried to sign a binding agreement, but then explicitly rejected it in favour of an informal agreement.

[15] Proof available on request.

Rejecting binding agreements in favour of non-binding ones

When do we see parties meet, try to sign a binding agreement, then walk away with a non-binding agreement? As a possible example, take the 2015 UN Climate Change Conference in Paris (aka COP21/CMP11). Many commentators have pointed out that the agreement is not binding, in the sense that there are no penalties for non-compliance (e.g. Clémençon (2016), Jacquet and Jamieson (2016)). Arguably, negotiators could have made a binding, or formal, commitment to co-finance each other's efforts to reduce greenhouse gas emissions, but instead they struck a non-binding MOU to exert such effort anyway, without co-financing mandates (at least between industrial countries). This may be an outcome along the lines that our Theorem 1 points to. Agents do not sign a formal co-financing agreement, but rather reject the agreement and coordinate on full investment nonetheless. The "soft-touch" Paris agreement may be the kind of situation pointed to by Theorem 1: informal deals gain traction as other formal deals are shunned. And institutions that allow formal agreements for co-financing to be signed, even when they are not, are critical for spurring informal agreements to invest in public goods.

Binding a few hands

Despite the usefulness of non-binding agreements, binding ones are often also signed in reality. Where such an agreement is signed, only a subset of potential signatories would typically actually sign. Strictly, our Theorem 1 is silent about binding agreements with few signatories. However, as argued for in Section 4, it is reasonable to view the result as illustrating a more general point that higher investment is feasible when few people sign a co-financing agreement than when many do. This interpretation suggests that reciprocity motivations may be able to explain why agreements with few signatories nonetheless manage to achieve desirable outcomes.

Focalising investment

Rather than offer the possibility to sign binding co-financing agreements, institution designers may wish to allow parties to directly discuss MOUs. In Section 6.2 we illustrated that such a meeting with cheap-talk messaging can lead to full investment via many different action profiles, this may make coordination difficult. By contrast, the number of routes to full investment is smaller with a co-financing agreements institution, as if there are many signatories, full investment is impossible. This difference would presumably make

coordination easier with a co-financing mechanism, a potentially important advantage of using co-financing agreements.

The overall lesson is clear: Institutions that offer opportunities for binding agreements may be important even if such agreements are not struck or there is low uptake. In our case, creating the possibility of a formal co-financing agreement may promote actual investment in public goods, even if a co-financing agreement is not signed.

Appendix

Proof of Observation 1 (Agreements game SPE)

Apply backward induction to identify the SPE of Γ_A. At $h = a^1$, non-signatories do not invest and signatory i invests iff $\beta \geq \frac{\gamma}{m(a^1)}$. Since $\frac{\gamma}{\beta}$ non-integer, write the condition as iff $m(a^1) \geq \lceil \frac{\gamma}{\beta} \rceil$. Given optimal behaviour at all a^1, consider the first-stage. First suppose there are less than $\lceil \frac{\gamma}{\beta} \rceil - 1$ signatories. Player i does not deviate in the first-stage as $\pi_i(.) = 0$ regardless. Thus this is a SPE. Second suppose there are $\lceil \frac{\gamma}{\beta} \rceil - 1$ signatories. Non-signatory i deviates to signing if $0 < \beta \lceil \frac{\gamma}{\beta} \rceil - \gamma$, which is true. Thus this is not a SPE. Third suppose there are $\lceil \frac{\gamma}{\beta} \rceil$ signatories. Signatory i does not deviate to not signing if $\lceil \frac{\gamma}{\beta} \rceil \beta - \gamma \geq 0$, which is true. Non-signatory i does not deviate to signing if $\lceil \frac{\gamma}{\beta} \rceil \beta \geq (\lceil \frac{\gamma}{\beta} \rceil + 1)\beta - \gamma$, which is also true. Thus this is a SPE. Fourth suppose there are more than $\lceil \frac{\gamma}{\beta} \rceil$ signatories. Signatory i deviates to not signing as other players invest regardless of his choice. Thus this is not a SPE. The four cases are exhaustive. ■

Proof of Theorem 1

(a) Take the set of strategy profiles that involve n players signing and investing on path. Partition this set into 3 subsets of profiles distinguished by behaviour following a history of a^1 such that $m(a^1) = n - 1$: *subset 1*, all invest; *subset 2*, all signatories invest only; and *subset 3*, all remaining profiles. We take each subset in turn and demonstrate that no profile in the subset is a SRE if β is sufficiently high and $Y < Y^*$.

Subset 1: Consider any candidate SRE profile s^* such that each $i \in N$ signs, then invests if a^1 is such that $m(a^1) \geq n - 1$. Reason as follows to

show that there is no behaviour at histories such that $m(a^1) < n-1$ that would imply s^* is a SRE.

Consider $h = a^1$ such that $m(a^1) = n-1$, so all players invest. Non-signatory i has the same material incentive to deviate to not investing as in Γ_P. Given $Y < Y^*$, a necessary condition for i not to deviate is that $\lambda_{iji}\left(s_j^*, .\right) > \frac{\beta}{2}$ (recall $\lambda_{iji}(1, .) = \frac{\beta}{2}$ in $\widehat{\Gamma}_P$). The value of $\lambda_{iji}\left(s_j^*, .\right)$ at h, depends on the action choices s^* prescribes at history h' where all except i and j sign. If s^* were such that n invest or all except j invest at h', then $\lambda_{iji}\left(s_j^*, .\right) = n\beta - \gamma - \frac{1}{2}(n\beta - \gamma + (n-1)\beta - \gamma) = \frac{\beta}{2}$ at h, thus i would deviate at h. If s^* were such that all except i invest or all except i and j invest at h', then $\lambda_{iji}\left(s_j^*, .\right) = n\beta - \gamma - \frac{1}{2}((n-1)\beta + (n-2)\beta) = \frac{3}{2}\beta - \gamma < \frac{\beta}{2}$, thus i would deviate at h. Therefore a necessary condition for i to not deviate at h, is that s^* must be such that some signatory l does not invest at h'.

Consider h' and suppose s^* prescribes signatory l does not invest. Signatory l has a material incentive to deviate to invest at h'. We now show that for β sufficiently high and all $Y < Y^*$, l's reciprocity incentives are insufficient to prevent deviation at h'. The change in signatory l's reciprocity payoff from playing s_l^* rather than $s_l'(h', s_l^*)$ equals $Y\Psi$, where

$$\Psi := \sum_{k \in N \setminus \{l\}} \left(\kappa_{lk}\left(s_l^*, .\right) - \kappa_{lk}\left(s_l'\left(h', s_l^*\right), .\right)\right) \lambda_{lkl}\left(s_k^*, .\right).$$

If $\Psi \leq 0$, then l deviates at h'. Suppose $\Psi > 0$. Signatory l does not deviate at h' if

$$Y \geq \frac{(n-2)\beta - \gamma}{(n-2)\Psi}. \tag{3}$$

Let $\widehat{Y}(\beta)$ denote the RHS of (3) as a function of β. For $Y < Y^*$ we require

$$\widehat{Y}(\beta) = \frac{(n-2)\beta - \gamma}{(n-2)\Psi} < \frac{2(\gamma - \beta)}{\beta^2 (n-1)} = Y^*.$$

Now argue that for sufficiently high β either $Y < Y^*$ does not hold or l has an incentive to deviate at h'. Note that $\lim_{\beta \to \gamma} Y^* = 0$. Therefore for β in the neighbourhood of γ, $Y < Y^*$ requires $\lim_{\beta \to \gamma} \widehat{Y}(\beta) \leq 0$. Evaluating $\widehat{Y}(\gamma)$, note that the numerator of $\widehat{Y}(\gamma)$ is positive thus it must be that $\Psi \leq 0$. However if $\Psi < 0$ then l would deviate at h' for β slightly lower than γ as already argued, thus $\Psi = 0$ when $\beta = \gamma$. That we have supposed that $\Psi > 0$ for sufficiently high β and deduced $\Psi = 0$ at $\beta = \gamma$, implies that the

denominator of $\widehat{Y}(\beta)$ approaches zero from above and hence the one-sided limit $\lim_{\beta\to\gamma^-}\widehat{Y}(\beta)=+\infty$ which is greater than $\lim_{\beta\to\gamma}Y^*=0$, violating $Y<Y^*$. Thus for all $Y<Y^*$ and β sufficiently high, l would deviate to investing at h'.

Subset 2: Consider a candidate SRE profile s^* such that each $i\in N$ signs, then invests if a^1 is such that $m(a^1)=n$, and if a^1 is such that $m(a^1)=n-1$, all except the non-signatory invest. At the initial node, i's incentives are identical to those in the full investment profile in $\widehat{\Gamma}_P$. Thus i deviates to not signing at the initial node for all $Y<Y^*$. Hence s^* is not a SRE.

Subset 3: Consider any remaining candidate SRE profile s^* such that each $i\in N$ signs, then invests if a^1 is such that $m(a^1)=n$, it must be that for history $h''=a^1$ such that $m(a^1)=n-1$, there exists some signatory r who does not invest. Reasoning analogous to that used to show signatory l deviates to investing at h' (end of subset 1) establishes that signatory r deviates to investing at h''. Hence s^* is not a SRE.

(b) Consider s^* such that each $i\in N$ does not sign, then invests if a^1 is such that $m(a^1)=0$ or $a_i^1=1$ and $m(a^1)\geq 2$, and does not invest otherwise. We demonstrate that there exists $Y''<Y^*$ such that no player deviates at any h if $Y\in[Y'',Y^*)$ and β is sufficiently large.

First consider $h=a^1$ such that $m(a^1)=0$. Player i has a material incentive to deviate to not investing and a reciprocity incentive to not do so. His increase in reciprocity payoff from playing s_i^* instead of $s_i'(h,s_i^*)$ is

$$\begin{aligned}&\left(n-m(a^1)-1\right)\cdot Y\cdot\left(\kappa_{ij}\left(s_i^*,.\right)-\kappa_{ij}\left(s_i'\left(h,s_i^*\right),.\right)\right)\cdot\lambda_{iji}\left(s_j^*,.\right)\\&+m(a^1)\cdot Y\cdot\left(\kappa_{il}\left(s_i^*,.\right)-\kappa_{il}\left(s_i'\left(h,s_i^*\right),.\right)\right)\cdot\lambda_{ili}\left(s_l^*,.\right),\end{aligned}\tag{4}$$

where j is a non-signatory and l a signatory. Since $\kappa_{ij}\left(s_i^*,.\right)=\lambda_{iji}\left(s_j^*,.\right)=\frac{1}{2}\left(n\beta-\gamma\right)$ and $\kappa_{ij}\left(s_i'\left(h,s_i^*\right)\right)=\frac{1}{2}\left(n\beta-\gamma\right)-\beta$, this increase in reciprocity payoff is larger than the reduction in material payoff if $Y\geq\Phi$, where

$$\Phi:=\frac{2\left(\gamma-\beta\right)}{\left(n-1\right)\beta\left(n\beta-\gamma\right)}.$$

Note that if $\beta>\frac{\gamma}{n-1}$, then $\Phi<Y^*$. Thus i does not deviate for $Y\in[\Phi,Y^*)$.

Now consider $h=a^1$ such that $m(a^1)>0$. No player has a material incentive to deviate at a^1 since for $\beta>\frac{\gamma}{2}$, $\lceil\frac{\gamma}{\beta}\rceil=2$, thus action choices

following all a^1 are identical to SPE profiles. We will demonstrate for sufficiently high β and Y (but less than Y^*), any reciprocity incentive to deviate is less than the material incentive to not do so.

At $h = a^1$ such that $m(a^1) = 1$, player i has no reciprocity incentive to deviate to investing, thus does not deviate. At $h = a^1$ such that $m(a^1) = 2$, the change in signatory i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$\begin{aligned}&\left(n - m(a^1)\right) \cdot Y \cdot \left(\kappa_{ij}\left(s_i^*, .\right) - \kappa_{ij}\left(s_i'\left(h, s_i^*\right), .\right)\right) \cdot \lambda_{iji}\left(s_j^*, .\right) \\ &+ \left(m(a^1) - 1\right) \cdot Y \cdot \left(\kappa_{il}\left(s_i^*, .\right) - \kappa_{il}\left(s_i'\left(h, s_i^*\right), .\right)\right) \cdot \lambda_{ili}\left(s_l^*, .\right), \end{aligned} \tag{5}$$

where j is a non-signatory and l a signatory. For non-signatory j, $\kappa_{ij}\left(s_i^*, .\right) = \beta$, $\kappa_{ij}\left(s_i'\left(h, s_i^*\right), .\right) = 0$ and $\lambda_{iji}\left(s_j^*, .\right) = -\frac{\beta}{2}$. For signatory l, $\kappa_{il}\left(s_i^*, .\right) = \frac{3}{2}\beta - \gamma$, $\kappa_{il}\left(s_i'\left(h, s_i^*\right), .\right) = \frac{1}{2}\left(\beta - \gamma\right)$ and $\lambda_{ili}\left(s_l^*, .\right) = \frac{3}{2}\beta - \gamma$. Substituting into (5) gives $Y\left(\left(\beta - \frac{\gamma}{2}\right)\left(\frac{3}{2}\beta - \gamma\right) - (n-2)\frac{\beta^2}{2}\right)$, which is negative for sufficiently large β. Signatory i's reciprocity incentive to deviate to not investing at h is no larger than the material incentive to not do so if

$$Y \leq \left[\frac{\frac{\gamma}{2} - \beta}{\left(\beta - \frac{\gamma}{2}\right)\left(\frac{3}{2}\beta - \gamma\right) - (n-2)\frac{\beta^2}{2}}\right].$$

For β sufficiently large, there exists Y satisfying the inequality and that $Y \geq \Phi$ (as $\lim_{\beta \to \gamma}[.] > 0$ and $\lim_{\beta \to \gamma} \Phi = 0$).

Using (4), non-signatory i's change in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $\frac{1}{2} Y \beta^2 (n-7)$. This is non-negative for $n \geq 7$, thus i does not deviate at h. For $n \in [4, 6]$, i does not deviate at h if $Y \leq \frac{2(\gamma - \beta)}{\beta^2(7-n)}$. There exists Y satisfying the inequality and that $Y \geq \Phi$, since the RHS of this inequality is greater than Φ if $\left(n^2 - 7\right)\beta + (1-n)\gamma > 0$, which holds given $n \geq 4$.

Consider $h = a^1$ such that $m(a^1) \in [3, n-1]$. Using (5), signatory i's change in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $Y\Xi$ where

$$\Xi := \left(m\left(a^1\right) - 1\right)\left(\beta - \frac{\gamma}{m\left(a^1\right)}\right)\frac{\beta}{2} - \left(n - m\left(a^1\right)\right)\frac{\beta^2}{2}.$$

If $\Xi \geq 0$, then i has no reciprocity incentive to deviate at h. Suppose $\Xi < 0$, then i does not deviate if $Y \leq \left[\frac{\gamma - \beta m(a^1)}{\Xi m(a^1)}\right]$. For β sufficiently large, there exists Y satisfying the inequality and that $Y \geq \Phi$ (as $\lim_{\beta \to \gamma}[.] > 0 > \lim_{\beta \to \gamma} \Phi = 0$).

Non-signatory i's change in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $\frac{1}{2}Y\beta^2(n - 2m(a^1) - 1)$ (use (4)). If $n - 2m(a^1) - 1 \geq 0$, then $\frac{1}{2}Y\beta^2(n - 2m(a^1) - 1) \geq 0$, thus i has no reciprocity incentive to deviate at h. If $n - 2m(a^1) - 1 < 0$, then i does not deviate at h if

$$Y \leq \frac{2(\gamma - \beta)}{\beta^2(2m(a^1) + 1 - n)}.$$

The RHS is strictly greater than Φ if $(n^2 - 2m(a^1) - 1)\beta + (1 - n)\gamma > 0$ which is true as β tends to γ.

Finally, at $h = a^1$ such that $m(a^1) = n$ and the initial node, player i has neither material nor reciprocity incentives to deviate. ■

Proof of Proposition 2 (3-players)

(a) Let $n = 3$. Consider s^* such that each $i \in N$ signs, then does not invest if a^1 is such that $m(a^1) = 1$ and does invest otherwise. Reason as follows to verify the profile is SRE for an interval of Y less than Y^*. Consider $h = a^1$ such that $m(a^1) = 3$, so all invest. Signatory i has neither material nor reciprocity incentive to deviate to not investing. Now consider $h = a^1$ such that $m(a^1) = 2$, so all invest. Signatory i has neither a material nor a reciprocity incentive to deviate to not investing at h. Non-signatory i has a material incentive to deviate to not investing at h and a reciprocity incentive to not do so. Reason as follows to identify Y such that i does not deviate. Using (4), non-signatory i's increase in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is no less than his reduction in material payoff if $Y \geq Y''(\beta, \gamma)$, where

$$Y''(\beta, \gamma) := \frac{\gamma - \beta}{\beta(3\beta - \gamma)}.$$

Note that $Y''(\beta, \gamma) \geq Y^*$ iff $\frac{\gamma}{\beta} + 1 > 3$, however given $\lceil \frac{\gamma}{\beta} \rceil < n = 3$, then $Y''(\beta, \gamma) < Y^*$. Thus i does not deviate at h if $Y \in (Y'', Y^*)$. Now consider $h = a^1$ such that $m(a^1) = 1$, so zero invest. Player i has neither a material nor a reciprocity incentive to deviate to investing. Then consider $h = a^1$ such that $m(a^1) = 0$, so all players invest. Non-signatory i, faces identical incentives as a non-signatory at a history with 2 signatories, i does not deviate at h if $Y \in (Y'', Y^*)$. Finally, at the initial node, i has neither reciprocity nor material incentives to deviate.

(b) Let $n = 3$. Consider s^* such that each $i \in N$ does not sign, then does not invest if a^1 is such that $m(a^1) = 1$ and does invest otherwise. Reason as follows to verify the profile is SRE for an interval of Y less than Y^*. Stage 2 behaviour is optimal (part (a) of this proof). At the initial node, i has neither reciprocity nor material incentives to deviate. ■

Proof of Proposition 3 (2-players)

(a) Let $n = 2$. Consider s^* such that each $i \in N$ signs, then invests iff a^1 is such that $m(a^1) = 1$. Note that i has no material incentive to deviate at any history. Furthermore if $\beta \geq \frac{2}{3}\gamma$, i has no reciprocity incentive to deviate either.

(b) Let $n = 2$. Consider any s^* such that each $i \in N$ does not sign, then invests on path. Consider $h = a^1$ such that $m(a^1) = 0$, so all invest. Non-signatory i has a material incentive to deviate to not investing. Non-signatory i's increase in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is (4). We now demonstrate that this increase is reciprocity payoff is not sufficient to prevent deviation for $Y < Y^*$. Consider the following 4 exhaustive cases.

Case (i): s^* is such that j does not invest if only i signs and i does not invest if only j signs. Note that (4) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y > \frac{2(\gamma-\beta)}{\beta(2\beta-\gamma)}$. However the RHS is less than Y^* iff $\beta > \gamma$, which is false.

Case (ii): s^* is such that j does not invest if only i signs and i does invest if only j signs. Note that (4) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y \geq Y^*$, which is false.

Case (iii): s^* is such that j does invest if only i signs and i does not invest if only j signs. Note that (4) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y > \frac{2(\gamma-\beta)}{\beta(2\beta-\gamma)}$. However the RHS is less than Y^* iff $\beta > \gamma$, which is false.

Case (iv): s^* is such that j does invest if only i signs, i does invest if only j signs. Note that (4) is no less than the reduction in i's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ iff $Y \geq Y^*$, which is false.

Thus i deviates at h. ■

Proof of Proposition 4 (High reciprocity)

We demonstrate that a particular profile implying full investment, s^*, is a SRE of $\widehat{\Gamma}_A$ for all $Y \geq Y^*$. Consider s^* such that each $i \in N$ signs, then does not invest if a^1 is such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil - q$ where $q > 0$ and odd, and does invest otherwise. Reason as follows to confirm that for all $Y \geq Y^*$, players have no incentive to deviate at any history.

Consider $h = a^1$ such that $m(a^1) \in (\lceil \frac{\gamma}{\beta} \rceil, n]$. Signatory i has neither a material nor a reciprocity incentive to deviate to not investing at h. Non-signatory i faces identical incentives to the full investment profile in $\widehat{\Gamma}_P$, thus i does not deviate to not investing at h if $Y \geq Y^*$.

Now consider $h = a^1$ such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil$. Signatory i has neither a material nor a reciprocity incentive to deviate to not investing at h. Non-signatory i has a material incentive to deviate to not investing at h and a reciprocity incentive to not do so. Reason as follows to identify Y such that i does not deviate. Non-signatory i's increase in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is (4). This is no less than the reduction in i 's material payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ if $Y \geq \overline{Y}(n, \beta, \gamma, m(a^1))$, where

$$\overline{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) := \frac{2(\gamma - \beta)}{\beta\left(\left(n\beta - \gamma - \beta\right) m(a^1) + (n-1)\beta\right)}.$$

Note that $\overline{Y}(n, \beta, \gamma, m(a^1)) \geq Y^*$ iff $\frac{\gamma}{\beta} + 1 \geq n$, however by assumption $\lceil \frac{\gamma}{\beta} \rceil < n$, therefore $\overline{Y}(n, \beta, \gamma, m(a^1)) < Y^*$. Thus $Y \geq Y^*$ is sufficient to prevent non-signatory i deviating at h.

Now consider $h = a^1$ such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil - q$ where $q > 0$ and even, so all players invest. Non-signatory i, faces identical incentives as a non-signatory at a history with $\lceil \frac{\gamma}{\beta} \rceil$ signatories, thus i does not deviate to not investing if $Y \geq \overline{Y}(n, \beta, \gamma, m(a^1))$, which holds for $Y \geq Y^*$.

Signatory i has a material incentive to deviate to not investing. Using (5), signatory i's change in reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is strictly positive iff $\frac{Y\Omega}{2} > 0$ where

$$\Omega := \left(m\left(a^1\right) - 1\right)\left(\beta - \frac{\gamma}{m(a^1)}\right)\Delta\lambda_S + \left(n - m\left(a^1\right)\right)\beta\Delta\lambda_N,$$

$\Delta\lambda_S := (n+1)\beta - \frac{m(a^1)+1}{m(a^1)}\gamma$ and $\Delta\lambda_N := (n-1)\beta - \frac{m(a^1)}{m(a^1)+1}\gamma$. Note that $\Delta\lambda_S < \Delta\lambda_N$. To determine the sign of Ω reason as follows. Clearly $m(a^1) -$

$1 > 0$, $\beta - \frac{\gamma}{m(a^1)} < 0$ and $n - m\left(a^1\right) > 0$, thus we need only sign $\Delta\lambda_S$ and $\Delta\lambda_N$. Given $n > \left\lceil \frac{\gamma}{\beta} \right\rceil$, it follows that $(n-1)\beta > \gamma$, which implies that $\Delta\lambda_N > 0$. Consider how $\Delta\lambda_S$ influences the sign of Ω. If $\Delta\lambda_S \leq 0$, then $\Omega > 0$. If $\Delta\lambda_S > 0$, since $\Delta\lambda_S < \Delta\lambda_N$ we can write

$$\Omega > \left(\left(m\left(a^1\right) - 1 \right) \left(\beta - \frac{\gamma}{m\left(a^1\right)} \right) + \left(n - m\left(a^1\right) \right) \beta \right) \Delta\lambda_S > 0.$$

where the final inequality follows from $\left(m\left(a^1\right) - 1\right)\left(\beta - \frac{\gamma}{m(a^1)}\right) + \left(n - m\left(a^1\right)\right)\beta > \Delta\lambda_N > 0$. Therefore $\Omega > 0$ and signatory i's reciprocity payoff is strictly higher playing s_i^* instead of $s_i'\left(h, s_i^*\right)$. This increase in reciprocity payoff is no less than i's reduction in material payoff from playing s_i^* rather than $s_i'\left(h, s_i^*\right)$ if $Y \geq \widehat{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$ where

$$\widehat{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) := \frac{2\left(\gamma / m\left(a^1\right) - \beta\right)}{\Omega}.$$

Now argue that $\widehat{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) < Y^*$. To do so, take a function, $\tilde{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$, such that $\tilde{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) > \widehat{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$. To identify an appropriate function, reason as follows. For a given $\Delta\lambda_N$, Ω is decreasing in $\Delta\lambda_S$, and $\Delta\lambda_S$ is bounded by $\Delta\lambda_N$, thus let $\Delta\lambda_S = \Delta\lambda_N$ to minimise Ω. Furthermore, note that Ω is increasing in $\Delta\lambda_N$, and that $\Delta\lambda_N$ is strictly greater than β. To see this, note that $\beta n - \gamma > \beta$ since $n > \frac{\gamma}{\beta} + 1$ by assumption. Also note that $\frac{\gamma}{m(a^1)+1} - \beta > 0$ for all $m\left(a^1\right) \in \{1, \ldots, \left\lceil \frac{\gamma}{\beta} \right\rceil - 2\}$. Putting this together gives $\Delta\lambda_N > \beta$. Overall then, substitute $\Delta\lambda_S = \Delta\lambda_N = \beta$ into $\widehat{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$ to give

$$\tilde{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) := \frac{2(\frac{\gamma}{m(a^1)} - \beta)}{\beta((n-1)\beta - \frac{m(a^1)-1}{m(a^1)}\gamma)}.$$

Suppose that $\tilde{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) > Y^*$. This requires

$$(\gamma - \beta)\left((n-1)\beta - \frac{m(a^1)-1}{m(a^1)}\gamma\right) < \left(\frac{\gamma}{m(a^1)} - \beta\right)\beta(n-1).$$

Note that the LHS is increasing in $m\left(a^1\right)$ and that the RHS is decreasing in $m\left(a^1\right)$. Substituting $m\left(a^1\right) = 1$ gives $(\gamma - \beta)(n-1)\beta < (\gamma - \beta)(n-1)\beta$, a contradiction. Therefore $\tilde{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) \leq Y^*$. Overall, $Y^* \geq \tilde{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right) > \widehat{Y}\left(n, \beta, \gamma, m\left(a^1\right)\right)$, thus $Y \geq Y^*$ is sufficient to prevent signatory i deviating from s_i^* to $s_i'\left(h, s_i^*\right)$ at h.

Now consider $h = a^1$ such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil - q$ where $q > 1$ and odd, so zero players invest. Non-signatory i has neither material nor reciprocity incentives to deviate to investing at h. Signatory i has no material incentive to deviate to investing at h. Using (5), the change in signatory i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $Y\Omega/2$, which is strictly positive as already established, thus i does not deviate at h.

Now consider $h = a^1$ such that $m(a^1) = \lceil \frac{\gamma}{\beta} \rceil - 1$, so zero invest. Non-signatory i has neither a material nor a reciprocity incentive to deviate to investing. Signatory i has no material incentive to deviate to investing. Using (5), the change in signatory i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $\frac{Y}{2}(\Omega + (n - m(a^1))\beta(\beta - \gamma/(m(a^1) + 1)))$, which is strictly positive as we know $\Omega > 0$ and that $\frac{\gamma}{\beta} < m(a^1) + 1$, implies $\frac{\beta - \gamma}{m(a^1)+1} > 0$, thus i does not deviate at h.

Now consider $h = a^1$ such that $m(a^1) = 0$, so zero invest for $\lceil \frac{\gamma}{\beta} \rceil$ odd and n invest otherwise. For $\lceil \frac{\gamma}{\beta} \rceil$ odd, player i has neither material nor reciprocity incentives to deviate to investing. For $\lceil \frac{\gamma}{\beta} \rceil$ even, player i faces identical incentives as a non-signatory following a history of $\lceil \frac{\gamma}{\beta} \rceil$ signatories, thus does not deviate if $Y \geq \overline{Y}(n, \beta, \gamma, m(a^1))$, which is satisfied for all $Y \geq Y^*$.

Finally consider the initial node. Player i has neither material nor reciprocity incentives to deviate. Hence s^* is a SRE. ■

Proof of Proposition 5 (Low reciprocity)

(a) Consider s^* such that $m(Y)$ sign, then i invests iff i signed and there are at least $m(Y)$ signatories. For β sufficiently high, we first identify non-deviation conditions for signatories in the investment stage, then do the same for non-signatories, and then consider the sign-up stage. Using these conditions, we show that for all $Y \in (0, Y''')$ where $Y''' \in (0, Y^*)$, some s^* is a SRE.

Consider $h = a^1$ such that $m(a^1) > m(Y)$. Signatory i has no material incentive to deviate to not investing. Using (5), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$Y\frac{\beta}{2}\left[(\beta - \frac{\gamma}{m(a^1)})(m(a^1) - 1) - \beta(n - m(a^1))\right]. \tag{6}$$

Note that if $m(a^1) = \frac{\gamma}{\beta}$ then (6) < 0, if $m(a^1) = n$ then (6) > 0 and that (6) is strictly increasing in $m(a^1)$. There must then exist some $\widetilde{m} \in (\frac{\gamma}{\beta}, n)$

such that if $m(a^1) = \widetilde{m}$ then (6) $= 0$. For $m(a^1) \geq \widetilde{m}$, signatory i does not deviate at h. For $m(a^1) \in [m(Y)+1, \widetilde{m})$, signatory i does not deviate to not investing at h if $Y \leq Y_1(m(a^1))$, where

$$Y_1(m(a^1)) \equiv \frac{\beta - \frac{\gamma}{m(a^1)}}{-\frac{\beta}{2}\left[(\beta - \frac{\gamma}{m(a^1)})(m(a^1)-1) - \beta(n - m(a^1))\right]}.$$

As $Y(m(a^1))$ is strictly increasing in $m(a^1)$, $Y \leq Y_1(m(Y)+1)$ is a sufficient condition for signatory i to not deviate to not investing at h.

Consider $h = a^1$ such that $m(a^1) = m(Y)$. Signatory i has a material incentive to not deviate to not investing. Using (5), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$f(m(a^1)) \equiv \begin{cases} (\beta - \frac{\gamma}{m(a^1)})\frac{m(a^1)\beta - \gamma}{2}(m(a^1)-1) - \frac{\beta^2}{2}(n - m(a^1)) & \text{if } m(a^1) \geq 3, \\ (\beta - \frac{\gamma}{m(a^1)})(m(a^1)\beta - \frac{\beta}{2} - \gamma)(m(a^1)-1) - \frac{\beta^2}{2}(n - m(a^1)) & \text{if } m(a^1) = 2. \end{cases}$$

Note that $f(m(a^1))$ is strictly increasing in $m(a^1)$. If $f(m(a^1)) \geq 0$, signatory i does not deviate to not investing at h. If $f(m(a^1)) < 0$, signatory i does not deviate to not investing at h if $Y \leq Y_2(m(a^1))$, where

$$Y_2(m(a^1)) \equiv \frac{\beta - \frac{\gamma}{m(a^1)}}{-f(m(a^1))}.$$

If $m(Y) \geq 3$, then for $m(a^1) \geq m(Y)$, signatory i does not deviate to not investing if $Y \leq min\{Y_1(m(Y)+1), Y_2(m(Y)\}$. This can be rewritten as $Y \leq Y_1(m(Y))$ since for $m(a^1) \geq 3$, $Y_1(m(a^1)) < Y_2(m(a^1))$ and both are strictly increasing in $m(a^1)$. If $m(Y) = 2$, then for $m(a^1) \geq 2$, signatory i does not deviate to not investing if $Y \leq Y_2(m(Y))$ (since $Y_2(m(a^1)) < Y_1(m(a^1)) < Y_1(m(a^1)+1)$).

Consider $h = a^1$ such that $m(a^1) = m(Y) - 1 \geq 2$. Signatory i has a material incentive to deviate to investing. Using (5), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$Y\frac{\beta}{2}\left[(n - m(a^1))((m(a^1)+1)\beta - \gamma) - (m(a^1)-1)(\frac{\gamma}{m(a^1)} - \beta)\right]. \qquad (7)$$

Note that (7) > 0. Thus signatory i does not deviate to not investing at h if $Y \geq Y_3(m(a^1))$, where

$$Y_3(m(a^1)) \equiv \frac{\beta - \frac{\gamma}{m(a^1)}}{\frac{\beta}{2}\left[(n - m(a^1))((m(a^1)+1)\beta - \gamma) - (m(a^1)-1)(\frac{\gamma}{m(a^1)} - \beta)\right]}.$$

Consider $h = a^1$ such that $m(a^1) \in [2, m(Y)-1)$. Signatory i has a material incentive to deviate to investing. Using (5), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is

$$Y\frac{\beta}{2}\left[(n - m(a^1))\beta - (m(a^1) - 1)(\frac{\gamma}{m(a^1)} - \beta)\right]. \tag{8}$$

Note that $(8) > 0$. Thus signatory i does not deviate to not investing at h if $Y \geq Y_4(m(a^1))$, where

$$Y_4(m(a^1)) \equiv \frac{\beta - \gamma/m(a^1)}{\frac{\beta}{2}\left[(n - m(a^1))\beta - (m(a^1) - 1)(\frac{\gamma}{m(a^1)} - \beta)\right]}.$$

Since $Y_4(.)$ is increasing in $m(a^1)$, for all $m(a^1) \in [2, m(Y) - 1)$, signatory i does not deviate if $Y \geq Y_4(m(Y) - 2)$.

Consider $h = a^1$ such that $m(a^1) = 1$. Signatory i has no material incentive to deviate to investing. Using (5), the change in i's reciprocity payoff from playing s_i^* rather than $s_i'(h, s_i^*)$ is $Y\frac{\beta^2}{2}(n-1) > 0$, thus i has no reciprocity incentive to deviate either. Finally, Consider $h = a^1$ such that $m(a^1) \in \{0, 1\}$, i has neither material nor reciprocity incentive to deviate to investing.

Now consider non-signatories. Clearly non-signatory i has no material incentive to invest. For all $h = a^1$ such that $m(a^1) \notin \{m(Y)-1, m(Y)\}$, non-signatory i perceives others as no more kind than in the full investment profile in $\widehat{\Gamma}_P$, thus does not deviate to investing for $Y < Y^*$. For $h = a^1$ such that $m(a^1) = m(Y) - 1$, all other players are unkind to i so he has no reciprocity incentive to deviate to investing. For $h = a^1$ such that $m(a^1) = m(Y)$, using (4), non-signatory i does not deviate to investing if

$$\beta - \gamma + Y\frac{\beta^2}{2}\left[(m(a^1))^2 + m(a^1) + 1 - n\right] < 0.$$

If [.] is non-positive then the inequality holds. If [.] is positive, then i does not deviate if $Y \leq Y_5(m(a^1))$, where

$$Y_5(m(a^1)) \equiv \frac{2(\beta - \gamma)}{\beta^2\left[(m(a^1))^2 + m(a^1) + 1 - n\right]}.$$

Note that if

$$m(a^1) \leq \frac{\sqrt{8n-7} - 1}{2}, \tag{9}$$

then $Y_5(m(a^1)) > Y^*$. In this case it must be that $Y < Y_5(m(a^1))$ since $Y < Y^*$. If $m(a^1) > (\sqrt{8n-7}-1)/2$, then i does not deviate if $Y \leq Y_5(m(a^1))$.

Now consider the sign-up stage. Signatory i has identical incentives to a signatory's incentives at $h = a^1$ such that $m(a^1) = m(Y)$. Non-signatory i has identical incentives to a non-signatory's incentives at $h = a^1$ such that $m(a^1) = m(Y)$. Thus if players have no incentive to deviate in the investment stage, they also have no incentive to deviate in the sign-up stage.

In sum, if Y is sufficiently small such that $m(Y) \leq (\sqrt{8n-7}-1)/2$, then there exists a SRE where $m(Y) = 2$ if $Y \in I_2 \equiv \{Y : Y < Y_2(m(Y))\}$; $m(Y) = 3$ if $Y \in I_3 \equiv \{Y : Y \in [Y_3(m(Y)-1), Y_1(m(Y))]\}$; $m(Y) \in [4, \widetilde{m})$ if $Y \in I_{[4,\widetilde{m})} \equiv \{Y : Y \in [Y_4(m(Y)-1), Y_1(m(Y))]\}$ and $m(Y) \geq \widetilde{m}$ if $Y \in I_{\geq \widetilde{m}} \equiv \{Y : Y \geq Y_4(m(Y)-1)\}$. To show that there exists a SRE for all $Y \in (0, Y''')$, verify that $I_2 \cup I_3 \cup I_{[4,\widetilde{m})} \cup I_{\geq \widetilde{m}}$ covers $\mathbb{R}^+$ as follows. First, $I_2 \cap I_3 \neq \varnothing$ as $Y_3(m(Y)-1) < Y_2(m(Y)-1) < Y_2(m(Y))$. Second, $I_3 \neq \varnothing$ since $Y_3(m(Y)-1) < Y_1(m(Y)-1) < Y_1(m(Y))$. Third, since $Y_4(m(Y)) < Y_1(m(Y))$ and both increase in $m(Y)$, it holds that $Y_4(m(Y)-1) < Y_4(m(Y)) < Y_1(m(Y)) < Y_1(m(Y)+1)$. Therefore the intersections for the intervals for all $m(Y) \geq 3$ are also non-empty.

(b) See final paragraph of part (a) of this proof. ■

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