Phase-Measurement Interferometry

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Introduction

Phase-measurement interferometry is a way to measure information encoded in interference patterns generated by an interferometer. Fringe patterns (fringes) created by interfering beams of light are analyzed to extract quantitative information about an object or phenomenon. These fringes are localized somewhere in space and require a certain degree of spatial and temporal coherence of the source to be visible. Before these techniques were developed in the late 1970s and early 1980s, fringe analysis was done either by estimating fringe deviation and irregularity by eye or by manually digitizing the centers of interference fringes using a graphics tablet. Digital cameras and desktop computers have made it possible to easily obtain quantitative information from fringe patterns. The techniques described in this article are independent of the type of interferometer used.

Interferometric techniques using fringe analysis can measure features as small as a micron wide or as large as a few meters. Measurable height ranges vary from a few nanometers up to 10s of mm, depending upon the interferometric technique employed. Measurement repeatability is very consistent, and typically repeatability of 1/100 rms of a fringe is easily obtainable while 1/1000 rms is possible. Actual measurement precision depends on what is being measured and how the technique is implemented, while accuracy depends upon comparison to a sanctified standard.

There are many different types of applications for fringe analysis. For example, optical surface quality can be determined using a Twyman–Green or Fizeau interferometer. In addition, wavefront quality measurements of a source or an optical system can be made in transmission, and the index of refraction and homogeneity of optical materials can be mapped out. Many nonoptical surfaces can also be measured. Typically, surface topography information at some specific spatial frequency scale is extracted. These measurements are limited by the resolution of the optical system and the field of view of the imaging system. Lateral and vertical dimensions can also be measured. Applications of nonoptical surfaces include disk and wafer flatness, roughness measurement, distance and range sensing. Phase measurement techniques used in holographic interferometry, TV holography, speckle interferometry, moiré, grating interferometry, and fringe projection are used for nondestructive testing to measure surface structure as well as displacements due to stress and vibration. This article outlines the basics of phase measurement interferometry (PMI) techniques as well as the types of algorithms used. The bibliography lists a number of references for further reading.

Background

Basic Parts of a Phase-Measuring Interferometer

A phase-measuring interferometer consists of a light source, an illumination system (providing uniform illumination across the test surface), a beamsplitter (usually a cube or pellicle so both beams have the same optical path), a reference surface (needs to be good because these techniques measure the difference between the reference and test surface), a sample
fixture, an imaging system (images a plane in space where the surface is located onto the camera), a camera (usually a monochrome CCD), an image digitizer/frame grabber, a computer system, software (to control the measurement process and calculate the surface map), and often a spatial or temporal phase shifter to generate multiple interferograms.

**Steps of the Measurement Process**

To generate a phase map, a sample is placed on a sample fixture and aligned, illumination levels are adjusted, the sample image is focused onto the camera, the fringes are adjusted for maximum contrast, the phase shifter or fringe spacing is adjusted or calibrated as necessary, a number of images is obtained and stored with the appropriate phase differences, the optical path difference (OPD) is calculated as the modulo $2\pi$ phase and then is unwrapped at each pixel to determine the phase map.

To make consistent measurements, some interferometers need to be on vibration isolation systems and away from heavy airflows or possible acoustical coupling. It helps to cover any air paths that are longer than a few millimeters. Consideration needs to be made for consistency of temperature and humidity.

The human operating the interferometer is also a factor in the measurement. Does this person always follow the same procedure? Are the measurements sampled consistently? Are the test surfaces clean? Is the sample aligned the same and correctly? Many different factors can affect a measurement. To obtain repeatable measurements it is important to have a consistent procedure and regularly verify measurement consistency.

**Common Interferometer Types**

One of the most common interferometers used in optical testing is the Twyman–Green interferometer (Figure 1). Typically, a computer controls a mirror pushed by a piezo-electric transducer (PZT). The test surface is imaged onto the camera and the computer has a frame grabber that takes frames of fringe data.

The most common commercially available interferometer for optical testing is the Fizeau interferometer (Figure 2). This versatile instrument is very insensitive to vibrations due to the large common path for both interfering wavefronts. The reference surface (facing the test object) is moved by a PZT to provide the phase shift.

Interference microscopes (see Microscopy: Interference Microscopy. Interferometry: White Light Interferometry) are used for looking at surface roughness and small structures (see Figure 3). These instruments can employ Michelson, Mirau, and Linnik interference objectives with a laser or white light source, or Fizeau-type interference objectives that typically use a laser source because of the unequal paths in the arms of the interferometer. The phase shift is accomplished by moving the sample, the reference surface, or parts of the objective relative to the sample. Figure 3 shows a schematic of a Mirau-type interferometric microscope for phase measurement.
Determination of Phase

The Interference Equation

Interference fringes from a coherent source (e.g., a laser) are theoretically sinusoidal (Figure 4a), while fringes from an incoherent source (e.g., white light) are localized in a wavepacket at a point in space (Figure 4b) where the optical paths of the arms of the interferometer are equal and marked on Figure 4 as scanner positions \( z = 0 \) (see Interferometry: White Light Interferometry). For generality, this analysis considers the determination of phase within the wave packet for fringes localized in space.

Interference fringes at any point in the wavepacket can be written in the following form:

\[
I(x, y) = I_0 \{1 + \gamma(x, y, z) \cos[\phi(x, y)]\}
\]

where \( I_0 \) is the dc irradiance, \( \gamma \) is the fringe visibility (or contrast), \( 2I_0 \gamma \) is the modulation (irradiance amplitude or the ac part of the signal), and \( \phi \) is the phase of the wavefront as shown in Figure 5. For simplicity, this drawing assumes that the interference fringe amplitude is constant.

Fringe visibility can be determined by calculating

\[
\gamma = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

where \( I_{\text{max}} \) is the maximum value of the irradiance for all phase values and \( I_{\text{min}} \) is the minimum value. The fringe visibility has a real value between 0 and 1 and varies with the position along the wavepacket. The fringe visibility, as defined here, is the real part of the complex degree of coherence.

Types of Phase Measurement Techniques

Phase can be determined from either a number of interference fringe patterns or from a single interferogram with appropriate fringe spacing. Temporal techniques require an applied phase shift between the test and reference beams as a function of time while multiple frames of interference fringe data are obtained. Spatial techniques can obtain data from a single interferogram that requires a carrier pattern of almost straight fringes to either compare phases of adjacent pixels or to separate orders while performing operations in the Fourier domain. Spatial techniques may also simultaneously record multiple interferograms with appropriate relative phase shift differences separated spatially in space. Multiple frame techniques require more data than single frame techniques. Temporal techniques require that the interference fringes be stable over the time period it takes to acquire the number of images. While single-frame spatial techniques require less data and can be done with a single image, they generally have a reduced amount of resolution and less precision than temporal techniques.

There are literally hundreds of algorithms and techniques for extracting phase data from interference fringe data. The references listed in the Further Reading offer more details of these techniques.

Temporal Phase Measurement

Temporal techniques use data taken as the relative phase between the test and reference beams is modulated (shifted). The phase (or OPD) is calculated at each measured point in the interferogram. As the phase shifter is moved, the phase at a single point in the interferogram changes. The effect looks like the fringes are moving across the interferogram, and because of these techniques are sometimes called fringe scanning or fringe shifting techniques. However, the fringes are not really moving; rather the irradiance at a single detector point is changing (hopefully sinusoidally) in time (see Figure 6). A \( 180^\circ \) or \( \pi \) phase causes a bright fringe to become a dark fringe.

Phase Modulation Techniques

There are many ways to introduce a phase modulation (or shift). These include moving a mirror or the
sample, tilting a glass plate, moving a diffraction grating, rotating a polarizer, analyzer, or half-wave plate, using a two-frequency (Zeeman) laser source, modulating the source wavelength, and switching an acousto-optic Bragg cell, or magneto-optic/electro-optic cell. While any of these techniques can be used for coherent sources, special considerations need to be made for temporally incoherent sources (see Interferometry: White Light Interferometry). Figure 7 shows how moving a mirror introduces a relative phase shift between object and reference beams in a Twyman–Green interferometer.

**Extracting Phase Information**

Including the phase shift, the interference equation is written as

\[
I(x, y) = I_0(x, y)[1 + \gamma_0(x, y)\cos(\phi(x, y) + \alpha(t))] \tag{3}
\]

where \(I(x, y)\) is the irradiance at a single detector point, \(I_0(x, y)\) is the average (dc) irradiance, \(\gamma_0(x, y)\) is the fringe visibility before detection, \(\phi(x, y)\) is the phase of the wavefront being measured, and \(\alpha(t)\) is the phase shift as a function of time.

Since the detector has to integrate for some finite time, the detected irradiance at a single point becomes an integral over the integration time \(\Delta\) (Figure 8) where the average phase shift for the \(i\)th frame of data is \(\alpha_i\):

\[
I_i(x, y) = \frac{1}{\Delta} \int_{\alpha_i-(\Delta/2)}^{\alpha_i+(\Delta/2)} I_0(x, y)[1 + \gamma_0(x, y)\cos(\phi(x, y) + \alpha(t))]d\alpha(t) \tag{4}
\]

After integrating over \(\alpha(t)\), the irradiance of the detected signal becomes

\[
I_i(x, y) = I_0(x, y)\left\{1 + \gamma_0(x, y)\sin\left(\frac{\Delta}{2}\right)\right\} \times \cos(\phi(x, y) + \alpha_i) \tag{5}
\]

where \(\sin(\Delta/2) = \sin(\Delta/2)/(\Delta/2)\), which reduces the detected visibility.

**Ramping Versus Stepping**

There are two different ways of shifting the phase; either the phase shift can be changed in a constant and linear fashion (ramping) (see Figure 8) or it can be stepped in increments. Ramping provides a continuous smooth motion without any jerking motion. This option may be preferred if the motion does not wash out the interference fringes. However, ramping requires good synchronization of the camera/digitizer and the modulator to get the correct OPD (or phase) changes between data frames. Ramping allows faster data taking but requires electronics that are more sophisticated. In addition, it takes a finite time for a mass to move linearly. When ramping, the first frame or two of data usually needs to be discarded because the shift is not correct until the movement is linear.

The major difference between ramping and stepping the phase shift is a reduction in the modulation of the interference fringes after detection (the \(\sin(\Delta/2)\) term in eqn [5]). When the phase shift is stepped \((\Delta = 0)\), the \(\sin(\Delta/2)\) term has a value of one. When the phase shift is ramped \((\Delta = \alpha)\) for a phase

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**Figure 5** Variable definitions for interference fringes.

**Figure 6** Interference fringe patterns corresponding to different relative phase shifts between test and reference beams.

**Figure 7** Moving a mirror to shift relative phase by \(\lambda/4\).

**Figure 8** Change in OPD integrating over a \(\Delta\) phase shift with sample separation \(\alpha\).
shift of $\alpha = 90^\circ = \pi/2$, this term has a value of 0.9. Therefore, ramping slightly reduces the detected fringe visibility.

**Signal Modulation**

Phase measurement techniques assume that the irradiance of the interference fringes at the camera covers as much of the detector’s dynamic range as possible and that the phase of the interference fringe pattern is modulating at each individual pixel as the phase between beams is modulated. If the irradiance at a single detector point does not modulate as the relative phase between beams is shifted, the height of the surface cannot be calculated. Besides the obvious reductions in irradiance modulation due to the detector sampling and pixel size or a bad detector element, scattered light within the interferometer, and defects or dirt on the test object, can also reduce signal modulation. Phase measurement techniques are designed to take into account the modulation of the signal at each pixel. If the signal does not modulate enough at a given pixel, then the data at that pixel are considered unusable, flagged as ‘bad’ and are often left blank. Phase values for these points may be interpolated from surrounding pixels if there are sufficient data.

**Three-Frame Technique**

The simplest method to determine phase uses three frames of data. With three unknowns, three sets of recorded fringe data are needed to reconstruct a wavefront providing a phase map. Using phase shifts of $\alpha_1 = \pi/4$, $3\pi/4$, and $5\pi/4$, the three fringe measurements at a single point in the interferogram may be expressed as

$$I_1 = I_0 \left[ 1 + \gamma \cos \left( \phi + \frac{\pi}{2} \right) \right]$$

$$= I_0 \left[ 1 + \frac{\sqrt{2}}{2} \gamma (\cos \phi - \sin \phi) \right] \quad [6]$$

$$I_2 = I_0 \left[ 1 + \gamma \cos \left( \phi + \frac{3\pi}{2} \right) \right]$$

$$= I_0 \left[ 1 + \frac{\sqrt{2}}{2} \gamma (-\cos \phi - \sin \phi) \right] \quad [7]$$

$$I_3 = I_0 \left[ 1 + \gamma \cos \left( \phi + \frac{5\pi}{2} \right) \right]$$

$$= I_0 \left[ 1 + \frac{\sqrt{2}}{2} \gamma (-\cos \phi + \sin \phi) \right] \quad [8]$$

Note that $(x, y)$ dependencies are still implied. The choice of the specific phase shift values is to make the math simpler. The phase at each detector point is

$$\phi = \tan^{-1} \left( \frac{I_1 - I_2}{I_1 - I_3} \right) \quad [9]$$

In most fringe analysis techniques, we are basically trying to solve for terms such that we end up with the tangent function of the phase. The numerator and denominator shown above are respectively proportional to the sine and cosine of the phase. Note that the dc irradiance and fringe visibility appear in both the numerator and denominator. This means that variations in fringe visibility and average irradiance from pixel to pixel do not affect the results. As long as the fringe visibility and average irradiance at a single pixel is constant from frame to frame, the results will be good. If the different phase shifts are multiplexed onto multiple cameras, the results will be dependent upon the gain of corresponding pixels.

Bad data points with low signal modulation are determined by calculating the fringe visibility at each data point using:

$$\gamma = \frac{(I_3 - I_2)^2 + (I_1 - I_2)^2}{2I_0} \quad [10]$$

It is simpler to calculate the ac signal modulation $(2I_0\gamma)$ and then set the threshold on the modulation to a typical value of about 5–10% of the dynamic range. If the modulation is less than this value at any given data point, the data point is flagged as bad. Bad points are usually caused by noisy pixels and can be due to scratches, pits, dust and scattered light.

Although three frames of data are enough to determine the phase, this and other 3-frame algorithms are very sensitive to systematic errors due to nonsinusoidal fringes or nonlinear detection, phase shifter miscalibration, vibrations and noise. In general, the larger the number of frames of data used to determine the phase, the smaller the systematic errors.

**Phase Unwrapping**

The removal of phase ambiguities is generally called phase unwrapping, and is sometimes known as integrating the phase. The phase ambiguities owing to the modulo $2\pi$ arctangent calculation can simply be removed by comparing the phase difference between adjacent pixels. When the phase difference between adjacent pixels is greater than $\pi$, a multiple of $2\pi$ is added or subtracted to make the difference less than $\pi$. For the reliable removal of
discontinuities, the phase must not change by more than \( \pi \) \((\lambda/2 \text{ in optical path (OPD)})\) between adjacent pixels. Figure 9 shows an example of wrapped and unwrapped phase values.

Given a trouble-free wrapped phase map, it is enough to search row by row (or column by column) for phase differences of more than \( \pi \) between neighboring pixels. However, fringe patterns usually are not perfect and are affected by systematic errors. Some of the most frequently occurring error sources are noise, discontinuities in the phase map, violation of the sampling theorem and invalid data points (e.g., due to holes in the object or low modulation regions). Some phase maps are not easily unwrapped. In these cases different techniques like wave packet peak sensing in white light interferometry are used.

Figure 9 Fringes, wrapped, and unwrapped phase maps.

From Wavefront to Surface

Once the phase of the wavefront is known, surface shape can be determined from the phase map. Surface height \( H \) of the test surface relative to the reference surface at a location \((x, y)\) is given by

\[
H(x, y) = \frac{\phi(x, y)\lambda}{2\pi(\cos \theta + \cos \theta')}
\]  

[11]

where \( \lambda \) is the wavelength of illumination, and \( \theta \) and \( \theta' \) – the angles of illumination and viewing with respect to the surface normal – are shown in Figure 10. For interferometers (e.g., Twyman–Green or Fizeau) where the illumination and viewing angles are normal to the surface \((\theta = \theta' = 0)\), the surface height is simply:

\[
H(x, y) = \frac{\lambda}{4\pi}\phi(x, y)
\]  

[12]

Since the wavefront measured represents the relative difference between the interfering reference and test wavefronts, this phase map only directly corresponds to the surface under test when the reference wavefront is perfectly flat. In practice, the shape of the reference surface needs to be accounted for by measuring it using a known test surface and subtracting this reference measurement from subsequent measurements of the test surface.

Phase Change on Reflection

Phase shifting interferometry measures the phase of reflected light to determine the shape of objects. The reflected wavefront will represent the object surface (within a scaling factor) if the object is made of a single material. If the object is comprised of multiple materials that exhibit different phase changes on reflection, the measured wavefront needs to be corrected for these phase differences (see Interferometry: White Light Interferometry).

Overview of Phase Measurement Algorithms and Techniques

There are literally hundreds of published algorithms and techniques. The optimal algorithm depends on...
the application. Most users prefer fast algorithms using a minimal amount of data that are as accurate and repeatable as possible, immune to noise, adaptable, and easy to implement. In practice, there are trade-offs that must be considered when choosing a specific algorithm or technique. This section provides an overview of the types of algorithms to aid the reader in sifting through published algorithms.

**Synchronous Detection**

One of the first techniques for temporal phase measurement utilized methods of communication theory to perform synchronous detection. To synchronously detect the phase of a noisy sinusoidal signal, the signal is first correlated (or multiplied) with sinusoidal and cosinusoidal reference signals (signals in quadrature) of the same frequency and then averaged over many periods of oscillation. This method of synchronous detection as applied to phase measurement can be extracted from the least squares estimation result when the phase shifts are chosen such that $N$ measurements are equally spaced over one modulation period. With phase shifts $\alpha_i$ such that:

$$\alpha_i = \frac{2\pi}{N} i, \quad \text{with } i = 1, \ldots, N$$  \hspace{1cm} \text{[13]}

the phase can be calculated from

$$\phi(x, y) = \tan^{-1}\left[ \frac{\sum_i I_i(x, y) \sin \alpha_i}{\sum_i I_i(x, y) \cos \alpha_i} \right]$$  \hspace{1cm} \text{[14]}

Note that $N$ can be any number of frames (or samples). The more frames of data, the smaller the systematic errors. This technique does not take large amounts of memory for a large number of frames, because only the running sums of the fringes multiplied by the sine and cosine of the phase shift need to be remembered. The 4-frame algorithm from Table 1 is an example of a direct adaptation of synchronous detection where simple values of $1, -1$ or 0 for every $\pi/2$ phase shift can be assigned to the sine and cosine functions.

**Algorithm Design**

In the last ten years, a lot of work has been done to generalize the derivation of fringe analysis algorithms. This work has enabled the design of algorithms for specific applications, which are insensitive to specific systematic error sources.

Most algorithms use polynomials for the numerator and denominator. Given fringe data:

$$I_i = I_0[1 + \gamma \cos(\phi + \alpha_i)]$$  \hspace{1cm} \text{[15]}

the phase is calculated using

$$\phi = \tan^{-1}\left[ \frac{\sum_i n_i I_i}{\sum_i d_i I_i} \right]$$  \hspace{1cm} \text{[16]}

The numerator and denominator of the arctangent argument are both polynomials. The numerator is a sum proportional to the sine (imaginary part) and the denominator is a sum proportional to the cosine (real part),

$$\text{num} = 2kI_0 \gamma \sin \alpha_i = \sum_i n_i I_i$$  \hspace{1cm} \text{[17]}

$$\text{den} = 2kI_0 \gamma \cos \alpha_i = \sum_i d_i I_i$$  \hspace{1cm} \text{[18]}

where the constant $k$ depends on the values of coefficients. From this the fringe visibility is given by

$$\gamma = \sqrt{(\text{num})^2 + (\text{den})^2} / 2kI_0$$  \hspace{1cm} \text{[19]}

The coefficient vectors for the numerator and denominator are window functions. For an algorithm such as the 4-frame technique, the weights of all samples are equal $[1, 1, 1, 1]$. This makes the coefficients all equal. For other algorithms, such as the 5-frame technique, the weights are larger on the middle frames than on the outer frames. The weights for the 5-frame technique are $[1, 2, 2, 2, 1]$. A property of the coefficient vectors is that the sum of the coefficients for each, the numerator and denominator, should be zero. Examples of coefficient vectors for a few selected algorithms are given in Table 1.

| Table 1 Sampling function weights for a few selected algorithms |
|---|---|---|
| $N$ frames | Phase shift | Coefficients |
| 3 | $\pi$ | 1, −1, 0 |
| 2 | $\pi/2$ | 0, 1, −1 |
| 4 | $\pi/8$ | 0, −1, 0, 1 |
| 2 | | 1, 0, −1, 0 |
| 5 | $\pi/5$ | 0, −2, 0, 2, 0 |
| 2 | | $1, 0, 2, 0, 1$ |
| 7 | $\pi/7$ | $\sqrt[3]{0, 1, 1, 0, 1, -1, 0}$ |
| 3 | | −1, −1, 1, 2, 1, −1, −1 |
| 8 | $\pi/8$ | 1, 5, −11, 15, 15, −11, −5, 1 |
| 2 | | 1, 5, −11, 15, 15, −11, −5, 1 |
| 12 | $\pi/12$ | $\sqrt[3]{0, -3, -3, 3, 9, 6, -6, -9, -3, 3, 3, 0}$ |
| 3 | | 2, 1, −7, −11, −1, 16, 16, −1, −11, −7, 1, 2 |

Note that $I_0$ can be any number of frames (or samples). The more frames of data, the smaller the systematic errors. This technique does not take large amounts of memory for a large number of frames, because only the running sums of the fringes multiplied by the sine and cosine of the phase shift need to be remembered. The 4-frame algorithm from Table 1 is an example of a direct adaptation of synchronous detection where simple values of $1, -1$ or 0 for every $\pi/2$ phase shift can be assigned to the sine and cosine functions.
Heterodyne Interferometry

Historically, heterodyne techniques were developed and used before temporal phase-shifting techniques. These techniques generally determine the phase electronically at a single point by counting fringes and fractions of fringes. Areas are analyzed by scanning a detector.

Phase shifts are usually obtained using two slightly different frequencies in the reference and test beams. The beat frequency produced by the interference between the reference and test beams is compared to a reference sinusoidal signal, which may be produced either optically or electronically. The time delay (or distance traveled) between the crossing of the zero phase points of the test and reference sinusoidal signals is a measure of the phase. Every time the test signal passes through another zero in the same direction as the test surface is moved, another fringe can be counted. This is how fringe orders are counted. If the beam is interrupted as the detector is scanned across the interferogram, the fringe count is corrupted and the measurement needs to be started again. Frequency multiplication (harmonics) can also be used to determine fractions of fringes. Today, heterodyne techniques are used mainly in distance measuring interferometers. The precision and accuracy of distance measuring interferometers is at least on the order of 1 part in 10⁶.

Fourier-Transform Technique

The Fourier-transform technique is a way to extract phase from a single interferogram. It is used a lot in nondestructive testing and stellar interferometry where it is difficult to get more than a single interferogram. The basic technique is shown schematically in Figure 11. The recorded interferogram distribution is Fourier transformed, and one order (usually the + or − first order) is either isolated and shifted to zero frequency or filtered out using a rectangular window. After an inverse Fourier transform, the result is the phase.

To illustrate this technique mathematically, the interference equation is rewritten as

\[ I(x, y) = I_0(x, y) + c(x, y) \exp(i2\pi f_0x) + c^*(x, y)\exp(-i2\pi f_0x) \]  

where \( c(x, y) = I_0(x, y)\gamma(x, y) \exp[i\phi(x, y)] \) and the * indicates a complex conjugate. The term \( c(x, y) \) contains the phase information we wish to extract. After performing a one-dimensional Fourier transform:

\[ I(\xi, y) = I_0(\xi, y) + c(\xi - f_0, y) + c^*(\xi - f_0, y) \]  

where \( \xi \) is the spatial frequency in the x direction, and italics indicate Fourier transforms. The next step is to filter out and isolate the second term, and then inverse Fourier transform to yield \( c(x, y) \). The wavefront modulo 2π is then given by

\[ \phi(x, y) = \tan^{-1}\left\{ \frac{\text{Im}[c(x, y)]}{\text{Re}[c(x, y)]} \right\} \]  

where Re and Im refer to the real and imaginary part of the function.

This technique has limitations. If the fringes are nonsinusoidal, there will not be a simple distribution in the frequency space; there will be many orders. Another problem is overlapping orders. There needs to be a carrier frequency present that ensures that the orders are separated in frequency space. This carrier frequency is produced by adding tilt fringes until the orders are separated. This means that the aberration (fringe deviation) has to be less than the fringe spacing. Another problem is aliasing. If the interferogram is not sampled sufficiently, there will be aliasing and it will not be possible to separate the orders in frequency space. Finally, large variations in average fringe irradiance and fringe visibility across the interferogram can also cause problems.

Spatial Carrier-Frequency Technique

This is essentially the equivalent of the Fourier transform technique but is performed in the spatial domain. It is also used when there is only one interferogram available and its major applications include nondestructive testing and measurement of large optics.

These techniques relate closely to the temporal phase-measurement methods; however, instead of using a number of interferograms they can obtain all the information from a single interferogram. As an example, let’s assume that the fringes are vertical and parallel to the columns on the detector array.

Figure 11 Fourier transform technique.
The carrier frequency (i.e., the number of tilt fringes) is adjusted so that there is an \( \alpha \) phase change from the center of one pixel to the next. As long as there is not much aberration (deviation), the phase change from pixel to pixel across the detector array will be approximately constant.

When the fringes are set up this way, the phase can be calculated using adjacent pixels (see Figure 12). If one fringe takes up 4 pixels, the phase shift \( \alpha \) between pixels will be 90°. An algorithm such as the three-frame, four-frame, or five-algorithm can be used with adjacent pixels as the input. Therefore, 3, 4, or 5 pixels in a row will yield a single-phase point. Then, the analysis window is shifted sideways one pixel and phase is calculated at the next point. This technique assumes that the dc irradiance and fringe visibility do not change over the few pixels used to calculate each phase value.

**Spatial Multichannel Phase-Shift Techniques**

These techniques detect all phase maps simultaneously and multiplex the phase shift using static optical elements. This can be done by using either separate cameras as illustrated below or by using different detector areas to record each of the interferograms used to calculate the phase. The phase is usually calculated using the same techniques that are used for the temporal phase techniques.

As an example, a four-channel interferometer can be made using the setup shown in Figure 13 to record four interferograms with 90° phase shifts between them. Camera 1 will yield fringes shifted 180° with respect to camera 2, and cameras 3 and 4 will have phase shifts of 90° and 270°. The optical system may also utilize a holographic optical element to split the beam to multiplex the four phase shifts on four quadrants of a single camera.

**Signal Demodulation Techniques**

The task of determining phase can be broadened by looking toward the field of signal processing. For communication via radio, radar, and optical fibers electrical engineers have developed a number of ways of compressing and encoding a signal as well as decompressing a signal and decoding it. An interference fringe pattern looks a lot like an am radio signal. Thus, it can be demodulated in similar ways. In recent years many new algorithms have been developed by drawing on techniques from communication theory and applying them to interferogram processing. Many of these use different types of transforms such as Hilbert transforms for straight fringes or circular transforms for closed fringes.

**Extended Range Phase Measurement Techniques**

A major limitation of phase measurement techniques is that they cannot determine surface discontinuities larger than \( \lambda/4 \) (\( \lambda/2 \) in optical path (OPD)) between adjacent pixels. One obvious solution is to use longer wavelength sources in the infrared where optically rough surfaces look smooth and their shape can be measured. An alternative is two-wavelength interferometry where two measurements at different wavelengths are taken and the measurable height limitation is now determined by the equivalent wavelength:

\[
\lambda_{eq} = \frac{\lambda_1 \lambda_2}{|\lambda_1 - \lambda_2|}
\]

Another method that allows for measurement of smooth surfaces with large height discontinuities that is limited only by the working distance of the objective combines a white light and a phase measurement interferometric technique in one long scan. The position of the wave packet resolves 2\( \pi \) ambiguities that result from the arctangent function (see Phase unwrapping section above). The phase of the fringe close to the wave packet maximum is determined without ambiguity. Sometimes it is not the step height discontinuity that is a problem but rather the high slope of the measured surface. If we know that the surface is continuous, then the...
unwrapping procedure can take advantage of this a priori information to look for a continuous.

Techniques for Deformation Measurement

Some of the techniques for deformation measurement have already been described earlier (see sections on Single frame techniques and Spatial multichannel phase-shift techniques). However, fringes in any interferometer can be analyzed not only in the \(x, y\) planes but also in the \(x, z\) or \(y, z\) planes for which the carrier frequency of the fringes is introduced. Analysis of fringes in any plane has the same restrictions.

If the initial static object shape is measured using conventional phase measurement techniques, not only can the motion of each object point be determined but also the deformation of the whole object in time. If the motion of the object is periodic, then the object motion can be ‘frozen’ by using stroboscopic illumination of the same frequency as the object’s motion. Once the motion is frozen, any temporal technique can be used to measure object shape at some phase of its motion. Changing the time offset between stroboscopic illumination and the periodic signal driving the motion, the object can be ‘frozen’ and thus measured at different phases of its periodic motion.

Systematic Errors

Noise Sensitivity

Measurement noise mostly arises from random fluctuations in the detector readout and electronics. This noise reduces precision and repeatability. Averaging multiple measurements can reduce effects of these random fluctuations.

Phase Shifter Errors

Phase shifter errors can be due to both miscalibration of the system and nonlinearities in the phase shift. It is possible to purchase very linear phase shifters. It is also possible to correct nonlinearities by determining the voltage signal making the phase shifter provide a linear phase shift.

Linear phase shifter errors (miscalibration) of the phase shift have an error signature that is at twice the frequency of the fringes. If there are two fringes across the field of view, the error signature will have four across the field of view. Figure 14 shows the difference between a calibrated and an uncalibrated phase shifter as well as the difference in error for two different phase measurement algorithms. Some algorithms are obviously more sensitive than others to this type of error.

Other Types of Systematic Errors to Consider

Other types of errors to consider are detector nonlinearities, quantization errors due to analog-to-digital converters, and dissimilar materials (see Phase change upon reflection). For temporal phase measurement techniques errors due to vibration and
air turbulence need to be considered as well. Spatial phase measurement techniques are sensitive to mis-calibrated tilt (wrong carrier frequency), unequally spaced fringes, and sampling and windowing in Fourier transform techniques.

**Choosing an Algorithm or Technique**

Each algorithm and type of measurement technique is sensitive to different types of systematic errors. Choosing a proper algorithm or technique for a particular type of measurement depends on the specific conditions of the test itself and reducing the systematic errors for a particular type of measurement. This is the reason that so many algorithms exist. The references in the bibliography will help the reader determine what type of algorithm will work best for a specific application.

**Examples of Applications**

Phase shifting interferometry can be used for measurements such as hard disk flatness, quality of optical elements, lens curvature, dimensions and quality of air-bearing surfaces of magnetic read/write heads, cantilevers, and semiconductor elements. Figure 15 shows results for measurements of a hard disk substrate and a roughness grating.

**Conclusions**

Phase measurement interferometry techniques have increased measurement range and precision enabling the production of more complex and more precise components. As work continues on development of interferometric techniques, phase measurement techniques will continue to become more robust and less sensitive to systematic errors. Anticipated advances will enable measurements of objects that were unimaginable 20 or 30 years ago.

**See also**


**Further Reading**


