This article focuses on interferometric techniques, which are various methods for obtaining and analyzing an interferogram (fringe pattern) in an interferometer (see INTERFEROMETERS AND INTERFEROMETRY). An interferogram is the recorded interference signal between two beams of light exiting from the same radiation source. An interferogram carries information about the phase (the difference between the two wave fronts of the beams), the spectral content of the source, and its spatial distribution. These three phenomena can be analyzed using interferometric techniques.
The techniques that measure the phase across the plane employ a two-beam interferometer to record an interference signal in the form of an interferogram (fringe pattern). These interferograms, consisting of fringes of equal thickness called Fizeau fringes (see INTERFEROMETERS AND INTERFERENCE), carry information about the phase, which corresponds to the relative difference between the test and the reference wavefront. Phase variation across the plane can represent surface shape, material homogeneity or thickness, or optical-component quality. A simple way to approximate the maximum deviation of phase from the plane involves introducing tilt between the test and reference wavefronts. In this way almost straight and parallel fringes are introduced into the interferogram, Fig. 1(a). Phase deviation from the plane is seen as a departure from straight lines in the fringe pattern, Fig. 1(b). In the two interferometers most commonly used for surface contouring, the Fizeau and Twyman–Green (see INTERFERENCE AND INTERFEROMETRY), the spacing between two neighboring fringes corresponds to a surface height difference of $1/2$ of the source wavelength used for testing ($1$ fringe $= \lambda/2$). The maximum surface height deviation from the plane, Fig. 1(b), can be found by measuring the maximum fringe deviation from a straight line $|\Delta(x, y)|$ and the average spacing between two neighboring fringes ($S$). However, the relationship between fringe spacing and surface height depends not only on the wavelength used for measurement but also the interferometric configuration and the indices of refraction of the media the beam passes through. Interferometric techniques used to measure the phase across the plane typically use a coherent light source such as a laser so as to achieve good-contrast fringes; these techniques may involve single or multiple interferogram analysis to retrieve the phase.
1.1 Fringe Tracking

Until the late 1970s, fringe tracking (Yatagai, 1993) was the main way to analyze interferograms quantitatively, and although this method required only a single interferogram, it was relatively imprecise; the accuracy obtained was only one tenth of the fringe spacing. Fringe tracking works by recording the positions of the fringe intensity maxima and minima; the rest of the data are interpolated in order to create a phase map across the plane. Fringe tracking requires that the fringes be ordered so as to reflect changes in the phase between fringes, a process simplified by introducing tilt between the reference and test wave fronts. However, the direction of the tilt must be known beforehand in order to determine the shape of the tested surface correctly, i.e., concave or convex (Fig. 2). If closed fringes are present in the interferogram, then fringe ordering also requires some knowledge about the tested element.

1.2 Interferogram Plus Intensities

The phase across the plane can be determined at each point without interpolation if the intensity distribution of each beam is recorded separately and one interferogram is taken. After these three intensity distributions have been determined, the phase variation can be calculated from the following equation that describes the intensity distribution $I$ in an interferogram:

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos \phi(x, y), \quad (1)$$

where $I_1$ and $I_2$ are the intensities of the two beams, $2(I_1 I_2)^{1/2}$ is the interference visibility (modulation) and $\phi(x, y)$ is the measured phase. Michelson or Mach–Zehnder interferometers (see INTERFERENCE AND INTERFEROMETRY) can use this method since the beams in the reference and test arms can be blocked separately. However, blocking each beam separately is not always possible, as when using a Fizeau interferometer, and alternative methods for retrieving the phase must be used.

1.3 Temporal Phase Measurement—Temporal Heterodyning

Temporal phase-measurement techniques (also known as temporal heterodyning, phase stepping, or phase shifting) introduce multiple times a known increment in the relative difference between the test and reference beams, called a phase shift. At least three interferograms are necessary to determine the phase encoded in the intensity distribution, Eq. (1), that can be rewritten as

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos[\phi(x, y') + m\tau] \quad (2)$$

where $\tau$ is the phase shift induced $m$ times. The
most common way to accomplish this phase shift is by changing the optical path difference between the interfering beams through a shift of the reference mirror along the optical axis, Fig. 3, although other ways exist—for example, by tilting a glass plate, moving a grating, rotating a half-wave plate or analyzer, or using an acousto-optic modulator (Creath, 1993). While some of these ways of shifting the phase alter the optical path between the interfering beams, others change the optical phase of one of the beams; nevertheless, both methods lead to an incremental difference (phase shift) between beams. Figure 4 shows three typical interferograms registered in the temporal phase measurement technique.

The measurement precision can be improved using more samples. An example of a common algorithm employing five intensity samples \( I_1, \ldots, I_5 \) with a relative \( \pi/2 \) phase shift to retrieve the phase \( \phi(x, y) \) is (Creath, 1993)

\[
\phi(x, y) = \arctan \left( \frac{I_1 + I_5 - 2I_3}{2I_2 + 2I_4} \right)
\]  

(3)

Because an arctangent function is not continuous and gives multiple solutions, an unwrapping procedure is employed to remove the discontinuities in the retrieved phase and essentially automatically orders the fringes (Robinson, 1993). Unwrapping procedures assume that the maximum phase discontinuity equals the difference between two neighboring solutions \( 2\pi \), which is a property of the arctangent function. The values of the phase, which are measured in radians, are then converted into the values that the phase represents—for example, surface height or thickness variations. Temporal phase measurement is 10–100 times more precise than tracking the position of the fringe maxima and minima, since the repeatability of temporal phase measurement is \( 1/100–1/1000 \) of a wavelength.

Temporal phase-measurement techniques generally require that each interference fringe be sampled at least twice per fringe spacing; this constraint limits the slope of the phase that can be measured. However, a few techniques work around this limitation (Greivenkamp and Brunning, 1992). An often-used solution, white-light interferometry (see Sec. 1.5), is suitable when measuring rough surfaces or surfaces with discontinuities where sampling twice per fringe is not possible (Larkin, 1996).

Finally, although temporal phase-measurement techniques usually capture interferograms sequentially in time, and only phase not varying in time can be analyzed, a few techniques modify the configuration of the interferometer so that time-varying phase can be measured, e.g., gas flow. These techniques record interferograms simultaneously using multiple cameras. However, accomplishing
this end requires complicated hardware and alignment is critical (Kujawinska, 1993).

1.4 Single Interferogram Analysis—Spatial Heterodyning

Measuring time-varying phase can also be accomplished through the analysis of a single interferogram. Two common techniques discussed below require that a large tilt be induced between the reference and test wave fronts and that the direction of the tilt or knowledge about the tested object be known beforehand. In addition, like the temporal phase-measurement techniques described above, these techniques also require a phase unwrapping procedure in order to obtain a continuous phase map, and each fringe must be sampled at least twice per fringe spacing.

1.4.1 Fourier-Transform Technique In the Fourier-transform technique (Takeda, 1990) the discrete Fourier transform of a single interferogram is calculated revealing three distinct spectral orders, typical for a consinusoidal function, which also represents fringes (see FOURIER AND OTHER MATHEMATICAL TRANSFORMS; Ramirez, 1985). The number of almost parallel fringes from the induced tilt between the reference and test wave fronts must be large enough to separate the spectral orders to enable filtering one of these orders at the spatial frequency of the fringes. The inverse Fourier transform is then performed and the phase encoded in the interferogram is retrieved from the arctangent function of the real and imaginary parts of the inverse Fourier transform.

1.4.2 Synchronous Detection Synchronous detection (Womack, 1984) calculates the phase by sequentially applying a temporal phase-measurement technique algorithm to a few consecutive intensity samples across the plane of a single interferogram. A known phase shift is induced between adjacent sampling points, rather than between interferograms as discussed above, by tilting one wave front with respect to the other. This phase shift manifests itself as a set of straight parallel fringes. Fringe spacing must be chosen carefully so that the phase difference between adjacent sampling points is close to the phase shift required by the algorithm employed. Uniform sample reflectivity and intensity of both beams are assumed in this technique.

1.5 White-light Interferogram Analysis

Interference fringes can also be observed by use of a light source with a low degree of coherence such as a halogen lamp. Interferograms obtained with these “white light” sources show fringes of good modulation only when the optical paths of the two beams match closely. This modulation characteristic becomes important when one is trying to determine large discontinuities in phase where temporal phase-shifting interferometry is unable to assign the correct order to the fringes. White-light interferometry (also known as low-coherence interferometry) allows for the easy identification of the fringe with the best visibility because the contrast falls off so quickly. For example, Figs. 5(a) and 5(b) show interferograms of a step object; the interferogram in Fig. 5(a) is achieved with a laser as a source, and in Fig. 5(b) a white light source is used. In Fig. 5(a) there are multiple solutions to the ordering of the fringes. Thus determining the height of the step (size of the phase discontinuity) is impossible without additional information. In Fig. 5(b) the fringes can be ordered fairly simple by tracking the highest visible fringe, and the step height can then be easily determined. Whitelight interferometry is generally used for measuring rough surfaces or any surface with discontinuities where sampling twice per fringe is not possible.

To measure a surface using white-light interferometry, the surface is scanned in height relative to the interferometer while frames of data are acquired. As the separation between the surface and the interferometer changes, the interference fringes will intersect heights on the test surface where the optical paths are equal. A complete scan conceptually includes a three-dimensional volume of data corresponding to the location of the white-light fringes in space. The highest-contrast fringe or the peak of the fringe modulation can be used to determine the height at a single point on the test surface. This simple approach has spurred a number of new techniques for determining surface structure (Larkin, 1996). Some techniques follow
the highest-contrast fringe and others look at the peak of the fringe modulation. The relative position of these two quantities depends upon the phase change upon reflection of the light from the test surface.

1.6 Summary

The various interferogram techniques described in this section complement one another. The temporal technique and the single-interferogram techniques are used for high-precision measurement of smooth surfaces, while the fringe-modulation peak-sensing technique can examine very rough and discontinuous surfaces. Moreover, interferometric techniques analyze not only static but also time-varying phase. Techniques such as synchronous detection, Fourier transform, and temporal-phase measurement employing a spatial separation of interferograms can measure time-varying phase.

2. MEASUREMENT SENSITIVITY AND LIMITATIONS

2.1 Measurement Sensitivity

The sensitivity of an interferometric measurement can be changed by altering the test wavelength, the angle of incidence of the test beam illumination, and the number of passes through or reflections from the tested element. Varying the sensitivity of the interferometer allows for the measurement of rougher or smoother test objects. For example, a source that emits a shorter wavelength will produce a greater number of fringes in an interferogram; this choice of source wavelength increases the sensitivity of the interferometer and allows for the testing of smoother objects with high precision. A large angle of incidence of test beam illumination decreases sensitivity by producing fewer fringes in the interferogram and allows for rougher test-object measurement.

2.2 Removal of Systematic Errors

The reference wavefront in an interferometric setup is not ideal, for it carries information about imperfections in both the reference mirror and the optical system of the interferometer. If an ideal surface were available, all systematic errors could be measured and then subtracted. However, because such a surface does not exist, a set of multiple measurements is necessary in order to subtract these systematic errors (Schultz and Schwider, 1976). In addition, because various interferogram analysis techniques contain systematic sources of error, ways have been developed to decrease this error. For example, phase-shifting interferometry is vulnerable to phase-shifter miscalibration. Methods of calibrating the phase shifter precisely and the use of error-reducing algorithms help reduce any systematic errors (Creath, 1993).
2.3 Reduction of Nonsystematic Errors

High-precision interferometric measurements require special laboratory conditions. Care needs to be taken to ensure vibration and acoustic isolation as well as temperature stability and controlled air flow within the exposed optical path. If the laboratory conditions are not well controlled, nonsystematic errors may dominate a measurement. However, as long as these errors are random, statistically independent measurements can be averaged to reduce their effects and improve measurement precision and accuracy.

3. FORM MEASUREMENT

Testing form requires the measurement of phase across the plane. Form is defined as object shape or optical-system quality, in order to differentiate it from other measurable quantities such as object displacement and vibration. This section divides the examination of form measurement into optical-component testing and engineering-surface testing. The optical-component testing section is broken down into the types of components tested because similar components are tested in similar interferometric configurations. The engineering-surface testing section is organized around measurement techniques because the wide range of engineering surfaces requires a variety of different testing methods. The interferometric technique used to measure form depends on both the features of the tested object, such as its size, shape, or roughness, and the type of data that needs to be obtained.

3.1 Optical-System and -Component Testing

Interferometric techniques have proven extremely useful in testing optical systems and components, especially with the strict tolerances imposed on high-precision components and the smooth wave fronts generated by optical systems. The techniques detailed below for testing optical elements are applicable to most types of interferometers but are mainly used with the Fizeau or Twyman–Green interferometers (see INTERFEROMETERS AND INTERFEROMETRY, Secs. 2.1 and 2.4). The two interferometers have different strengths. The Fizeau interferometer is less sensitive to vibration than the Twyman–Green because the two optical paths of the Fizeau interferometer are virtually common. However, because the test and reference optical paths differ in length, the Fizeau requires a source with higher coherence than a Twyman–Green does.

3.1.1 Flats Optical flats are typically measured against a reference flat in a collimated beam. Often the flatness is specified as the maximum surface irregularity over a given surface area. Commercially available instruments can routinely measure surface irregularities as small as 1/100 of the source wavelength. If more accurate measurements are needed, flat surfaces can be tested relative to a liquid surface (Bünning, 1956); the surface of the liquid, which must be clear and highly viscous, works as a reference flat because its radius equals that of the Earth’s and for a diameter of 0.5 m the maximum error is \( \frac{\lambda}{100} \) for \( \lambda = 500 \text{ nm} \). However, practicalities such as the influence of vibration and dust settling on the surface are problems. If a good flat is not available to test against, three flats can be tested in triple combinations in order to obtain absolute line profiles of each flat (Dew, 1966; Fritz, 1984).

3.1.2 Spherical and Aspheric Surfaces Spherical surfaces may be tested for their departure from spherical shape by comparing them with an ideal spherical reference wave front. When a spherical surface is tested in a Fizeau interferometer, Fig. 6, a transmission reference sphere is placed in the common path of the beams and the radius of the beam incident on the tested spherical surface must fill and match the radius of that surface.

Aspheric surfaces are difficult to test against a spherical wave front because they may depart too much from a spherical shape and generate too many fringes to resolve the interferogram. Null lenses and computer-generated holograms or real holograms (Offner and Malacara, 1992) are used to produce a wave front matching a perfect aspheric wave front, thereby reducing the number of fringes in the interferogram. Shearing interferometry reduces the number of fringes by interfering the tested wave front with its own replica, which can be
displaced laterally, rotated, radially expanded or contracted. The number of fringes varies with the amount of shear, and fringes do not represent the shape of the aspheric surface directly (Malacara, 1992; Mantravadi, 1992).

### 3.1.3 Prisms

Although prism angles may be measured with an autocollimator and a goniometer, interferometers are the most accurate method for measuring angle deviations. The procedure simply involves inserting the test prism and return flat into the test arm of the interferometer. Prisms with roof angles of 90° may be put into the test arm with (Fig. 7) or without the return mirror, as the beam can be retroreflected back to the system. The maximum deviation from 90° that a phase measuring interferometer can measure is about 1 min of arc.

### 3.1.4 Light Sources

The quality of an optical wave front exiting from a light source may be tested using a point-diffraction interferometer or a special configuration of a Mach–Zehnder interferometer (Greivenkamp and Bruning, 1992). Both setups induce a spherical reference wave front through the use of a pinhole in the reference arm.

### 3.1.5 Homogeneity

Homogeneity can be measured by preparing a plane-parallel sample and placing it in the test arm of an interferometer. A sample does not have to be specially prepared if it is submerged in a refractive-index–matching oil. Systematic errors can be subtracted by making a measurement before placing the sample in the test arm. Surface-independent measurements (Schwider, 1990) require multiple takes to remove variations due to the test sample's surface; this is done to isolate the refractive-index variations in the sample.

### 3.2 Testing Engineering Surfaces

Interferometry can also be used to measure the shape and properties of surfaces not used as optical components, such as machined metal or molded plastics. Because testing engineering surfaces is rather a specialized application of interferometric techniques, this section focuses on the methods developed to test these surfaces. The types of surfaces to be tested vary from very large ones that require overall testing of shape to very small ones that require a high resolving power to measure their micro-roughness. Speckle interferometry or holography may also be used for testing engineering surfaces, but since these methods can also be used for determining object displacement and vibration, two parameters important in experimental mechanics, they are discussed in the nondestructive testing section.

#### 3.2.1 Fringe Projection

Many engineering surfaces are too complex to be measured with standard interferometric techniques. Rowe and Welford (1967) devised a simple approach for measuring surface contours by projecting interference fringes or a grating onto an object and then viewing the fringes from another angle. Fringe projection is related to optical triangulation, which uses a single point of light and light sectioning; a single line is projected onto an object and then viewed from a different direction.
3.2.2 Moiré  In optics the term moiré (Rayleigh, 1874) refers to a pattern arising when two gratings of approximately equal spacing are superimposed. Interferometry using moiré (Patorski, 1993) can be implemented in a number of ways. A common approach is to project interference fringes (or a grating) onto an object and view them through a second grating placed in front of the viewer; see Fig. 8.

These techniques produce fringes corresponding to contours of equal height on the object. The contour interval is determined by the fringe (grating line) spacing and the angle between the illumination and viewing directions. Phase shifting can be introduced by shifting one grating or the projected fringes. Moiré and fringe-projection interferometry complement conventional interferometry, as they can contour objects with an effective contour interval of 10 μm and larger.

3.2.3 Interference Microscopy  Interference microscopy combines an interferometer and a microscope into one instrument and is used for measuring engineering surfaces that demand testing with high resolving power. Interference microscopy uses three types of interferometric objectives: Michelson, Mirau, and Linnik (all are modifications of the Michelson interferometer). In a Michelson interferometer objective, which is used for small magnifications, a beam-splitter cube and reference mirror are inserted between the objective and the tested surface. At magnifications above five times, the working distance, which refers to the space between the last surface of the lens and the tested surface, becomes too small to squeeze in a beam splitter cube; instead a Mirau interferometric objective (described in Optical Instrumentation, Sec. 3.2.1) is employed. For magnifications of one hundred times and above, a Linnik interferometric objective (Linnik, 1933) is used since the working distance for such high magnification is even smaller and the Mirau design is impossible to implement. In a Linnik configuration a beam-splitter cube placed before the light reaches the objective directs the beam onto two objectives; one beam is directed to the reference mirror and the other is directed to the test surface. Any type of phase-across-the-plane analysis technique discussed in this article may be used with these microscopes. Other interference microscopes are reviewed by Steel (1983).

3.2.4 Fringes of Equal Chromatic Order Interferometry  Fringes of equal chromatic order (FECO) interferometry (Fig. 9) is based on multiple-beam interference between two highly reflective and nearly parallel plates and results in very sharp fringes (Tolansky, 1970). Multiple-beam interference magnifies the extent of the deformation, thereby making it easier to detect small imperfections on the tested surface. FECO uses a white-light source and is combined with a spectrometer to test thickness variation and surface roughness, which are determined from the width of the fringes. Small sections of the test surface are observed through a spectrometer that displays fringes of equal chromatic order. A FECO interferometer can achieve a precision level of λ/500 and is capable of distinguishing hills from valleys by use of the direction of the bending of the colored fringes. However, the test surface must be overlaid with a highly reflective coating and the reference surface rms (root mean square) roughness should be better then λ/20.

4. PHASE AT ONE POINT

Phase-at-one-point interferogram detection is commonly used to measure length, the spectral content of a source (see Sec. 6), or its polarization (see Sec. 8.4). For length measurement these interferometric techniques
Interferometric Techniques

Hariharan, 1985; Steel, 1983) generally utilize a single detector to measure the phase at a single point of the interferogram where the fringe spacing is much larger than the size of the detector. The interferometric techniques that test the phase at one point measure length with very high precision, up to $1/10^4$ of a fringe, and are extensively used to measure relative and absolute distance. A few common single-point–phase detection techniques are described below. While measuring length is an important application of these techniques, the phase at a single point may represent other parameters, such as changes in pressure, temperature, or rotation.

4.1 Heterodyne Interferometry

Heterodyne interferometry observes interference between two beams with slightly different optical frequencies. This frequency difference is usually achieved by introducing a frequency shift into one of the beams. This frequency shift may be a result of a linear change in the phase difference and is achieved by rotating a radial grating, combining stationary and rotating quarter-wave plates, or moving a mirror in one of the arms of the interferometer. When the two beams interfere, a photodetector records a traveling interference signal at the beat frequency, which is equal to the difference in the two optical frequencies of the beams.

Heterodyne interferometry is often used to detect small changes in length. This detection is accomplished by evaluating changes in the beat frequency between the stable laser and the test (slave) laser frequency. Heterodyne interferometry measures changes in length by measuring changes in frequency.

4.2 Fringe Counting

These techniques essentially count the changes in intensity as they scan across a point detector while increasing the optical path difference between two beams. If the signal is lost for any reason, the measurement must be repeated. Two basic approaches to phase detection using fringe counting are often used. The first is based on observing the intensity changes in two interferograms. Into one interferogram an additional $\pi/2$ phase shift is introduced between interfering beams in order to determine the direction of the phase-difference changes (signals are in quadrature). The $\pi/2$ phase shift is induced by use of a special beam splitter or through polarization effects. The phase change can be detected to within a fraction of the fringe by observing the circular pattern formed by the two intensity signals mapped onto the horizontal and vertical axes on an oscilloscope. A full circle is traced each time one full fringe is scanned. The drawback to this method is that any changes in the intensity signal (such as source intensity variations) could be interpreted as a fringe.

The second approach avoids this problem; it is based on a heterodyne system that detects the interference of beams of different
optical frequencies. Figure 10 shows a distance-measuring interferometer based on a heterodyne system that uses fringe counting. The differing frequencies can be obtained by using either a laser with two output frequencies or a single-frequency laser with two acousto-optic modulators. Either way the beams need to be orthogonally polarized. The first detector records the traveling interference reference signal at the beat frequency, which is equal to the difference in the two optical frequencies of the source. A polarizing beam splitter directs the beam of $v_1$ into the reference arm and $v_2$ into the test arm of the interferometer. Corner cubes return these beams to the second detector where the polarizer allows them to interfere. When the test corner cube is stationary, an intensity signal at the reference beat frequency is observed; when the test corner cube is in motion, it introduces a Doppler frequency shift in the returning beam and a different beat frequency is observed at the second detector. Signals from both detectors are forwarded to a differential counter and the change in the length of the test arm is determined.

4.3 Phase Locking

Phase-locking interferometry measures small changes in phase by locking the phase difference between the beams while detecting changes in the intensity signal (Moore, 1979). Locking the phase difference is accomplished by offsetting the phase of one of the beams and then subjecting that phase to sinusoidal modulation introduced by an oscillating mirror. This procedure induces intensity modulation that can be expressed as

$$I = I_1 + I_2 + 2(I_1 I_2)^{1/2} \cos(\phi + \delta + a \sin \omega t).$$

(4)

The amplitude value $a$ of the phase oscillation $\sin \omega t$ is much smaller than the source wavelength. The offset phase $\delta$ is set so that the average phase difference satisfies

$$\phi + \delta = m\pi,$$

(5)

where $m$ is an integer. Under this condition the intensity signal goes to zero and there is no intensity modulation due to mirror oscillation. When the average phase difference changes because of changes in tested phase $\phi$, the intensity signal starts to modulate with its magnitude being proportional to the changes in the phase. This signal is then filtered and used to produce a feedback signal that tells the phase modulator how to alter the phase offset $\delta$ so that the condition $\phi + \delta = m\pi$ is satisfied and the intensity modulation due to mirror oscillation is eliminated.
5. APPLICATIONS OF PHASE AT ONE POINT

5.1 Length Measurement

Phase-at-one-point detection in interferometry is very often applied to the precise measurement of length (Steel, 1983). In fact, interferometric techniques really established their value when they were applied to the measurement of material standards such as length, wavelength, line, and gauging.

5.2 Gravity-Wave Detection

Single-point phase detection has recently been employed to detect gravity waves. Much as accelerating charges produce electromagnetic waves, accelerating masses emit gravity waves. Gravity waves produced by collapsing supernovas, binary systems of neutron stars, and black holes affect the dimensions and spacing of all material objects. The movements that signal the existence of gravity waves can be detected by observing the small changes in phase at a single point in an interferometer. To register these changes requires an interferometer of very high sensitivity. A simple Michelson interferometer would need unrealistically long arms or high laser power to achieve the required accuracy. A number of new interferometers have been devised that should have sufficient sensitivity to detect gravity waves (Saulson, 1994). One of these has been coined the Laser Interferometric Gravitational-wave Observatory (LIGO). Its interferometric setup (Fig. 11) places identical Fabry-Pérot resonant optical cavities into each arm of a Michelson interferometer. The Fabry-Pérot arm cavities effectively increase the optical path length of each arm on account of the multiple reflection of light inside the cavities. The Fabry-Pérot mirrors act as inertial test masses suspended like pendulums from seismically isolated platforms. Passing gravity waves change the relative length of the resonant Fabry-Pérot cavities; this change alters the phase of the light and the phase change is picked up by the detector, thus revealing the existence of passing gravity waves. In order to detect the phase, the frequency of the laser is locked so that it is in resonance with the length of one cavity and then the length of the other cavity is adjusted to achieve the same resonance. The system detects the dark interference fringe, which allows most of the light to be reflected back to the source and back out again into the system with a recycling mirror. This additional mirror and the first surfaces of the test masses create the cavities where the light builds up, which effectively increases the optical path length and the illuminating power of the laser. The LIGO interferometer allows the detection of movements as slight as 1/1000 the diameter of a proton \((10^{-18} \text{ m})\) and enables the identification of the presence of matter at distances of more than \(70 \times 10^6\) light years from Earth. The distance separating the mirrors, more than 4 km, makes the LIGO interferometer the world’s largest precision optical instrument and is pushing the limits of technology.

5.3 Other

Interferometry is used for monitoring processes such as plasma etching, film etching rate, uniformity of etching, and substrate temperature (see Plasma Etching). Phase-at-one-point detection can be also used for measuring refractive index and electron density of a plasma. Temperature, pressure, strain, and rotation can be detected by optical-fiber sensors (see Sec. 8.2). Phase detection has also been used to measure the speed of light (Steel, 1983).
6. INTERFERENCE SPECTROSCOPY

Interferometric spectrometers analyze the spectral distribution of a radiating source rather than examining the phase difference as the techniques discussed previously do. Generally, spectrometers measure not just the spectrum of radiation but rather the combination of the radiating source, the detector sensitivity, and the transmittance of the interferometer. In addition, a perfectly monochromatic spectral line will result in different spectral energy distributions when detected by different types of spectrometers. This characteristic of spectrometers is called the response of the system, and it is dependent on the spectrometer’s instrumental function, which determines when two spectral lines of frequencies \( v_1, v_2 \) (or \( \lambda_1, \lambda_2 \) in terms of wavelength) can be still resolved by the instrument. The resolving power of the spectrometer is then determined by

\[
R = v/\Delta v = \lambda/\Delta \lambda.
\]  

An important characteristic of a spectrometer, especially when faint spectra are studied, is its “light-gathering power”, which is defined by the invariant known as \( \text{étendue} \). For example, dispersive spectrometers, like grating or prism spectrometers, generally use a narrow slit; these systems have smaller étendue than those spectrometers that do not use this type of slit. Two types of interferometric spectrometers and their techniques for recovering the spectral content of a radiating source are discussed below, a Fabry–Pérot etalon and a version of a Michelson interferometer.

6.1 Fabry–Pérot Spectrometer

The Fabry–Pérot interferometric spectrometer (Fabry and Pérot, 1899; Vaughan, 1989) employs multiple interference between two high-quality and highly reflective plates, which can be either fixed (etalon) or variably separated. Fixed plates yield a set of circular (spatial) sharp fringes for different wavelengths of the source. This setup is often used to measure the separation of two planes in etalons (standards), the thickness of a thin film, or the wavelength of a source (Hariharan, 1985).

The instrumental function of Fabry–Pérot plates separated by distance \( d \) is the Airy function

\[
A(v) = T^2/((1 - R)^2 + 4R\sin^2(\psi/2)),
\]

where \( R \) and \( T \) are the reflectance and transmittance of the plates and \( \psi = (4\pi nd/c)\cos\theta \), where \( n \) is index of refraction of the medium between the plates and \( \theta \) is the angle of incidence. The Airy function has a set of narrow peaks that are separated by a frequency difference equal to \( c/2dn \); this frequency difference describes the free spectral range where the fringes will not overlap. The half-width of the peaks \( \Delta v \) is described by

\[
\Delta v = c/2ndF,
\]

where \( F \) is the finesse of the system,

\[
F = \pi\sqrt{R/(1 - R)}.
\]  

The limited spectral range of the etalon can be overcome by separating the ranges in order to determine the frequency they represent without any ambiguity; however, this procedure is accomplished by imaging fringes onto a slit of the spectrometer, which then limits the étendue of the system.

The scanning Fabry–Pérot interferometric spectrometer has significantly higher étendue and is often used as a spectrometer for testing faint light sources. In this system the optical separation between the two plates changes linearly and the transmitted intensity signal (section of spectrum) is recorded by a point detector placed in the focal plane of the lens, which is placed behind the interferometer. Many modifications can be introduced to increase the performance of this spectrometer. For example, spherical mirrors may replace plates and thus increase light usage efficiency.

6.2 Fourier-Transform Spectroscopy

Interferometry may also be used to obtain the spectral energy distribution of a source through a technique called Fourier-transform spectroscopy (Michelson, 1902; Vanasse and Sakai, 1967; Bell, 1972). Fourier spectroscopy records the variation in intensity at a single point while varying the optical path difference between interfering wavefronts in a two-beam
interferometer, which is typically a Michelson with the effective source at infinity. The optical path difference is realized by moving one of the mirrors, as in phase-measurement techniques. The detected intensity of the interference signal is the sum of independent interference signals for each spectral component and differs with the spectral content of the tested source. This intensity can be represented by

\[ I(\tau) = \int G(\nu)[1 + \cos(\tau 2\pi \nu)] \, d\nu \quad (10) \]

where \( \tau \) is the introduced optical path difference and \( G(\nu) \) is the spectral distribution of the source. Since the recorded intensity signal is not the direct spectrum signal as in the Fabry–Pérot interferometer but rather the cosine Fourier transform of the spectrum, the Fourier transformation of the recorded signal must be calculated to achieve the spectrum of the source. The intensity of the interference signal must be sampled at least twice per fringe (as in phase-detection techniques). Because the fringe spacing in an interference signal corresponds to the wavelengths of the source, the minimum wavelength that can be detected is limited (as the slope is limited in phase-across-the-plane detection). Unequal sampling intervals and finite changes in optical path difference will introduce errors into the calculated spectrum. A process called apodization, where a measured interference signal is multiplied by a symmetrical signal with gradually decreasing values, like a triangle function, reduces the magnitude of errors in the calculated spectrum.

Fourier spectroscopy may be relatively difficult to implement; but when compared with prism or grating spectroscopy, Fourier spectroscopy has a very high étendue and better resolution than either of the other two methods. Another advantage of Fourier spectroscopy is its multiplex advantage: the signal-to-noise ratio is significantly improved as a result of simultaneous registration of all spectral components of radiation. Major applications of this technique include detection of emission spectra and absorption spectra of aqueous solutions, gases, and transient species. Because of its efficient use of light, Fourier spectroscopy is used often in astronomy to record spectra (usually infrared—see SPECTROMETERS, INFRARED) from very faint sources.

Fourier transformation is a powerful tool in interferometry; it can be used with either a coherent or an incoherent spectral source to analyze the phase across the plane. In addition, Fourier techniques are used for analyzing both the spatial and spectral distribution of a radiating source.

6.3 Spectral Imaging

The interferometric instruments described so far have been able to acquire either phase or spectral information about an object. However, many disciplines such as astronomy, robotic vision, or remote sensing are interested in obtaining both types of information at once. Recent progress in optics and electronics has allowed researchers to register multiple spectral images, thus spurring the development of new spectral imaging techniques (Itoh, 1996). Many of these techniques combine interferometric methods or couple interferometry with other optical testing procedures. For example, a double Fourier spatio-spectral interferometer merges a Michelson interferometer for Fourier spectrometry with a Michelson stellar interferometer for star diameter determination.

7. INTERFERENCE IMAGERY

Interference imagery (Steel, 1983) is an interferometric technique for testing the spatial distribution of a radiating source and is often used in astronomy to measure stellar diameters that are nearly impossible to resolve with conventional telescopes. In 1921 Michelson first developed a method for testing stellar diameters; his stellar interferometer could resolve distant objects with angular diameters on the order of \( 10^{-2} \) arc s (Michelson, 1921; see also INTERFEROMETERS AND INTERFERENCE, Sec. 1.3.2). Since that time, interferometric techniques have become more precise, using visible light as well as radio waves to measure spatial distributions. The resolution of stellar interferometers depends on both the distance between the mirrors (antennas) and the number of mirrors (antennas) used. Stellar interferometers are sometimes referred to as long-baseline or very-long-baseline (VLBI) interferometers or aperture-synthesis arrays.
Stellar interferometers today test many parameters in order to achieve maximum precision, including the phase across the plane for fringe tracking and heterodyne techniques for delay.

7.1 Michelson Stellar Interferometry

A Michelson stellar interferometer is a two-beam interferometer that uses wave-front division rather than amplitude division as the interferometers and techniques described thus far do. A Michelson stellar interferometer (Fig. 12) consists of two widely spaced mirrors (whose separation can be changed) pointed at the same stars. Light from the two mirrors is combined to form interference fringes on the image plane of a telescope. If the interferometer is pointing at two stars, two point sources, then two slightly shifted interference patterns of mean wavelength $\lambda$, one from each star, are formed. Because the source radiation from the stars is spatially incoherent, the sum of the interference patterns’ intensities observed on the detector forms a new fringe pattern. The visibility of these new fringes depends on the shift between the interference patterns from each star. This shift is dependent on both the angular separation $\theta$ of the stars, which is fixed, and the separation $d$ between the mirrors, which is referred to as the baseline and varies. The visibility of the fringes versus the mirror separation is plotted in Fig. 13(a). This visibility of the fringes is an envelope function of the Fourier transform of the spatial distribution of the source. Thus, if the mirror separation $d$ is found for which the visibility of fringes equals zero, then the distance between the stars can be calculated.

When one is measuring the diameter of a star, the star is treated not as a point source but as a set of incoherent point sources that fills the star’s diameter. The visibility of the fringes, Fig. 13(b), changes in a different fashion than for two point sources. By adjusting the separation of the two input mirrors until the fringe visibility reaches zero, which occurs when the spacing between the two mirrors is approximately $1.22\lambda \theta$, where $\theta$ is the stellar diameter, the diameter of the star can be determined but only in the direction of the mirror separation. Another procedure records the fringe pattern and then calculates the zero fringe visibility in order to determine the stellar diameter. In practice stellar interferometers are difficult to build, since the optical paths between the arms of the interferometer need to match as stars emit incoherent light. Stabilizing a system such as this requires very stable mechanical construction of the interferometer.

![FIG. 12. Michelson stellar interferometer for measuring stellar dimensions.](image)

![FIG. 13. Fringe visibility in a Michelson stellar interferometer for (a) two point sources and (b) a disk source.](image)
and correction for atmospheric turbulence. In addition, if a star is not radially symmetrical, the mirrors need to be rotated or an additional set of mirrors implemented in order to determine the diameter of the star in a different direction. Finally, if radiation from the stellar source is not uniform, measurements that employ multiple separations of the mirrors need to be taken.

7.2 Stellar Intensity Interferometry

A major step in the development of stellar interferometers was the invention of the intensity interferometer by Hanbury Brown and Twiss (1954). The intensity interferometer is essentially a Michelson stellar interferometer that observes the intensity variation coming from each mirror separately; these intensity variations are then electronically correlated. The detectors in the intensity interferometer can be separated by thousands of kilometers, thereby significantly increasing the resolution of the system.

7.3 Radio Interferometry

Radio astronomy uses radio waves with wavelengths ranging from millimeters to hundreds of meters for observations in a different spectral range. Similarly, as stellar interferometry increases the resolution of visible light telescopes, radio interferometry, which is based on stellar interferometry (Steel, 1983), improves the resolution of radio telescopes (see Radio Telescopes). Radio interferometry may use antennas for radiation collection instead of lenses and mirrors, but its principle of operation is still based on interference phenomena. Radio interferometry has many uses, ranging from listening for signs of earthquakes and volcanic activity to measuring the thickness of ice sheets and monitoring sea levels (see Geodesy).

7.4 Stellar Speckle Interferometry

Stellar diameters can be determined using a single telescope and a technique developed by Labeyrie (Labeyrie, 1976; Dainty, 1984). This technique allows a telescope to measure angular distances at its diffraction limit regardless of the light conditions. Individual speckles formed at the focus of the telescope have angular sizes determined by the diffraction limit of the telescope. When short exposures are made, the speckle pattern can be frozen in time. By taking the square modulus of the Fourier transform of a speckle pattern and averaging a number of these, a fringe pattern will be seen whose spacing is proportional to the stellar diameter or separation.

8. OTHER APPLICATIONS

In addition to the applications discussed so far in this article, there are many other applications that do not fit into the previous categories. These include the nondestructive testing of mechanical displacement and strain, the use of optical fibers in interferometers, the measurement of polarization and material properties, and interference between particles rather than waves. Since it is not possible to describe every application, this section highlights a few of these other applications.

8.1 Nondestructive Testing with Interferometry

Nondestructive evaluation of the mechanical properties of technical components is increasingly important for process control during manufacturing. Information about components’ deformation due to an applied force or a change in temperature can be delivered by different interferometric techniques. As these techniques are applied to experimental mechanics, changes in phase can represent the displacement both in and out of the plane of the object.

8.1.1 Holographic Interferometry

Holographic interferometry is an often-used technique because both amplitude and phase and in- and out-of-plane displacement information can be obtained about a surface as a known force is applied to it. These results can be compared with simulations generated by finite-element analysis to see whether mechanical parts behave as they are modeled. For example, holographic interferometry can provide information about whether a part in an air-
plane motor will perform as expected when it is stressed and loaded while in operation. The mathematical theory and basics of holographic (hologram) interferometry are outlined in *Optical Holography*, Sec. 4.3.

Recent developments in nondestructive testing using holographic interferometry have enabled holograms to be made in media that are erasable and reusable (Hariharan, 1994). Phase-measurement techniques can be applied to holographic interferometers to make quantitative analysis possible. Quantitative data enable a better comparison with simulation data for structural analysis. Applications include the testing of airplane tires, audio speakers, turbine blades, satellite fuel tanks, musical instruments, and infrastructure such as pipes and bridges.

### 8.1.2 Speckle Interferometry
Speckle interferometry is similar in many ways to holographic testing; the difference is that information about object displacement is derived from the speckle patterns. Correlating speckle patterns before and after a force is applied yields information about the surface displacements. When speckle techniques were first developed, they relied on film as the recording medium. Sensor arrays have replaced film as the recording media, and advances in signal processing have allowed for fast quantitative analysis of object displacements (Jones and Wykes, 1983).

### 8.1.3 TV Holography
By combining holographic and speckle interferometry, it is possible to record holograms on a CCD (charge-coupled device) sensor array and do image processing essentially to create digital holograms (Creath and Wyant, 1992; Kreis, 1996; Rastogi, 1997). Because the hologram must be resolvable with a TV camera, the frequency information content has to be within the Nyquist sampling limit of the sensor. Another consideration is that image processing must remove the self-interference terms from the hologram so that the cross-interference term is not corrupted. Many different approaches have been developed to perform this processing. Displacement measurements are easily obtained by computationally subtracting two holograms. Vibration analysis can be carried out stroboscopically or by adding a known vibration to the system and looking at the difference in frequency. Instruments using these techniques can be made to be quite insensitive to vibration. These instruments are often portable so that measurements are not limited to laboratory settings.

### 8.1.4 Grating Interferometry
High-sensitivity grating interferometry (Patorski, 1993; Post *et al.*, 1994), has become an important method in experimental mechanics to measure, with submicrometer sensitivity, the in-plane displacements of nearly flat objects under load. In conventional high-sensitivity grating interferometry, the object under test with a reflection-type grating adhered to it is illuminated by two coherent beams symmetrical to the grating normal. The interference between the two diffraction orders reflected from the grating is recorded, resulting in a map of the in-plane displacements. Polarization techniques can be used in this system for phase shifting.

### 8.2 Optical-fiber Interferometry
Optical-fiber interferometers do not differ in principle from conventional two-beam interferometers; the difference is that in optical-fiber interferometers the beams travel through single-mode optical fibers, allowing for compact construction of an interferometer (Hariharan, 1985; also see *Sensors, Optical*, Sec. 2.8).

Optical-fiber interferometers are often used as optical sensors to detect changes in pressure or temperature, as these effects alter the optical path length of the fiber. The sensitivity of the interferometric system increases as longer fibers are used; thus optical-fiber interferometers find their advantage in combining compactness with sensitivity, since long optical paths can be contained in compact areas.

The optical-fiber interferometer can use a Sagnac-type (Vali and Shorthill, 1976) interferometric configuration to sense rotation (see *Interferometers and Interferometry*). This is done by replacing the ring cavity of a conventional Sagnac interferometer with a multi-turn loop made of single-mode optical fiber. The phase of the counterpropagating beams can then be detected using a phase-sensitive detector (Ezekiel, 1984).
8.3 Refractometry

Refractometers use interferometric techniques to measure changes in the refractive index of solids, liquids, or gases more precisely than any other method (see Refractometers, Optical). The first interferometer to measure a refractive index was devised by Michelson. Later Rayleigh and Jamin interferometers were developed specifically for this purpose (Interferometers and Interferometry).

8.4 Ellipsometry

Ellipsometry (Azzam and Bashara, 1987; Ellipsometers) employs light polarization to characterize surfaces, interfaces, and thin films. In an ellipsometer linearly polarized incident light reflects from the test surface as elliptically polarized light. By superimposing two orthogonal components of an electric field, the elliptically polarized light can then be measured. Ellipsometry is a variation of common-path polarization interferometry where one of two copropagating orthogonally polarized waves acts as the reference for the other. Some ellipsometers use interferometric arrangements (mainly Michelson) to measure the index of refraction or film thickness over an entire surface, not just at a single point.

8.5 Atom Interferometry

Waves are characterized by phase and amplitude while particles are characterized by position and momentum. Both atoms and light can be treated as waves and particles, and both can be described by similar sets of equations. However, atoms have mass; therefore atomic waves obey a different dispersion relationship than does light. As the interference of wavefronts is an inherent feature of interferometry, atom interferometry (Adams et al., 1994; Berman, 1996; see also Interference and Interferometry) observes atom interference patterns. (Similarly, electron and neutron interference patterns are observed in electron and neutron interferometry.) Using methods similar to classical interferometry, atomic interference patterns are analyzed for modulation and fringe spacing and position. With much better precision than other interferometric methods, atom interferometry is able to detect and test gravitational and inertial effects, atomic properties, and some effects in quantum mechanics. Atom interferometry replaces the optical elements from conventional interferometers with microfabricated elements that diffract or split thermal atom beams, and tunable lasers that manipulate the trajectories of neutral atoms.

GLOSSARY

Fringe Order: The number assigned to a fringe in a single interferogram; it represents the direction of the changes in the recorded phase wavefront.

Optical Path: The length of the path that light takes through the medium multiplied by the index of refraction of the medium.

Optical Phase: Optical path of the light modified by phase changes on reflection of the light from the medium, where the phase changes depend on angle of incidence of the light and on the medium’s complex index of refraction.

Phase Shift: The controlled change of the optical phase between interfering wavefronts.

Phase: Optical phase difference between test and reference wavefronts. The phase retrieved from an interferogram may represent the shape of the tested surface, its slope, or its displacement, among other things.

Works Cited


Further Reading


