Simultaneous registration of in- and out-of-plane displacements in modified grating interferometry

Joanna Schmit,* MEMBER SPIE
University of Arizona
Optical Sciences Center
Tucson, Arizona 85721
E-mail: jschmit@wyko.com

Krzysztof Patorski, MEMBER SPIE
Warsaw University of Technology
Institute of Micromechanics and Photonics
Warsaw, Poland

Katherine Creath,† MEMBER SPIE
University of Arizona
Optical Sciences Center
Tucson, Arizona 85721

Abstract. We recently presented a novel approach using high-sensitivity grating interferometry to measure in- and out-of-plane displacements. The interference of each of the specimen grating diffraction orders with the reference beam was recorded separately but not simultaneously. Computer subtraction and addition of the phase maps obtained from the interferograms yielded the in-plane and out-of-plane displacements (u and w). This article presents our continuing work with this interferometric setup, extending the application of this method to include the simultaneous registration of interferograms for analyzing time dependent events. The spatial separation of interferograms, accomplished by means of polarization techniques, allows for the simultaneous registration of interference patterns. The methods, employing two different sets of linear polarizations for the diffraction orders and two different polarization beamsplitting elements, are described, and the experimental results are presented. Further modifications for sequential and simultaneous monitoring of all three displacements u, v, and w are suggested. © 1997 Society of Photo- Optical Instrumentation Engineers. [S0091-3286(97)00909-4]

Subject terms: grating interferometry; in- and out-of-plane displacements; optical testing.

Paper 36076 received July 31, 1996; revised manuscript received Jan. 15, 1997; accepted for publication Mar. 27, 1997.

1 Introduction

High-sensitivity grating interferometry (also called the moiré interferometry1,2) has become an important method in experimental mechanics to measure, with submicron sensitivity, the in-plane displacements of nearly flat objects under load. In recent years the literature in the field has documented several efforts to simultaneously record in-plane displacements, u and v, in two orthogonal directions.3–5 Dadkhah et al.3 suggested that a specific polarization of the collimated beam, which is then reflected from a specimen grating with exactly 600 lines/mm and a three-mirror head, would result in the orthogonal polarization of diffracted beam pairs, which could then be separated by a polarization beamsplitter. Guo and Kobayashi7 avoided the possibility of ghost images caused by the eventual ellipticity of the beam polarizations by focusing each pair of diffracted beams for u and v displacements at different places in the spectrum plane. A little mirror placed in the spectrum served to direct one pair of beams to another camera. In Salbut’s5 approach, which led to the simultaneous registration of u and v displacements, the polarization states of different parts of the collimated beam corresponding to the u and v displacements were controlled by half-wave and quarter-wave plates, resulting in the orthogonal polarization of the pairs of diffracted orders corresponding to the u and v displacements. A polarizing beamsplitter separated and directed the interference patterns to two different cameras or two different parts of a single camera. Further, a cross pattern, the sum of two interferograms with carrier-fringe frequencies in orthogonal directions, could be obtained, avoiding the necessity for any beamsplitting element in the system. The cross pattern was then processed using a two-directional automatic fringe-pattern analysis technique.6

In most cases an object under load undergoes not only an in-plane displacement but an out-of-plane displacement as well. While information about the in-plane displacement is adequate for some applications, the out-of-plane displacement information is essential for a complete analysis of material- or object-structure behavior. In addition, the in- and out-of-plane displacements ideally should be registered in the same optical system. Several methods for registering both in- and out-of-plane displacements in this way have previously been described.2,5,7–13 Some register the out-of-plane displacements independently of the in-plane displacements, and others take advantage of the fact that the diffraction orders carry information about in-plane as well as out-of-plane displacements.

In a method developed by Basehore and Post,7 which allowed for simultaneous registration of u and w displacements, a reference beam was introduced that interfered with two diffracted orders reflected from the specimen grating. However, the resulting three-beam interference required further optical filtration, a significant disadvantage to the procedure. In addition, the v in-plane displacements had to be registered separately. Patorski8 proposed a photographic superimposition of two conjugate interferograms in order to...
obtain the out-of-plane displacements. The two diffraction orders were separated, and a tiny pinhole was inserted sequentially in the spectrum of each diffraction order so as to generate a reference beam for each interferogram. Asundi and Cheung \(^9\) used an additional reference grating, which allowed them to simultaneously register \(u\) and \(w\) displacements. Wang and Chiang \(^{10}\) proposed using a transmission grating that acted as a beamsplitter in a combined Twyman-Green and grating interferometer. Systems that also employed features of the Twyman-Green and grating interferometers were proposed by Czarnek \(^7\) and Salbut. \(^3, 11\) Their method used an additional collimated beam and beamsplitter. Our most recent work \(^{12}\) described a modified grating interferometer that extended the method proposed by Basheore and Post and avoided any cumbersome optical filtering.

All the methods discussed above, because they register interferograms sequentially, are best suited for analyzing object behavior under a static load. In addition, many of the methods developed earlier did not use automatic interferogram analysis. Only with its recent implementation has the retrieval of displacement information become rapid enough for on-line measurements and for full-field analysis, and the high accuracy of fringe-pattern analysis has opened new possibilities for using grating interferometry in a wide variety of applications.

In addition to detecting the in- and out-of-plane displacements for objects under a static load, recent interest in using grating interferometry to analyze objects under load in time-dependent events (for example, for relaxation or heat transfer processes) has necessitated developing techniques to analyze full-field displacements simultaneously. In responding to this need, a new group of methods has been devised that allows for dynamic measurements of full-field displacements. Methods for measuring time-dependent events are more challenging, as all three displacement maps, the orthogonal in-plane \((u, v)\) displacements and the out-of-plane \((w)\) displacements, must be registered simultaneously.

Only a few methods for simultaneous registration of all three displacements exist. Asundi et al. \(^{13}\) developed their earlier method to accommodate simultaneous registration of all three \(u\), \(v\), and \(w\) displacement maps. The drawback to this method is the difficulty in aligning the system with its object and reference gratings. In addition, the image of the fringes that represents the out-of-plane displacements is viewed at an oblique angle, requiring that the camera be properly inclined in order to allow for distortion. Salbut \(^5\) used part of the collimated beam in an additional Twyman-Green path where a reference beam was formed for measuring the out-of-plane displacements. This system separated the interferograms that contain the information about in-plane and out-of-plane displacements using polarization techniques or different wavelengths.

This article details our methods for simultaneously registering the in-plane displacements \(u\) and the out-of-plane displacements \(w\). For comparison, a review is presented of how in-plane displacements are retrieved using a conventional grating interferometer and how in-plane and out-of-plane displacements are obtained in the modified grating interferometer we previously developed. \(^{12}\) We describe the modified grating-interferometer experimental setups for simultaneous registration of two interferograms and detail some examples from our experimental results. Our technique seems to be easier to align than any other technique presented for simultaneously registering \(u\) and \(v\) displacements; in addition, our setup contains fewer optical elements. Finally we suggest a few ways that all three displacements \(u\), \(v\), and \(w\) could be registered simultaneously in the modified grating interferometer.

2 Background for Conventional and Modified Grating Interferometry

In conventional high-sensitivity grating interferometry, the object under test, with a reflection-type grating attached to it, is symmetrically illuminated by two coherent beams \(A\) and \(B\) (Fig. 1) at angles \(\alpha\) and \(-\alpha\) to the grating normal. The interference between the two diffraction orders \(A_{-1}\) and \(B_{+1}\) reflected from the grating is recorded, which results in a map of the in-plane displacements. However, each of the diffraction orders carries information about both in-plane and out-of-plane displacements, with one difference, namely that the out-of-plane displacements are of the same sign, while the in-plane displacements are of opposite sign. The complex amplitudes of the two interfering diffraction orders \(A_{-1}\) and \(B_{+1}\) in the plane conjugate to the surface of the specimen under load can be represented as (see, e.g., Ref. 14)

\[
E^A_{-1}(x,y) = \exp\left[-i \frac{2\pi}{d} \left[u(x,y) - kw'(x,y)\right]\right],
\]

\[
E^B_{+1}(x,y) = \exp\left[i \frac{2\pi}{d} \left[u(x,y) + kw'(x,y)\right]\right],
\]

where \(d\) is the grating period (in our experiment \(d = 0.833 \, \mu\text{m}\)), \(k = 2\pi/\lambda\), \(\lambda\) is the light wavelength (\(\lambda = 0.633 \, \mu\text{m}\)) and \(u(x,y)\) is the in-plane displacement function. The optical path change, \(w'(x,y)\), is related to the out-of-plane displacement function \(w(x,y)\) by

\[
w'(x,y) = w(x,y)(1 + \cos \alpha),
\]

where the amplitudes of the diffraction orders have been normalized to unity. Equations (1) and (2) correspond to the illumination angle \(\alpha\), such that the first diffraction orders reflected from the specimen grating (SG) propagate along the grating normal (i.e., the uniform-field detection mode without introducing carrier or reference fringes). Thus, by the interference of the diffraction orders the infor-
Spatial versions of the phase-shifting technique can be used. However, the focus of this paper is the simultaneous registration of displacements. The validity of this method was verified by comparing our results with data obtained using a conventional interferometer.

In the approach described previously, an additional reference beam is introduced so as to interfere with each of the diffraction orders (Fig. 2), and the interferograms are recorded separately. Two interferograms, described by

\[ I_{\text{UGL}} = |E^A_1 + E^B_{+1}|^2 = 2 \left( 1 + \cos \left[ \frac{4 \pi}{d} u(x,y) \right] \right). \]

For this reason, this interferometer is sensitive only to in-plane displacements. There is no influence of out-of-plane displacements if a very small out-of-plane object surface displacement (rigid or local) can be assumed.

In the approach described previously, an additional reference beam is introduced so as to interfere with each of the diffraction orders. The first interferogram carries information only about in-plane displacements, while computer addition gives the out-of-plane displacements. The retrieved phases will also have opposite signs, a fact that must be taken into account in further calculations. Neither case is preferred over the other; however, it may be easier to align the system if the two diffraction orders propagate along the grating normal, the case we employ in this article.

### 3 Description of Interferometric Setups for Simultaneous Registration of \( u \) and \( w \) Displacements

Figures 4 and 5 show the schematic representations for two of our experimental setups for the simultaneous registration of two interferograms in the modified grating interferometer. The first depicts the horizontal and vertical polarizations of the diffraction orders, while the other shows the 45- and −45-deg linear polarizations of those orders.

![Fig. 2 Schematic representation of modified grating interferometer for in-plane and out-of-plane displacements measurements.](Image 67x651 to 246x729)

![Fig. 3 Beam orientation: (a) reference beam along grating normal, (b) reference beam at angle to grating normal.](Image 313x90 to 526x191)

![Fig. 4 Modified grating interferometer for simultaneous registration of two interferograms using two cameras and employing horizontal and vertical linear polarizations of diffraction orders A\(_{+1}\) and B\(_{+1}\). See text for symbol explanation.](Image 340x652 to 500x729)
Fig. 5 Modified grating interferometer for simultaneous registration of two interferograms using a single camera and employing 45- and −45-deg linear polarizations of diffraction orders $A_{-1}$ and $B_{+1}$. See text for symbol explanation.

Fig. 6 Two sets of polarization states of three beams for simultaneous separation of interferograms in space: (a) horizontal and vertical and (b) 45- and −45-deg linear polarizations of diffraction orders $A_{-1}$ and $B_{+1}$.

3.1 Basic Setup of the Modified Interferometer

In both setups, as in a modified grating interferometer, one part of the illuminating plane wavefront, designated as I, from the collimator CO impinges on the mirror M1 and is directed to the specimen SG and mirror M2. This creates symmetrical illumination of the specimen grating by wavefront beams $A$ and $B$. Two diffraction orders $A_{-1}$ and $B_{+1}$, reflected from the grating, propagate along the grating normal.

The other part of the collimated beam, designated as II, serves as the reference beam and is directed through beamsplitter BS2 (PBS2) and BS1 to propagate also along the grating normal. This beam may be tilted if interferograms with carrier-frequency fringes are required. The beamsplitter BS2 (PBS2) is used instead of a mirror because it reduces the amplitude of the reference beam to match the amplitude of the diffraction orders; this ensures better fringe contrast (a density filter could be used for the amplitude reduction, but that would be an additional element in the system). If further improvement of fringe contrast is desired, an additional reflecting element may be introduced into the path of the reference beam, thus equalizing the reference-beam optical path with the optical paths of the diffraction orders. Lenses L1 and L2 image the fringes at the surface of the specimen onto a CCD camera.

A modified interferometer for simultaneously registering interferograms requires the addition of a few polarizing elements into the system in order to establish the appropriate polarizations of the interfering wavefronts. Our approach employed an orthogonal polarization of the diffraction orders while introducing a circular or linear polarization into the reference beam. If the reference beam is to be linearly polarized, it must be at an angle between the polarization angles of the diffraction orders. The beamsplitter (such as a polarizing beamsplitter cube PBSC as in Fig. 4 or a Wollaston prism WP as in Fig. 5), inserted into a common path of these three beams, has two tasks: first, it separates the two orthogonal polarizations (here the diffraction orders), and second, it splits the reference beam in two, creating a reference beam for each of the diffraction orders. Our systems using two different states of polarization of the diffraction orders are described below.

3.2 Horizontal and Vertical Polarizations

To obtain horizontal and vertical polarizations of the diffraction orders, we introduced two half-wave plates $H1$ and $H2$ into our original modified grating interferometer setup; see Fig. 4. The first half-wave plate $H1$, with the fast axis at the proper angle with respect to the angle of polarization of the light exiting the laser, must be placed between the laser and the pinhole to realize the horizontal polarization of the beam. The angle of beam polarization may be established by simply rotating the laser; however, in order to achieve a finer adjustment of the beam polarization state, rotating an inserted half-wave plate $H1$, instead of the laser, seems more practical, because it will not introduce any beam misalignment. The second half-wave plate $H2$ needs to be inserted in the path of either beam $A$ or beam $B$ somewhere between the collimator CO and the specimen SG (in Fig. 4 the half-wave plate $H2$ is inserted into a path of beam $A$). Its fast axis should be at a 45-deg angle with respect to the horizontal polarization of the beam, thereby rotating the polarization by 90 deg. These procedures result in the orthogonal polarization of beams $A$ and $B$. The horizontal and vertical polarizations incident on the specimen and mirror should remain unchanged in the reflected and diffracted $A_{-1}$ and $B_{+1}$ orders.

However, due to various imperfections (e.g., incorrect angles of polarizations, improper alignment of a polarizing beamsplitter, rotation of sample), an additional quarter-wave plate should be introduced into either beam $A$ or $B$ (or both) to compensate for the eventually arising elliptical states of polarization. The two beams are then separated by the polarizing beamsplitter cube PBSC and directed to two different CCD cameras. The reference beams for each of these diffraction orders are obtained by splitting the reference beam, designated as II, into two beams with the same polarization beamsplitter cube PBSC. Reference beam II can be either circularly polarized or linearly polarized at an angle of 45 deg. Circular polarization can be achieved by introducing a quarter-wave plate $Q1$ into the reference beam, as shown in Fig. 4, while linear polarization can be obtained by introducing a polarizer or half-wave plate (not shown in the figure). The further decrease in the amplitude of each of the reference beams allows us to match the am-
plitudes of the reference beams more closely with the diffrac tion orders, thereby obtaining better fringe contrast. The polarizing beamsplitter cube PBSC will direct two components of any of these polarization states to two different cameras, CCD1 and CCD2, where they will interfere with the diffraction orders. Two images from two cameras can be captured simultaneously and forwarded for further fringe-pattern analysis.

### 3.3 Polarizations at 45 and −45 deg

The second system, represented in Fig. 5, also utilizes the separation of the orthogonally polarized diffraction orders, but in this system the orders are polarized at 45 and −45 deg instead of being horizontally and vertically polarized. To achieve this state of polarization we rotated the laser beam’s linear polarization as it left the laser to one of these angles using half-wave plate H1, as in the previous system. The polarization of the diffraction order \( A_{-1} \) will be rotated 90 deg due to the single reflection of the specimen grating (reflection causes left-to-right polarization-plane reversal), while the state of the polarization diffraction order \( B_{+1} \) remains the same, since it undergoes an even number of reflections, once from mirror M2 and once from the specimen grating SG.

Unfortunately, the single half-wave plate H1 in this system of 45- and −45-deg polarizations is not enough to establish the ideally linear polarization of diffraction orders \( A_{-1} \) and \( B_{+1} \). The ellipticity of the diffraction orders is even larger than in the case of the horizontally and vertically polarized beams incident on the specimen, due to the new angles of polarization, which are different than 0 and 90 deg. Thus, the quarter-wave plate Q1, introduced before the pinhole, serves to compensate for the ellipticity of one of the diffraction orders, while the second quarter-wave plate Q2, introduced into the path of the beams, compensates for the ellipticity of the other diffraction order. This compensation is necessary to avoid spurious fringes, which would otherwise be noticeable in the interferograms.

The separation of the beams can be achieved through the use of a polarizing beam splitter cube (PBSC, as in Fig. 4) or a Wollaston prism (WP, as in Fig. 5) in both the 45- and −45-deg polarization system and the horizontal and vertical polarization system.

In the 45- and −45-deg polarization system described here, the Wollaston prism WP separates the two diffraction orders at a small angle and splits the reference beam into two reference beams. In this way, two interferograms can be recorded on a single camera. However, the prism then must be rotated 45 degrees about the optical axis, and the interferograms are thus separated not vertically or horizontally, but at a 45-deg angle. If the polarizing beamsplitter cube PBSC were used instead of the Wollaston prism WP, then PBSC would also have to be rotated 45 deg about the optical axis. If this were the case, the interferogram deviated by the beamsplitter would be sent at 45 deg to the \( x \) axis and rotated at 45 deg about its axis. This system requires that the CCD2 camera be awkwardly positioned.

To avoid this difficulty an additional half-wave plate H2 may be inserted in front of the polarizing beamsplitter, thereby rotating the linear polarizations 45 and −45 deg to linear horizontal and vertical polarizations. Then, when the polarizing beamsplitter cube is used, the cameras can be positioned simply at 90 deg with respect to each other as in Fig. 4. When the Wollaston prism is used, it need not be rotated, and the vertical or horizontal separation of interferograms can be achieved as in Fig. 5.

Certain advantages and disadvantages exist when using the Wollaston prism in either of the systems. First, the Wollaston prism’s primary advantage is a higher polarization efficiency, and second, the two interferograms are sheared with respect to each other and imaged on only a single camera. A disadvantage comes from the loss of resolution or the decreased area of the tested specimen, as we now register two images on one CCD array. We might have inserted the Wollaston prism into the focal plane between lenses L1 and L2; however, in this case significant spherical aberration may be introduced if the numerical aperture of the passing beams is too large. Further, lens L2 would have to be increased in diameter by a factor of two, and the images might need to be corrected by a factor of \( \cos \beta \), where \( \beta \) is half angle between deviated beams. Certainly, other polarizing beamsplitters such as a beam-displacing prism may be introduced, but in most cases the Wollaston prism is the most cost-effective.

### 3.4 Merits and Drawbacks of Each Setup

To reiterate, we have no preference in using one setup over the other. Several problems, introduced by the half- or quarter-wave plates inserted into the collimated beam, exist in both systems. These problems, which are the focus of our discussion in the next section, include nonuniform performance of the plate over the active area and the possibility of the plate limiting the size of the beam that illuminates the specimen and thus limiting the size of the tested area. For the system with 45- and −45-deg polarizations, it could be possible (but costly) to produce for mirror M2 a coating with the proper correction of the polarization for beam \( B \), while the correction for beam \( A \) would be established by retarders inserted at the pinhole. Such a system would not have these problems and thus would be preferred.

### 4 Error Sources in the Polarization Method

Besides the advantages that polarization techniques bring to the modified grating interferometer, we do encounter several disadvantages. First, because more elements are introduced into the optical system, a higher standard of performance is required of each of the elements. Next, the size of the retarders placed in the collimated beams may limit the field of view of the tested specimen, and the retarders need to be of good quality over their whole area. An additional difficulty accompanying the polarization technique is the possible presence of locally detectable low-contrast spurious fringes, as when we encounter a specimen with a large difference in local slopes. In this case the local angles of incidence are different, and thus a range of elliptical polarizations may be introduced, while compensations can be set for only a single state of polarization. Other problems that may be introduced by retarders are described, for example, in a recent article by Poirson et al.19

The polarization state of the beam reflected from the specimen grating depends not only on the material used to manufacture the reflective grating and the mirrors, the
angle of beam incidence, and the angle of polarization, but also on the spacing, depth, shape, and direction of the grooves of the specimen grating itself. Because of the number of factors that may influence the state of the polarization, experimentally aligning the angular positions of the half-wave and quarter-wave plates' fast axes is the best way to ensure linear and orthogonal polarizations.

5 Experimental Results

After aligning the system and properly setting the polarization state of each of the beams, two pairs of interferograms are recorded, the first before the sample is loaded and the second after it is loaded. The phase maps of the unloaded state are subtracted from the phase maps of the loaded state, removing in this way any systematic errors of the system. Systematic errors can arise from either system aberrations or specimen grating imperfections.

To analyze the fringes the error-reducing six-point method described by the expression

$$\varphi = \arctan \frac{3I_2 - 4I_4 + I_6}{I_1 - 4I_3 + 3I_5}$$

was chosen to obtain phase maps reasonably free from errors due to miscalibrated carrier frequency and unequally spaced fringes. Detailed analysis of the error sources of the n-point method and performance comparison of this method and the temporal phase-shifting method can be found in the literature. The analyzed loaded sample—a horizontal beam that undergoes four-point bending as represented in Fig. 7—is the same one we used in our initial investigation of the modified grating interferometer. Figure 8 shows two interferograms for a loaded sample registered in the modified grating interferometer with horizontal and vertical polarizations of the diffraction orders and registered by two cameras. Figure 9 represents the in- and out-of-plane displacements retrieved from those interferograms as well as from interferograms registered in an interferometer with the diffraction orders linearly polarized at 45 and -45 deg. The displacements obtained for two different polarization systems have the same character as we expected. The validity of the modified grating interferometer’s performance was previously verified by comparing the results obtained from it with results gathered from a conventional grating interferometer paired with a Twyman-Green interferometer to obtain both in- and out-of-plane displacements. In the systems described here, we only introduced different polarization states into the beams, thus not disturbing the validity of the method.

In the description of the experimental setup we emphasized the possibility of low-contrast spurious fringes appearing due to a range of elliptical polarizations of the diffraction orders. However, in most cases the influence of these fringes is insignificant when calculating phase maps using the n-point method. In Figs. 10 and 11 we present two pairs of displacement maps; the first is calculated from interferograms influenced by elliptical polarizations of the diffraction orders, and the second is obtained from interferograms showing no influence from elliptical polariza-
This was achieved by stopping each of the diffraction orders in turn. In the obtained phase maps, no measurable trace of spurious fringe influence is detectable. The two phase maps are comparable; however, the rms values for the displacement maps obtained from the system with residual polarization are slightly larger than the rms values for displacement maps obtained from the system without residual polarization, indicating that the maps are influenced by the residual polarization. The difference in the displacement maps is shown in Fig. 12, where the rms values for the difference are insignificant for the measurement about 2.5% and 0.4% of the rms values for in- and out-of-plane displacements respectively.

Our example shows fairly uniform distribution of displacement fields. However, if the singularity in the displacement field occurs, then for example the two-directional $n$-point method designed for closed fringes should be used. If this method does not provide a sufficient solution, then a phase-measuring technique not requiring carrier fringes needs to be used.

6 Registration of $u$, $v$, and $w$ Displacements

Generally, registering information about two mutually orthogonal in-plane displacements requires a reflective cross-type grating be attached to the object and symmetrical two-beam illuminations in orthogonal directions. This symmetrical illumination can be achieved in two or more different ways. A compact three-mirror head is one fairly common way to obtain this illumination. In this method three different parts of a single collimated beam are reflected from the mirrors onto the object, while one part of the beam impinges directly onto the object.
6.1 Sequential Registration

The conventional way to register in-plane \( v \) displacements is through the separate registration of the interference between the diffraction orders; this solution seems simplest. An alternate method places an additional set of polarization compensators in the collimated beam, thus establishing the orthogonal polarization (horizontal and vertical, or \( 45 \) and \(-45 \) deg) of the diffracted orders. Beams illuminating the specimen are reflected from the mirrors at an angle different from the beams for \( u \) in-plane displacements; therefore, a different polarization compensation is required. Two pairs of interferograms with encoded information \((u+w, -u + w)\) and \((v + w, -v + w)\), respectively) could be recorded sequentially with only a short time interval separating the registration of the interferograms. Properly positioned stops would result in only one pair of interferograms appearing in the system at a time.

Another possibility is to use a different combination of the beams’ polarization states. Beams \( u + w \) and \(-u + w \) could be linearly polarized in the vertical direction, and beams \( v + w \) and \(-v + w \) could be linearly polarized in the horizontal direction. In this case we would need a stop that allows either pair of beams, \( u + w \) and \( v + w \) or \(-u + w \) and \(-v + w \), to propagate in the system at the same time. A final possibility allows for the sequential registration of two cross-pattern interferograms instead of two pairs of interferograms; however, this method may result in larger errors. An advantage to these last two methods is that the out-of-plane displacements are retrieved twice, which provides verification of the measurements taken.

6.2 Simultaneous Registration

However, the simultaneous registration of all three displacement components, \( u, v, \) and \( w \), is necessary for the analysis of time-dependent events. We suggest three modifications to our system that allow for the accomplishment of this task.

First, we could have all five beams propagating simultaneously (four diffraction orders linearly polarized and the reference beam polarized, e.g., circularly). In this case the beamsplitter separates the diffraction orders and splits the reference beam in two, resulting in two sets of three beams of like polarization. Two fringe patterns of three-beam interference are then registered simultaneously. The relative angles of the beams could be carefully chosen so that we would have cross patterns for interference between the reference beam and each of the diffraction orders, but unfortunately our interferogram would then have a third set of fringes in the diagonal direction from the interference between diffraction orders. The Fourier transform fringe-pattern analysis technique could be used to calculate a phase map for each of the three sets of fringes, or we could filter out the diagonal fringes and process the resulting cross pattern using a two-directional \( n \)-point technique. The disadvantage here lies in the difficulty in analyzing a complicated fringe pattern with many sets of fringes. However, for simple cases this technique could be effective.

In a second approach, the object could be symmetrically illuminated in each of the orthogonal directions by a light source of a different wavelength. Then one pair of diffraction orders could be directed to one camera by a beamsplitter and an interference filter, and in this way separated from the interferograms with encoded information about the sum and difference of out-of-plane displacements and in-plane displacements in the other direction. This approach is similar to the one presented by Salbut.\(^7\) The disadvantage to this approach is that it requires additional optical elements and two light sources of different wavelengths.

A third approach to additional and simultaneous registration of in-plane \( v \) displacements bears some resemblance to the method presented by Guo and Kobayashi.\(^4\) The diffraction orders for one direction of object illumination can be focused at a place in the spectrum plane different from the focus of the diffraction orders for the other direction of object illumination and the reference beam. The positions of the diffraction orders’ spectra can be controlled by means of the angle of object illumination. For in-plane \( v \) displacement, this can be done by tipping and tilting the top and bottom mirrors. A little mirror inserted into the spectrum plane can then redirect the two separated diffraction orders onto another camera, where the interference fringes representing classical in-plane \( v \) displacements for one direction can be registered. At the same time two other interferograms with encoded information about in-plane \( u \) displacements and out-of-plane \( w \) displacements, as discussed in this paper, can be registered by a separate camera. The best fringe contrast for in-plane \( v \) displacements will be observed when the diffraction orders have the same polarization, which can be achieved\(^3^5\) when the illuminating beam is polarized at an angle of about \( 45 \) deg. For this reason the modified interferometer with \( 45 \)- and \(-45 \)-deg polarization angles of the diffraction orders for in-plane \( u \) displacements is preferred.

The above proposals for solving the problem of sequential and/or simultaneous monitoring of all three displacements in an interferometer using the three-mirror four-beam illumination system should be treated as introductory ones. The description of an optimized solution will be given in a separate paper.

7 Conclusions

This article has presented our continuing work on the modified grating interferometer. We showed that simultaneous registration of two interferograms giving the in- and out-of-plane \((u \text{ and } w)\) is possible by utilizing polarization techniques. Two systems with two different orthogonal states of polarization for diffraction orders were proposed. Interferograms can be registered by a single camera or a pair of cameras, depending on the polarization-splitting element used. Advantages and disadvantages of the introduced modifications were considered and experimental results presented. Further modifications of the interferometer for measurements of all three displacements \((u, v, w)\), simultaneous and sequential, were suggested.

References

4. Z. K. Guo and A. S. Kobayashi, “Simultaneous measurement of \( u, v, w \), and \( v \) displacements, respectively.”


