

# Thin Wall Structures – *Bring your own can!*

*ie Pepsi can lab*

## Background

In this lab, the application of a strain gauge to a soda can will be completed by each student. Once applied, the strain gauge will be used to measure the strain difference between an unopened and open can. By using some assumptions and the thin walled pressure vessel theory the original pressure can be determined.

## Objectives:

- Learn strain gauge application
- Using circuits developed in strain lap obtain strain difference between cans before and after opening
- Determine internal pressure using Hooke's law and thin-walled pressure vessel theory

## Pre-Lab

In what direction, hoop or axial, will you see larger strain values? (Hint: Look at the derived equations in this document!) What is the criterion for a pressure vessel to be considered thin walled? Derive the **strain-pressure** relationships and include drawing of the idealized volumes with proper labels. (Hint: Read this entire document).

## Theory and Discussion: Determination of Stresses in the Soda Can

In order to determine the pressure in the can we need to find a relationship between the strain values that we can experimentally measure and the pressure exerted by the carbonation of the soda. In your prelab, you will need to determine the criteria for a pressure vessel to be considered thin walled. By applying this criterion to a soda can you will see it is a thin walled pressure vessel. Additionally, the can is clearly a cylinder so we can model it as a cylindrical pressure vessel. The ends are problematic because they are either thicker due to necessity on the pop top side and complex shape on the bottom of the can. To make things easier we can make the assumption that the ends are spherical to simplify the stress-pressure relationships. With these assumptions we can now model our soda can as a cylindrical pressure vessel with spherical end caps.

To determine the stress-pressure relationships we need to assign a coordinate system to the pressure vessel. The logical choice is to align one axis along the long side of the cylinder (longitudinal) with the other being in the hoop direction around the cylinder. This results in the third axis in the radial direction. Since the hoop and longitudinal stresses are the principal stress, we know that there are no shear stresses in these directions.

With the axis assigned we can now determine the stress-pressure relationships by making cuts in the cylinder and performing a force balance. This will give us two equations that relate the stress to the pressure.

To determine the longitudinal stress, we cut across the cylinder section. Imagine that we cut the spherical end cap off the cylinder. The pressure exerted on the area created by the cut in the cylinder must equal the longitudinal stress on the wall of the cylinder.

$$P_{\sigma_l} \cdot \pi r^2 = \sigma_l \cdot t \cdot 2\pi r$$

To determine the hoop stress a cut can be made along the longitudinal axis of a section of the cylinder. For this calculation imagine that you take a slice of the cylinder with a length of dx and then slice this in half. A static equilibrium can once again be assumed and the forces balanced to yield the following relationship.

$$P_{\sigma_h} \cdot 2r \cdot dx = \sigma_h \cdot 2t \cdot dx$$

These formulas can be arranged to provide two separate ways to calculate the pressure in the can.

$$P_{\sigma_l} = \frac{2\sigma_l t}{r}$$

$$P_{\sigma_h} = \frac{\sigma_h t}{r}$$

The only problem is that we can't directly measure the stress in the can. With the strain gauges we can measure the strain. Once we have the strain we need to either use it to calculate the stress in can or modify our earlier formulas so we can just plug in the strain. Since we are dealing with a thin-walled vessel, we can make the assumption that the skin on the can is under plane stress. Make certain that you understand this concept because if we use the 1-D relationship our results will be wrong. Using Hooke's law for plane stress we can obtain the stiffness matrix that gives us relationships between stress and strain in the x,y,z coordinate system.

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix} = \frac{E}{1-\nu^2} \begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & 1-\nu \end{bmatrix} \begin{bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \end{bmatrix}$$

This can then be substituted into the pressure-stress relationships we derived to obtain pressure-strain relationships. This way, the strains obtained in the experimentation can be used to determine the pressure inside the cans.

$$P_{\sigma_l} = \left| \frac{2t}{r} \frac{E}{1-\nu^2} (\nu\epsilon_h + \epsilon_l) \right|$$

$$P_{\sigma_h} = \left| \frac{t}{r} \frac{E}{1-\nu^2} (\epsilon_h + \nu\epsilon_l) \right|$$

The subscripts indicate from which principal stress the pressure was determined. The absolute value bars appear because the gages were installed and "zeroed" on the cans in their pressurized state, and then the pressure released, thereby causing the walls to contract to give negative strain readings.

Although we have obtained the proper relationships and strains, certain material properties and can measurements are still needed to solve the equations. The material properties of the aluminum can be found on [www.matweb.com](http://www.matweb.com). The aluminum of a typical soda can is usually in the 1000 series (1050, 1060, 1070 or 80) and strain hardened from H12 to H18. Your job is to visit Matweb and find a data sheet with the material properties (Poisson's ratio and Young modulus) needed to complete the calculations. Cite the Matweb page and indicate what material you chose in your lab report.

Think about the strain value that you obtained and the loading conditions on the can. After deciding on how to apply your values to the equations, calculate the pressure that is present inside of the soda can. You might use a units system that you are comfortable with, which for me is looking at the values in psi. Is your calculated value what you thought it would be? Remember that a car tire is at about 35 psi, a road bike tire is usually filled to ~ 110 psi, and the orange nitrogen tanks in the lab are filled to 2000 psi. Does the pressure you calculated make sense compared to these values?

### Test Procedure

1. Sketch the can in your notes and obtain the necessary dimensions needed for the pressure calculation. Make certain to read through the lab, so that you get accurate dimensions in critical areas. You will have to wait until testing is completed to measure the thickness of the can.
2. Apply strain gauge to your soda can using procedure demonstrated in class and clearly described in the directions provided with the strain gauges.
3. Within your three person group make certain to apply one strain gauge in the vertical, one in the horizontal direction, and a rosette that is aligned with either axis. Identify which orientation will provide the longitudinal strain and which will provide the hoop strain. If any deviance between the proper coordinate systems and the applied strain gauge exists, make certain to note the offset.
4. Solder the wires to your coke can. The wires have a coating so even though they are twisted they will not short. Make sure that the ends have the coating removed to provide the proper conduction. **Be careful with the soldering irons because they are very hot.** Tape the wires to your can before soldering to provide strain relief or the strain gauge can be easily peeled from the can. Once you have soldered the wires onto the leads of your strain gauges check to make sure the resistance across the circuit is correct. (Most strain gauges are 120 or 350  $\Omega$  so if you measure something different from that there is either a short or an improper connection.)
5. Now that you have strain gauge wired you can connect it to the NI equipment. Since only one gauge is being used the configuration is known as a single bridge configuration. Read the label on the NI device for the proper wiring of your strain

- gauge (3 and 5 pin for a single bridge). Now you need to enter the resistance and gauge factor for your gauge. Using these values, the program can calculate the strain. When you run Labview Express you will notice that it reads a strain value rather than zero strain like you would expect. This is because the gauge could be slightly stretched in the application or the actual resistance (the wires resistance come into play) is different than the given value. So you need to remove this strain which can be done with a voltage offset or adjusting the resistance in the circuit. You can create a voltage offset using the Labview Express using the procedure shown in class.
6. Now that you have zero strain you can run the program and then carefully open your can taking care to not dent the can while opening it. Note the change in strain and the sign. Is this what you expected? Think about the change in loading to the can from the closed (loaded) and open (unloaded). Initially the can had pressure applied by the carbonated soda. Once the can was open the pressure was removed and the only force on the side of the cans is the weight of the soda which we will neglect for this experiment. The strain that the can underwent is the difference between these two values. Make sure and write down the values you obtain.
  7. Repeat for the additional cans with single strain gauges in your group. For the rosette strain gauged can make certain to sketch the orientation of the gauge and note the change of each of the different gauges on the rosette. Calculation of the principal strains from the rosette can be done using the guide on the website.
  8. Drink the beverage and then measure thickness of the can using the micrometer. Make certain to clean off the sticky residue before measuring.
  9. Enter your data in the class spreadsheet. Remember to ensure that the units you use are correct and make sense.

### **Data Analysis and Reporting (Memo Report)**

1. Prepare a table of the experimental results with data from all cans tested during the class. Include all the cans tested during the lab period, not just your groups. Calculate the hoop and longitudinal strain from the rosette cans (See the website for how to do this) in your class and see if the values compare to the single strain gauges. Calculate the mean and standard deviation for the hoop and longitudinal strain. If there are values that are significantly larger or smaller than the mean, then remove them from your calculations and indicate why they could be wrong.
2. Determine the internal pressure in the can using both of the equations. Do the pressures match? Is there a difference between the derivations of the equations? What do the calculated pressures tell you about your assumptions in those derivations? Does a soda can act like a thin walled cylindrical pressure vessel with spherical end caps?