

Efficiency in Second-Price Auctions:  
A New Look at Old Data\*

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February 9, 2004

Abstract:

The experimental economics literature on second-price sealed-bid private value auctions has established that subjects typically bid more than their value, despite the fact that value bidding is a dominant strategy in such auctions. Moreover, the laboratory evidence shows that subjects do not learn to bid their values as they gain more experience. In the present paper, we re-examine the second-price auction data from Kagel and Levin's (1993) classic paper. We find that auction efficiency is rising over time, even though the frequency of overbidding is unchanged. We argue that the rise in efficiency is due to a decline in the variability of overbidding. This is consistent with subjects learning to bid more like each other.

JEL Numbers: C72, D44

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\* We are grateful to John Kagel for providing us with data from experiments reported in Kagel and Levin (1993). We thank Dan Ackerberg for helpful comments.

## 1. Introduction

In a second-price sealed-bid auction with private values, it is a dominant strategy for a bidder to bid his value. One of the striking results to come from the experimental literature on second price auctions (SPAs) is that, in fact, subjects have a strong tendency to overbid. In 5-bidder auctions, for example, Kagel and Levin (1993) find that over two-thirds of all bids exceed the bidder's value. Kagel, Harstad, and Levin (1987) find that the market price (the second highest bid) exceeds the second highest value in 80% of their SPAs. Further evidence of overbidding is reported in Harstad (2000).

These papers report, moreover, that there is little evidence that subjects are learning to bid their values. Kagel and Levin (1993) note that "Learning to play the dominant strategy is quite limited as only 2 out of 21 subjects play it precisely for 5 or more consecutive periods starting from the end of the auction series."<sup>1</sup> Kagel, Harstad, and Levin (1987) report "... no obvious tendency for prices to converge to the dominant bid price over time was observed."<sup>2</sup>

In the present paper, we re-examine the data from Kagel and Levin (1993), focusing on auction efficiency. We define auction efficiency as the probability that the highest bidder has the highest value, given the empirical distribution of bid-value pairs. We show that there is a strong tendency for auction efficiency to rise over time. Hence, even though subjects are not learning to bid their values, bidding behavior is changing over time systematically, in a way that leads to higher efficiency.

We argue that the explanation for rising efficiency is that the variance of overbids is falling over time. Hence, even though bidders continue to overbid, the bidder with the highest value becomes increasingly likely to have the highest bid. A falling variance in overbids is consistent with bidders tending to all overbid by a similar amount – they learn to bid more like each other.

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<sup>1</sup> See p. 872, footnote 5, of Kagel and Levin (1993).

<sup>2</sup> See Chapter 5 of Kagel and Roth (1995) for a survey of the experimental literature on auctions.

## 2. Summarizing K&L's data on Second-Price Auctions

Our analysis is based on the second-price auction experiments reported in Kagel and Levin (1993).<sup>3</sup> Each experiment had 10 subjects. Each period, each subject's value was randomly drawn from the uniform distribution on \$0 to \$28.30. The subjects then participated in a second-price sealed-bid auction in a market with either 5 or 10 bidders. After each auction, profits (losses) were added (subtracted) from the subject's current cash balance. Subjects were given an initial cash balance of \$10. A subject who went bankrupt in the course of the experiment was removed.

The experimental design for the second price auctions is summarized in Table 1 below. Periods 13-24 of session 2.1 used a "dual market" procedure in which, using the same value, a subject simultaneously bid in a market with 5 bidders and a market with 10 bidders. Session 2.2 used a "cross over" procedure in which subjects first bid in a market with 5 bidders, and then bid in market with 10 bidders.

Auction series	Number of Subjects	Bidders per Auction	Auction Periods
2.1	10	5	1-12
		5 and 10	13-24
2.2	10	5	1-13
		10	14-25
		5	26-35

Table 1

We will focus on the data from markets with 5 bidders, since there is more data for these markets. (For markets with 5 bidders there were 240 bids in session 2.1 and 230 bids in session 2.2. For markets with 10 bidders there were only 240 bids in total over the two sessions.)

K&L documented that there was (i) a strong tendency for subjects to bid above their values, with nearly two-thirds of all bids being overbids, and there was (ii) no tendency for subjects to learn to bid their values over time. Figure 1 illustrates these conclusions by showing the frequency of “under bids,” “value bids,” and “over bids” across time. Here, as in K&L, a bid is classified as an overbid if it is more than 5 cents above the bidder’s value, it is an underbid if it is more than 5 cents below his value, and it is a value bid otherwise.

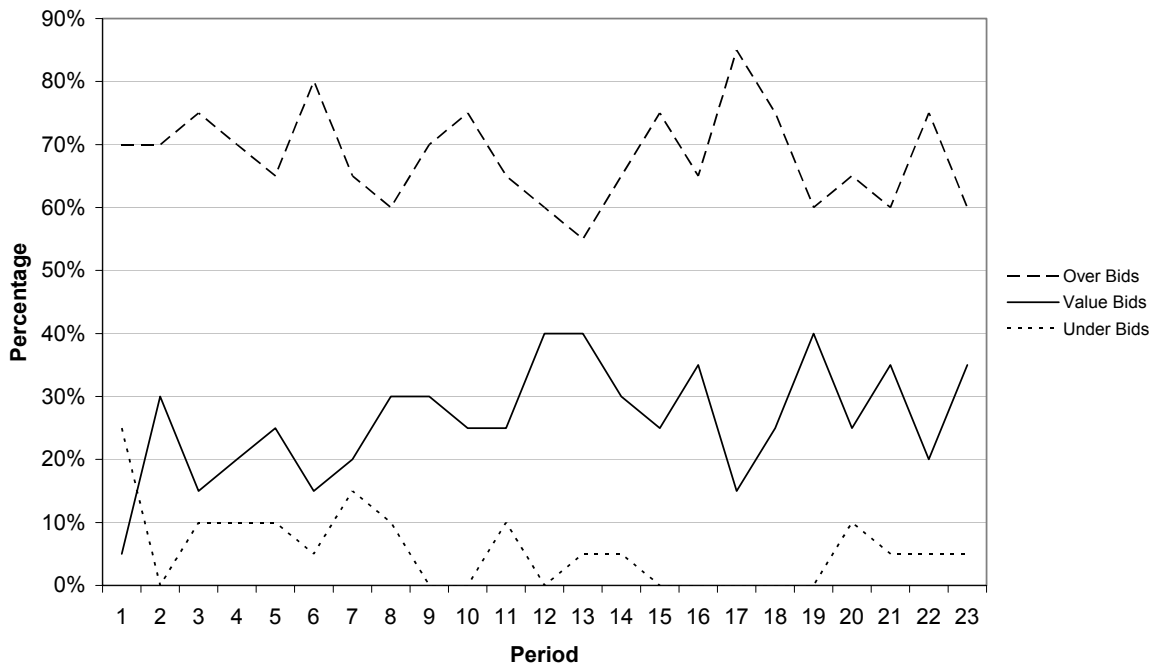


Figure 1: Frequency of under-, value-, and overbids in the 5 bidder markets of Sessions 2.1 and 2.2.

Figure 1 is obtained by pooling, period by period, the first 23 periods of bids in the 5 bidder markets of Sessions 2.1 with the 23 periods (1-13 and 26-35) of bids from 5 bidder markets of Session 2.2. (The bids from period 24 of Session 2.1 are dropped since they have no counterpart in Session 2.2.)

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<sup>3</sup> Kagel and Levin (1993) also report results from first and third price auctions, but here we restrict attention to second-price auctions.

K&L convincingly argue that the lack of convergence towards value bidding is a result of the weak learning incentives in second-price auctions. They report for the 5 bidder market that, conditional on winning the auction, a subject lost money with probability .25. Hence, subjects generally made money when they won, whether they had overbid or not.

### 3. Convergence to Efficiency

Figure 1 shows that subjects do not learn to value bid, at least as we have defined it here. However, examining auction efficiency reveals that subjects are bidding *differently* over time. Our measure of efficiency applies to an empirical distribution of bid-value pairs. Consider a sample of  $n$  bid-value pairs formed by pairing the bid of each of  $n$  subjects with the subject's value. Given this sample, auction efficiency in a market with  $k$  bidders is defined to be the probability that, in a random sample of  $k$  of the  $n$  bid-value pairs, the highest of the  $k$  bids is paired with the highest of the  $k$  values. It represents the probability the auction is efficient (i.e., the bidder with the highest value wins), when bidding behavior is described by the empirical distribution of bid-value pairs. An equivalent way to think of this efficiency measure is that it represents the fraction of the

$$J = \frac{n!}{k \times (n-k)!}$$

possible different groups of  $k$  bidders that, if formed, would yield an efficient outcome given their bids and values.

K&L's data for markets with 5 bidders provides, for each of the 23 periods, an empirical distribution of 20 bid-value pairs. Figure 2 plots auction efficiency, as defined above, in a market with 5 bidders across the 23 periods. It is visually apparent that efficiency tends to rise over time. In a regression of efficiency against time, the slope coefficient is statistically significant ( $p$ -value of .003), with the coefficient estimate suggesting that efficiency rises by approximately .76% each period. (The fitted regression line is also shown in Figure 2.)

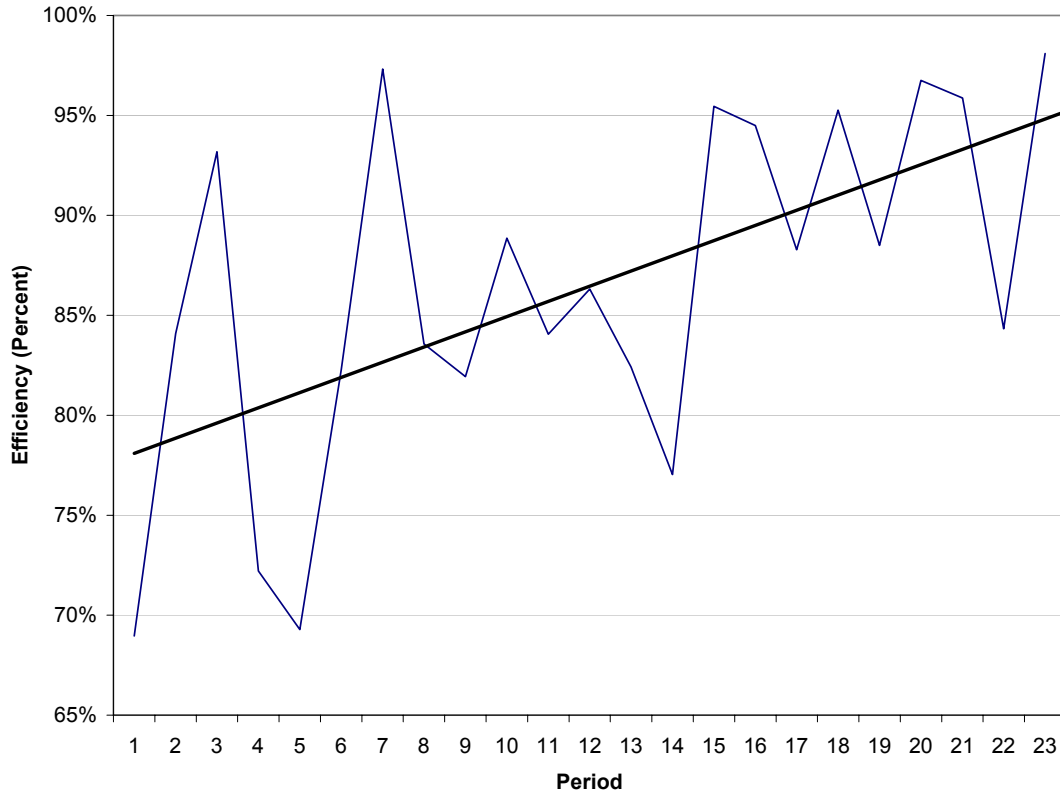


Figure 2: Efficiency over time in K&L's 5-Bidder Auctions

An alternative, but related, measure of efficiency is expected lost surplus. Expected lost surplus is the expected difference between the highest bidder's value and the highest value, in a random sample of 5 of the 20 bid-value pairs. It measures how much surplus is lost, on average, as a result of the item being allocated inefficiently. Once again, consider forming the  $J = 20!/[5!(20-5)!]$  possible groups of 5 bidders out of 20 bidders in each round, and index the different groups by  $j$ . For each group  $j$  ( $j= 1, \dots, J$ ) let  $v_j^*$  denote the highest value of a bidder in group  $j$ , and let  $v_j^{**}$  denote the value of the bidder with the highest bid in group  $j$ . Expected lost surplus for each round is given by

$$\frac{1}{J} \sum_{j=1}^J (v_j^* - v_j^{**}).$$

Figure 3 shows that expected lost surplus (measured on the left-vertical axis) falls quite quickly. In the first third, second third, and last third of the experiment the average expected loss is \$.67, \$.25, and \$.08. Hence, the expected loss from inefficient allocations becomes small.

The right-vertical axis of Figure 3 shows the (expected) surplus captured as a percentage of total surplus. This is defined by

$$\frac{1}{J} \sum_{j=1}^J \frac{v_j^* - v_j^{**}}{v_j^*},$$

and it is essentially above 99% over the last third of the experiment. Figure 3 shows that by the last third of the experiment the second-price sealed-bid auction is performing well, losing only a small amount of surplus in absolute terms, and capturing nearly all surplus in percentage terms. It shows that the failure over time of subjects to bid their dominant strategy largely does not undermine the efficiency of the second-price auction.

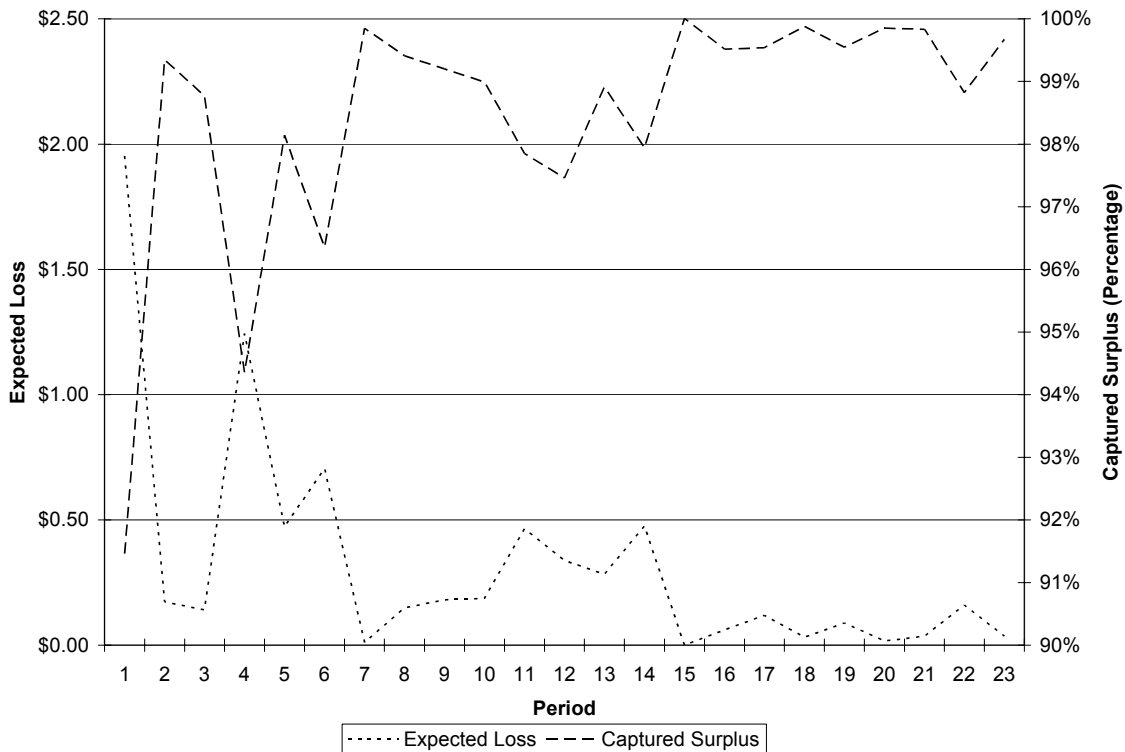


Figure 3: Expected lost surplus in K&L’s 5-bidder auctions.

#### 4. Understanding Rising Efficiency

While subjects do not learn to value bid, Figures 2 and 3 demonstrate that auction efficiency and the percentage of surplus captured are both rising, while expected lost surplus falls. In this section we investigate the changes in bidding behavior that explain these trends. In our analysis of the data we drop three very large overbids, made by the

same subject. Including them would have a large effect on the regression results since each of these overbids is more than \$26, while the mean absolute deviation from value bidding (excluding these overbids) is \$1.30 overall. Since the subject had high values when making these overbids, dropping them has little effect on the efficiency statistics displayed in Figures 2 and 3.<sup>4</sup>

The first feature of the data is that there is a small tendency for the mean overbid to decline over time. Column (b) of Table 2 reports the results of regressing overbids against time. It shows that overbids decrease by about 2.7 cents per period, and this effect is statistically significant. Declining overbids, however, do not explain rising efficiency. If, for example, every subject overbid by the same amount then efficiency would be 1, and constant across time, even if the amount of the overbid were falling over time.

More insight into rising efficiency can be obtained by examining the squared residuals of the regression from (b). The squared residuals are a measure of the variance of overbids, under the hypothesis that the mean overbid is declining according to the regression results reported in (b). Column (c) of Table 2 shows that the squared residuals, and hence the variance of overbids, are falling over time, and this effect is statistically significant. The auction allocation is inefficient when the bidder with the highest value doesn't have the highest bid. This becomes less likely as the variance of overbids falls, and hence the decline in the variance of overbids explains the rise in auction efficiency.

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<sup>4</sup> The three dropped bids (in periods 14, 16, and 17) were all of exactly \$55.00. Efficiency in these periods was, respectively, .77, .94, and .88. Excluding these bids has only a small effect on efficiency, yielding efficiencies of .82, .93, and .97, respectively.

Dependent Variable	Efficiency (a)	Overbid (b)	Squared Residuals from (b) (c)
Constant	0.773 (0.031)	1.295 (0.172)	6.632 (1.424)
Period	0.008 (0.002)	-0.027 (0.013)	-0.289 (0.104)
Observations	23	457	457
R squared	0.3130	0.0100	0.0174
Adj. R squared	0.0729	0.0078	0.0152

Standard errors are in parenthesis.

Table 2

## 6. Concluding Remarks

We have discovered a previously unrecognized feature of Kagel and Levin's second-price auction data: auction efficiency in a second-price sealed-bid auction rises over time. The explanation for this phenomenon appears to be that while bidders in second-price auctions do not learn to bid optimally over time they do learn to bid more like each other. It is the reduction in the variability of overbids that leads to increased efficiency.

Our findings call for further research into the robustness of learning and rising efficiency in second-price auctions. We took a step in this direction by conducting 4-bidder second-price auctions using the Veconlab software made available by Charles Holt at the University of Virginia (at the time the software did not allow 5-bidder markets). The results are very similar to the results described above for the Kagel and Levin data.<sup>5</sup>

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<sup>5</sup> For the interested reader, the data from our experiment, along with figures and a table analogous to those presented in this paper, are available online at <http://www.econ.ucsb.edu/~garratt/GW4bidder.xls>.

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