Chapter 4: Control Volume Analysis Using Energy

Conservation of Mass and Conservation of Energy
The **objective** of this chapter is to develop and illustrate the use of the control volume forms of the conservation of mass and conservation of energy principles. Mass and energy balances for control volumes are introduced in Secs. 4.1 and 4.2, respectively. These balances are applied in Sec. 4.3 to control volumes at steady state and in Sec. 4.4 for transient applications.

Although devices such as turbines, pumps, and compressors through which mass flows can be analyzed in principle by studying a particular quantity of matter (a closed system) as it passes through the device, it is normally preferable to think of a region of space through which mass flows (a control volume). As in the case of a closed system, energy transfer across the boundary of a control volume can occur by means of work and heat. In addition, another type of energy transfer must be accounted for- the energy accompanying mass as it enters or exits.
Examples of Open Systems
How do we get from the fixed mass (closed) systems that we know, to the open systems we are interested in?

Mass crosses boundary - What about energy?

Consequences of mass and energy crossing boundary?
Where does mass cross boundary?

Does total mass in the system change over time?
Where does **energy** cross boundary?

What is the purpose of this device?
What performance questions might we ask?
Which energies are most significant?
What is the relationship between these energies?
Does the total mass and/or energy of system change over time?
What **forms of energy** cross boundary?

- **Chemical**
  - **Thermal (In)**

- **Mechanical Work (Out)**
  - Rotational, Torque

- **Non-reacted Gas, Hot Gas (Out)**

- **Chemical Energy (In)**
  - (Stored in Chemical Bonds)
One Inlet, One Exit
Control Volume Boundary

Dashed line defines control volume boundary

Mass is shown in green
Developing Control Volume Mass Balance

Start with fixed mass (closed system) and allow an interaction with mass crossing a boundary. Transform to a form that accounts for flow across boundary.

Total mass is:

\[ m_{total} = m_i + m_{cv}(t) \]

At time \( t \)

Push mass \( m_i \) into CV during interval \( \Delta t \)
Developing Control Volume Mass Balance

At time $t + \Delta t$

$m_{\text{total}} = m_i + m_{cv}(t) = m_{cv}(t + \Delta t) + m_e$

$m_{cv}(t + \Delta t) - m_{cv}(t) = m_i - m_e$
Developing Control Volume Mass Balance

Mass balance

Express on rate basis over time interval $\Delta t$, as $\Delta t$ goes to zero:

$$m_{cv}(t + \Delta t) - m_{cv}(t) = m_i - m_e$$

$$\frac{m_{cv}(t + \Delta t) - m_{cv}(t)}{\Delta t} = \frac{m_i}{\Delta t} - \frac{m_e}{\Delta t}$$

$$\lim(\Delta t \to 0) \left[ \frac{m_{cv}(t + \Delta t) - m_{cv}(t)}{\Delta t} = \frac{m_i}{\Delta t} - \frac{m_e}{\Delta t} \right]$$

$$\frac{dm_{cv}}{dt} = \dot{m}_i - \dot{m}_e$$
Multiple Inlets and Exits
Control Volume Boundary
Dashed line defines control volume boundary

\[
d\frac{m_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]
Conservation of Mass for Control Volume

\[
\begin{bmatrix}
\text{time rate of change of mass contained within the control volume at time } t \\
\end{bmatrix} = \begin{bmatrix}
\text{time rate of flow of mass in across inlet } i \text{ at time } t \\
\text{time rate of flow of mass out across exit } e \text{ at time } t \\
\end{bmatrix}
\]

\[
\frac{dm_{cv}}{dt} = \sum_{i} m_i - \sum_{e} m_e
\]

Suppose

\[
\sum_{i} m_i > \sum_{e} m_e
\]

What happens to \( \frac{dm_{cv}}{dt} \)

\[\text{In} = \text{Stored} + \text{Out}\]
Conservation of Mass for Control Volume

\[
\begin{bmatrix}
\text{time rate of change of mass contained within the control volume at time } t
\end{bmatrix}
= \begin{bmatrix}
\text{time rate of flow of mass in across inlet } i \text{ at time } t
\end{bmatrix}
- \begin{bmatrix}
\text{time rate of flow of mass out across exit } e \text{ at time } t
\end{bmatrix}
\]

\[
\frac{dm_{cv}}{dt} = \sum_{i} m_{i} - \sum_{e} m_{e}
\]

What happens to \( \frac{dm_{cv}}{dt} \) if:

\[
\sum_{i} m_{i} < \sum_{e} m_{e} \quad \Rightarrow \quad \frac{dm_{cv}}{dt} < 0
\]

\[
\sum_{i} m_{i} = \sum_{e} m_{e} \quad \Rightarrow \quad \frac{dm_{cv}}{dt} = 0 \quad \text{Steady State}
\]
Evaluating Mass Flow Rate at Ports

\[
\text{amount of mass crossing } dA \text{ during the time interval } \Delta t = \rho(V_n \Delta t) dA
\]

\[
\text{instantaneous rate of mass flow across } dA = \rho V_n dA
\]

Integrate over port area:
\[
\dot{m} = \int_A \rho V_n dA
\]

\[
\frac{m}{L^3 \left( \frac{L}{t} \right)} L^2 \sim \frac{m}{t}
\]
Forms of Mass Rate Balance

\[
\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]

Alternative forms convenient for particular cases:

One-dimensional Flow

Steady State

Integral
One Dimensional Flow Model

How to handle non-uniform velocity/properties

Actual Two-Dimensional Flow:

\[ V = V(r) \]

\[ \dot{m} = \int_{A} \rho V dA = \int_{A} \rho(r) V(r) [2\pi r dr] \]

One-Dimensional Flow Model or Approximation:

\[ \dot{m} = \int_{A} \rho V dA = \bar{\rho} A \bar{V} \]

Note simplifying 2-D to 1-D
One Dimensional Flow Model

\[
\dot{m} = \rho A \overline{V}
\]

\[
\nu = \frac{1}{\rho}
\]

\[
\dot{m} = \frac{A \overline{V}}{\nu}
\]

\(A \overline{V}\) is the Volumetric Flow Rate (e.g. m³/kg)

\[
\frac{dm_{cv}}{dt} = \sum_i \frac{A_i \overline{V}_i}{\nu_i} - \sum_e \frac{A_e \overline{V}_e}{\nu_e} = \sum_i \rho_i \left( A_i \overline{V}_i \right) - \sum_e \rho_e \left( A_e \overline{V}_e \right)
\]

Convenient form when Volume Flow Rates known.
Other forms of Conservation of Mass

In many cases, the mass in balances the mass out

**Steady-State Flow:**

No changes in time

All derivatives with respect to time are zero

\[
\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]

What goes in, goes out!

\[
\sum_i \dot{m}_i = \sum_e \dot{m}_e
\]
Other forms of the Conservation of Mass

Fluid density may vary **within** the control volume
Fluid velocity and density may vary **across** flow area

\[
\frac{dm_{cv}}{dt} = \sum_i \dot{m}_i - \sum_e \dot{m}_e
\]

\[
m_{cv}(t) = \int_V \rho dV
\]

\[
\dot{m} = \int_A \rho V_n dA
\]

**Integral Form:**

\[
\frac{d}{dt} \int_V \rho dV = \sum_i \left( \int_A \rho V_n dA \right)_i - \sum_e \left( \int_A \rho V_n dA \right)_e
\]
Text Examples

Feedwater Heater at Steady State

Filling a barrel with water
Energy Conservation in Open Systems

- Mass crosses the system boundary, transporting
  - Internal thermal energy per unit mass, \( u \)
  - Kinetic energy per unit mass
  - Potential energy per unit mass
  - Chemical energy in chemical bonds

- Perform Transformation of Control Mass to Control Volume
Conservation of Energy for Control Volume

Approach similar to Mass Conservation derivation:

Closed system to open system

Mass balance to rate basis

Below:

\[ \text{Stored} = \text{Heat}_{\text{in}} - \text{Work}_{\text{out}} + \text{Net \ "Mass-Energy" In} \]

\[
\begin{align*}
\text{time rate of change of the energy contained within the control volume at time } t &= \text{net rate at which energy is being transferred in by heat transfer at time } t - \text{net rate at which energy is being transferred out by work at time } t + \text{net rate of energy transfer into the control volume accompanying mass flow} \\
\end{align*}
\]
Conservation of Energy for Control Volume

Single Inlet, Single Outlet

\[
\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \left[ m_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - m_e \left( u_e + \frac{V_e^2}{2} + gz_e \right) \right]
\]
Conservation of Energy for Control Volume

Energy transfers can occur by heat and work

\[ \frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \left[ m_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - m_e \left( u_e + \frac{V_e^2}{2} + gz_e \right) \right] \]

Note: Internal energy, Kinetic energy, Potential energy associated with fluid flow at inlet & exit
Conservation of Energy for Control Volume

Work term includes flow across boundary and all other forms

\[
\frac{dE_{cv}}{dt} = \dot{Q} - \dot{W} + \left[ m_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - m_e \left( u_e + \frac{V_e^2}{2} + gz_e \right) \right]
\]
Push mass $m_i$ into Control Volume

Force through a displacement is Work

Work IN, is done ON system at inlets
Concept of Flow Work

Push mass $m_e$ out of Control Volume

Force through a displacement is Work

Work OUT, is done BY system at outlets
Concept of **Flow Work Rate**

Work Rate: \( \frac{\delta W}{\delta t} = \dot{W} = F \frac{dx}{dt} = FV \)

Work Rate in terms of pressure & velocity

\[ \dot{W} = FV = (PA)V \]

\[
\text{time rate of energy transfer by work from the control volume at exit, e}
\]

\[ = (P_e A_e)V_e \]
Evaluating Rate of Flow Work

$$\dot{W} = \dot{W}_{cv} + (P_e A_e) V_e - (P_i A_i) V_i$$

$$\dot{m} = \rho A V$$ or $$\dot{m} = \frac{A V}{\nu}$$

$$\left( P A \right) V = \dot{m} (P \nu)$$

$$\dot{W} = \dot{W}_{cv} + \dot{m}_e \left( P_e \nu_e \right) - \dot{m}_i \left( P_i \nu_i \right)$$

\(\nu\) is specific volume

Rate of Flow Work & Rate of All other forms of work included (rotating shafts, displacement of boundary, electrical work, etc.)
Other forms of Conservation of Energy

Single Inlet, Single Outlet

\[ W = W_{cv} + \text{Flow Work} = W_{cv} + m_e (P_e v_e) - m_i (P_i v_i) \]

Recall: \[ h = u + P \cdot v \]

Energy Rate Balance:

\[ \frac{dE_{cv}}{dt} = Q_{cv} - W_{cv} + m_i \left( u_i + \frac{V_i^2}{2} + gz_i \right) - m_e \left( u_e + \frac{V_e^2}{2} + gz_e \right) \]
Multiple Ports

**Energy Rate Balance:**

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i m_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e m_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]

**Variations within CV and at boundaries:**

\[
E_{cv}(t) = \int_V \rho \dot{e} dV = \int_V \rho \left( u + \frac{V^2}{2} + gz \right) dV
\]

**Integral Form:**

\[
\frac{d}{dt} \int_V \rho \dot{e} dV = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \left[ \int_A \left( h + \frac{V^2}{2} + gz \right) \rho V_n dA \right]_i - \sum_e \left[ \int_A \left( h + \frac{V^2}{2} + gz \right) \rho V_n dA \right]_e
\]
Analyzing Control Volumes at Steady State

Transient start up and shutdown periods not included here
Steady-State Forms of Mass and Energy Rate Balances

Control volume at Steady-State (SS):

- **Conditions (states/properties)** of mass within CV and at boundaries do not vary with time
- No change of mass (+ or -) within CV
- Mass flow rates are constant
- Energy transfer rates by heat & work are constant
Steady-State Forms of Mass and Energy Rate Balances

Control volume at Steady-State (SS):

- Conditions (states/properties) of mass within CV & at boundaries do not vary with time

\[ \rho(x, y, z) \]
\[ T(x, y, z) \]
Steady-State Forms of Mass and Energy Rate Balances

Control volume at Steady-State (SS):

- No change of mass (+ or -) within CV
Steady-State Forms of Mass and Energy Rate Balances

Control volume at Steady-State (SS):

- Mass flow rates are constant

\[ \dot{m}_e \quad \text{Yes:} \]
\[ \dot{m}_i \quad \text{No:} \]
Steady-State Forms of Mass and Energy Rate Balances

Control volume at Steady-State (SS):

- Energy transfer rates by heat & work are constant
Steady State

Special case when no changes occur in time

\[
\frac{dm_{cv}}{dt} = \sum_i m_i - \sum_e m_e
\]

\[
\sum_i \dot{m}_i = \sum_e \dot{m}_e
\]

**Steady State Flow:**

\[
0 = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]

Net Energy Flow In = Net Energy Flow Out

\[
\dot{Q}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) = \dot{W}_{cv} + \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]
One Inlet, One Outlet Steady State

Special case when no changes occur in time

\[
\dot{m}_i = \dot{m}_e = \dot{m}
\]

\[
0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m} \left[ (h_i - h_e) + \frac{(V_i^2 - V_e^2)}{2} + g(z_i - z_e) \right]
\]

\[
0 = \frac{\dot{Q}_{cv}}{\dot{m}} - \frac{\dot{W}_{cv}}{\dot{m}} + (h_i - h_e) + \frac{(V_i^2 - V_e^2)}{2} + g(z_i - z_e)
\]

Note energy DIFFERENCES- Reference cancels out
Modeling Control Volumes at Steady State

Common modeling assumptions- The Art of Engineering

Heat Transfer- can be ignored (adiabatic) when:

- Outer surface is well insulated
- Outer surface too small for significant heat transfer
- Temperature difference between surface and surroundings so small that heat transfer can be ignored
- Fluid passes through CV so quickly that not enough time for significant heat transfer to occur

Work Transfer- can be ignored when:

- No rotating shafts, displacements of boundary, electrical effects or other work mechanisms
Modeling Control Volumes at Steady State

Common modeling assumptions- The Art of Engineering

Kinetic and/or Potential Energy- can be ignored when:

• Are small compared to other energy forms

Steady state may be OK if:

• Properties only fluctuate slightly about average values
• Periodic time variations are observed: time-average
Examples
Nozzles and Diffusers

\[ \dot{m} = \rho A V \]

\[ \dot{m}_1 = \rho_1 A_1 V_1 = \dot{m}_2 = \rho_2 A_2 V_2 \]

State & Process Information Needed to Determine Pressures, etc
Steady Flow Devices

Nozzles and Diffusers

Photo courtesy of NASA Ames
AME Windtunnels

Low-Speed Closed Return Wind Tunnel Facility
Nozzle & Diffuser Steady Flow Analysis

\[ \frac{dm_{cv}}{dt} = m_1 - m_2 \]

\[ \frac{dm_{cv}}{dt} = 0 \]

\[ m_1 = m_2 = \dot{m} \]

\[ \frac{dE_{cv}}{dt} = 0 = \dot{Q}_{cv} - \dot{W}_{cv} + \dot{m}_1 \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \dot{m}_2 \left( h_e + \frac{V_e^2}{2} + gz_e \right) \]

\[ 0 = \dot{Q}_{cv} \left( h_1 - h_2 \right) + \left( \frac{V_1^2 - V_2^2}{2} \right) + \frac{\dot{Q}_{cv}}{\dot{m}} \text{ Often negligible} \]
Nozzles and Diffusers

Common Form of 1\textsuperscript{st} Law:

\[ 0 = (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right) \]
Text Example

Calculate Exit Area of a Steam Nozzle
Turbines

1100kW Helicopter Engine

Photo courtesy of U.S. Military Academy

50 watt Microturbine

Photo courtesy of M.I.T. Microturbine lab

Turbine schematics courtesy of www.howstuffworks.com
Steam and Gas Turbines

Axial flow steam of gas turbine

Common Form of 1st Law:

\[
\frac{\dot{W}}{m} = (h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)
\]

Single Inlet, Single Outlet
Heat transfer is often small and neglected.
\(\Delta KE \ & \Delta PE\) often small and neglected
Work produced is equal to enthalpy difference

\[
\dot{w} = \frac{\dot{W}}{m} = (h_1 - h_2)
\]
Hydraulic Turbine in a Dam

Common Form of 1st Law:
\[
\frac{\dot{W}}{m} = (h_1 - h_2) + \left( \frac{V_1^2 - V_2^2}{2} \right) + g(z_1 - z_2)
\]

Heat transfer is small and neglected.
Work produced is associated primarily with change in PE
Text Example

Calculate Heat Transfer from a Steam Turbine
Compressors and Pumps

Compressors are devices where work is done on a GAS to raise pressure, change state.

Pumps are devices where work is done on a LIQUID to raise pressure, change state.
Compressors and Pumps

Rotating compressors

Reciprocating compressor

Common Form of 1st Law:

\[
\dot{W} = \frac{\dot{m}}{(h_1 - h_2) + \left(\frac{V_1^2 - V_2^2}{2}\right) + g(z_1 - z_2)} \quad [W < 0]
\]
Text Example

Calculate Compressor Power

Power Washer
Heat Exchangers

Heat exchangers are devices that exchange thermal energy between fluids at different temperatures by heat transfer, $Q$

Work, changes in KE and PE are not usually important

Changes in enthalpy are important

Balance is between $Q$ and changes in enthalpy
Heat Exchangers

- Direct Contact
- Tube-within-a-tube parallel flow
- Tube-within-a-tube counterflow
- Cross-flow

Common Form of 1st Law:

\[
\sum_i m_i \left( h_i + \frac{V_i^2}{2} + g\varepsilon_i \right) = \sum_e m_e \left( h_e + \frac{V_e^2}{2} + g\varepsilon_e \right)
\]
Text Example

Power plant condenser

Cooling computer components
Throttling Devices

These are devices designed to reduce the pressure of a fluid (gas or liquid)

\[
\sum_{i} m_i \left( h_i + \frac{V_i^2}{2} + g z_i \right) = \sum_{e} m_e \left( h_e + \frac{V_e^2}{2} + g z_e \right)
\]
Throttling Devices

Common Form of 1st Law: \( h_i = h_e \)

Work = 0

Adiabatic assumption often reasonable (why?)

Changes in KE and PE are not usually important (why not?)

Single inlet, single outlet implies enthalpy balance
Measuring steam quality
System Integration

Engineers creatively combine/integrate components into systems to achieve overall objective, subject to constraints such as cost, weight, pollution

Example of simple power plant given: (Turbine/generator, condenser, pump, boiler)
Text Example

Waste heat recovery system
Transient (Unsteady) Analysis

- Many devices undergo periods of transient operation in which state changes with time.
- Examples include startup and shutdown of turbines, compressors, and motors.
- Filling and emptying vessels are other examples.
- Steady state assumptions not valid.
- Special care in applying mass and energy analyses.
Transient Mass Balance

Recall that:

\[
\frac{dm_{cv}}{dt} = \sum_{i} m_{i} - \sum_{e} m_{e}
\]

Integrate mass balance from \( t = 0 \) to final time, \( t \):

\[
\int_{0}^{t} \left( \frac{dm_{cv}}{dt} \right) dt = \int_{0}^{t} \left( \sum_{i} m_{i} \right) dt - \int_{0}^{t} \left( \sum_{e} m_{e} \right) dt
\]

\[
m_{cv}(t) - m_{cv}(0) = \sum_{i} \left( \int_{0}^{t} m_{i} dt \right) - \sum_{e} \left( \int_{0}^{t} m_{e} dt \right)
\]
Transient Mass Balance

\[
\int_0^t \left( \frac{dm_{cv}}{dt} \right) dt = \int_0^t \left( \sum_i \dot{m}_i \right) dt - \int_0^t \left( \sum_e \dot{m}_e \right) dt
\]

\[
m_{cv}(t) - m_{cv}(0) = \sum_i \left( \int_0^t \dot{m}_i \, dt \right) - \sum_e \left( \int_0^t \dot{m}_e \, dt \right)
\]

\[
m_i = \int_0^t \dot{m}_i \, dt \quad \text{amount of mass entering the control volume through inlet } i, \text{ from time } 0 \text{ to } t
\]

\[
m_e = \int_0^t \dot{m}_e \, dt \quad \text{amount of mass exiting the control volume through exit } e, \text{ from time } 0 \text{ to } t
\]

Change of mass in CV = Total mass going IN minus Total mass going OUT
Transient Energy Balance

Ignore KE and PE and Integrate this wrt time:

\[
\frac{dE_{cv}}{dt} = \dot{Q}_{cv} - \dot{W}_{cv} + \sum_i \dot{m}_i \left( h_i + \frac{V_i^2}{2} + gz_i \right) - \sum_e \dot{m}_e \left( h_e + \frac{V_e^2}{2} + gz_e \right)
\]

\[
U_{cv}(t) - U_{cv}(0) = Q_{cv} - W_{cv} + \sum_i \left( \int_0^t \dot{m}_i h_i \, dt \right) - \sum_i \left( \int_0^t \dot{m}_e h_e \, dt \right)
\]

For special case where states at inlet & outlets are constant with time:

\[
\int_0^t \dot{m}_i h_i \, dt = h_i \int_0^t \dot{m}_i \, dt = h_i m_i \quad \int_0^t \dot{m}_e h_e \, dt = h_e \int_0^t \dot{m}_e \, dt = h_e m_e
\]

\[
U_{cv}(t) - U_{cv}(0) = Q_{cv} - W_{cv} + \sum_i m_i h_i - \sum_e m_e h_e
\]
Another special case where intensive properties within CV are uniform with position at each instant:

\[
m_{cv}(t) = \frac{V_{cv}(t)}{v(t)}
\]

\[
U_{cv}(t) = m_{cv}(t)u(t)
\]
Text Examples

Withdrawing steam from tank at constant pressure

Using Steam for Emergency Power Generation (new)

Storing compressed air in a tank

Temperature variation in a well-stirred tank
END