# The Impact of Market Structure and Learning on the Tradeoff between R&D Competition and Cooperation

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**Abstract**

This paper explores the trade-off between R&D cooperation and competition with learning. We develop a continuous time, two-armed bandit model in which firms can devote resources to a "safe" investment in an established market or to a risky R&D investment aimed at discovering a new product that is characterized by both "if" and "when" uncertainty. The firm that wins the race under R&D competition enjoys a period of monopoly profits. But, after this period, competition in the market for the new product occurs, which may also impair the profitability of each firm's established product. Post-patent market structure and the extent of the adverse impact of the new product on the profitability of established products play a key role in driving the tradeoff between R&D competition and R&D cooperation. Firms' incentives to invest in the non-cooperative regime differ from a consortium's investment incentives. Under non-cooperative R&D, a free rider problem can arise, which generally results in an equilibrium investment flow that is less than or equal to that of the research consortium. Expected ex ante undiscounted investment and welfare under R&D competition can also be shown to be less than what it would be under R&D cooperation. However, if the gain due to the oligopoly profit from the new product is less than the loss in profit from the established product, then the free rider problem does not arise. In this case, R&D cooperation results in the same or lower level of investment than arises under non-cooperative R&D. Thus, in contrast to the traditional literature, we show that technology spillover alone need not lead to higher R&D investment or higher social welfare under research cooperation. In fact, significant product market spillovers are necessary if the underlying R&D project exhibits both "if" and "when" uncertainty.

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# 1 Introduction

Since the passage of the National Cooperation Act in 1984, firms are encouraged to form research consortia to conduct R&D, so long as they maintain competition in product markets. In the 1990s, regulatory authorities further relaxed antitrust restraints on these consortia.[1] As a result, the past two decades have witnessed a surge of cooperative R&D, most especially in high technology industries such as pharmaceuticals, information technology (IT), and aerospace & defense (Hagedoorn, 2002). Traditional theories of R&D cooperation (e.g., D'Aspremont and Jacquemin, 1988; Kamien, *et al.*, 1992; Jin and Troege, 2006; Salant and Shaffer, 1998; Amir *et al.*, 2003) have focused on process innovations that are guaranteed to reduce production costs. But in practice, industries exhibiting the most significant growth in R&D cooperation are those in which firms typically engage in new product R&D and face significant uncertainties both about whether their R&D investment will ever result in a commercializable discovery, and if so, how long it will take. This paper presents a theory of R&D cooperation and competition in which there is not only uncertainty about how long it will take to discover a new product, but also uncertainty about whether a new discovery is even possible. The latter type of uncertainty makes strategic experimentation and learning a key feature of the R&D investment process: by devoting more resources to a risky R&D project, a firm not only can accelerate the time to discovery (if the project is indeed viable), but it can also increase the rate at which it updates its beliefs about the project's viability. The focus on "if" and "when" uncertainty, and the associated importance of strategic experimentation and learning, distinguishes this paper from previous studies of the trade-offs between R&D cooperation and R&D competition.

Specifically, we apply the exponential bandit framework of strategic experimentation developed by Keller, Rady, and Cripps (2005) to a setting in which firms can either devote resources to a "safe" investment in an established market or to a risky R&D investment aimed at discovery of a new product. Under R&D competition, firms independently make investments in the hope of discovering the technology to make the new product. The firm that makes the discovery enjoys a period of monopoly profits. But after a period of time (the duration of which we interpret as the length of a patent), competition in the market for the new product occurs.[2] This competition may involve new firms that enter the market in

[1] In 1993, the US enacted the National Cooperative Research and Production Act (NCRPA), which requires "rule of reason" rather than per se prohibition in analyzing possible antitrust challenges over those joint ventures. The European Union also has regulations addressing similar antitrust concerns.

[2] Alternatively, this time period could be thought of as the length of time it takes a firm to imitate

the post-patent period. In addition, the new product may also impair the profitability of each firm's established product. Firms observe their own and their competitors' investment levels and R&D outcomes, and as time passes without a discovery having been made by any firm, they become more pessimistic about the viability of the project.[3] Under cooperative R&D, firms coordinate their investment levels and share the monopoly profit for the new product. But after the patent period ends, firms compete in the market for the new product, as in the noncooperative case. And, as in the noncooperative case, the discovery of a viable new product will impair the profitability of the firms' established products. As we will show, the post-patent market structure and the extent of the adverse impact of the new product on the profitability of established products play a key role in driving the trade-off between R&D competition and R&D cooperation.

In our analysis, we explicitly characterize the symmetric equilibrium investment and value functions under cooperative R&D and noncooperative R&D. The state variable in our model is the firms' common posterior probability that the R&D project is viable. The optimal R&D investment decision for a research consortium is a cutoff rule: if the research consortium's belief about the viability of the R&D project is higher than a particular threshold, each firm in the consortium will invest the maximum amount possible in new product R&D. Otherwise, firms in the consortium will not invest. In other words, the consortium's equilibrium investment policy is "bang-bang" in the state variable.

By contrast, the (symmetric) equilibrium investment strategy under R&D competition may not be "bang-bang." This is because under R&D competition there may be a free-rider problem that creates a coordination issue. The free-rider problem can arise because once the patent period expires, firms other the discovering firm are assumed to be able to imitate the product and derive oligopoly profits from it. Thus, if a set of firms invest to the greatest possible extent, other firms may prefer not to invest at all, but if the firms in the initial set choose not to invest, the other firms may in fact invest. Thus, there may not be a symmetric equilibrium in which all firms invest fully or all do not. In these cases, which occur for intermediate beliefs about viability, the symmetric equilibrium involves both firms making investments at a level that is less than the maximum. Hence, noncooperative R&D results in equilibrium investment behavior that can be quite different from the investment behavior

around the patented product.

[3]The assumption that firms observe the R&D outcomes of their rivals, means that under R&D competition firms are, in effect, pooling their learning. Thus, what we refer to as non-cooperative R&D would be analogous to what Kamien et al. refer to as "research joint venture (RJV) competition." While what we refer to as cooperative R&D would be analogous to what Kamien et. al. refer to as "RJV cartelization."

of a research consortium. For example, a research consortium that has been unsuccessfully investing “flat out” to develop a new product will abruptly stop its investment activities once it crosses the discontinuation belief threshold. By contrast, noncooperative firms engaging in R&D competition would decrease their investment levels gradually over time.

When R&D competition entails a free-rider problem, equilibrium investment for any posterior is less than or equal to the equilibrium investment by the research consortium (and this inequality is strict for a range of posteriors with positive measure). In this case, expected *ex ante* undiscounted investment under R&D competition is also less than what it would be under R&D cooperation. Further, when the free-rider problem arises, R&D cooperation can be shown to be welfare improving. These results on the impact of R&D cooperation on investment and welfare are analogous to those in the traditional literature (e.g., d’Aspremont and Jacquemin, 1988; Kamien *et al.*, 1992). The difference, however, lies in the critical role of product market spillover. d’Aspremont and Jacquemin (1988) shows that significant technology spillover is sufficient to induce higher R&D investment. In contrast, we show that in addition to technology spillover, significant product market spillover is necessary to generate higher investment in cooperative R&D. This is because if there is both "if" and "when" uncertainty, a research consortium may terminate the R&D project earlier then noncooperative firms when there is no significant product market spillover.

Nevertheless, R&D competition may not always entail a free-rider problem. If the gain due to the oligopoly profit from the new product is less than the loss in profit from the established product — which could occur if competition in the post-patent market is sufficiently intense, if the patent period is sufficiently long and/or if firms cannot easily imitate the patented product — then a firm that does not invest in R&D may be harmed if another firm invests and discovers the new product. In this case, R&D competition is akin to a “winner-take-all” market, and unlike the Keller, Rady, and Cripps (2005) model, the (symmetric) equilibrium strategy is “bang-bang.”

We show that in this case, R&D competition results in the same or higher level of investment than arises under an R&D consortium and that the competitive level is unambiguously greater over a range of beliefs with positive measure. Thus, in contrast to the traditional literature, we show that research cooperation need not lead to more investment. Furthermore, in this case, research cooperation may result in a lower level of social welfare. Finally, the *ex ante* likelihood of making the new discovery in this case is shown to be lower under cooperative R&D than it is under noncooperative R&D. This implies that a necessary condition for cooperative R&D to result in a higher likelihood of discovery is that R&D competition be characterized by the free-rider problem. These results suggest that in a setting that generates aggressive R&D racing to develop a new product, the insights about R&D cooperation generated by the static models of process innovation may not apply.

Our results have relevance for the empirical study of the impact of R&D cooperation on R&D outcomes. Becker and Dietz (2004) show that across a large sample of German manufacturing industries, R&D cooperation is associated with greater R&D intensity and a greater probability of developing new products. Our results suggest that this aggregate result may not necessarily hold within specific industries. Our model would predict that in industries in which post-patent competition is intense and/or imitation of new product discoveries is very difficult, R&D competition is likely to be intense, and the formation of an R&D consortium would be expected to moderate investment incentives. Thus, empirical work that studies how R&D cooperation affects investment incentives should ideally take into account the impact of post-patent market structure on investment incentives.

This paper is related to the literature on R&D that focuses on the game theoretic study of new product or technology development, as pioneered by Reinganum (1982).[4] Recently, a small set of papers began incorporating belief updating in this type of R&D competition. Malueg and Tsutsui (1997) consider a differential game of R&D competition with belief updating, in which the equilibrium investment is a function of beliefs. Decamps and Mariotti (2004) assume that firms do not know the value of the project but can observe a signal before they enter the market. In addition, they can update their belief about the value when seeing other firms entering the market. Those papers study R&D competition as an isolated game while our paper analyzes how firms balance the existing market growth with potential market profit when participating in an R&D race. Our paper differs from this line of literature by incorporating market structure to study the trade-offs between cooperative and noncooperative research and development.

This paper is also related to several papers in the literature on strategic experimentation. Bergemann and Välimäki (1997, 2000) analyze a model in which buyers and sellers learn the value of the new product through experimentation. Bolton and Harris (1999) extend the classic two-armed bandit problem to a many-agent setup. In essence, a firm allocates resources between two arms. The "safe" arm generates a steady but low pay-

[4] See Reinganum(1989) for an excellent survey of this literature.

off, while the "risky" arm generates a random payoff that follows a geometric Brownian motion.[5] As noted above, our model uses the exponential bandit framework developed by Keller, Rady and Cripps (2005). In their model, the arrival of payoff from the risky arm follows a Poisson process, a specification that enables an elegant characterization of the symmetric equilibrium under both competition and cooperation. But our model differs from the Keller, Rady and Cripps model in an economically important way. Their model assumes that the success of the experiment leads to a higher expected payoff for each agent. However, as we note below, this may not fully characterize the environment of R&D competition where there are plausible circumstances under which firms will be worse off if the competitor makes the discovery. Furthermore, the focus of the Keller, Rady, Cripps model is an exploration of the general properties of the two-arm bandit problem with exponential bandits. Our paper aims to apply the exponential bandit framework to study the trade-off between R&D competition and cooperation. Thus our paper explicitly incorporates market structure in a way that the Keller, Rady, Cripps paper does not and demonstrates its critical role in affecting firms' incentives to invest when facing different research configurations.

The paper is divided into six sections. Section 2 describes the model. Section 3 analyzes the equilibrium in R&D competition and discusses a research consortium's optimal investment decision. Section 4 compares the cooperative and noncooperative research while section 5 studies the welfare implication. Section 6 concludes the paper.

## 2 The Model

We consider a continuous-time game with $N$ firms. Each firm considers investing in an R&D project that could result in the discovery of a new product. However, the viability of the R&D project is uncertain: no firm knows for sure whether this R&D project is indeed destined to succeed. If the project does succeed and the new product is discovered, the winner of the R&D competition will be awarded a patent of length $T$, during which it will earn a flow profit $\tilde{\pi}_m$. After the patent expires, other firms may imitate the new product. In the post-patent competition, each firm (including the original discoverers and each of the $N-1$ other firms that took part in the R&D race) will earn an oligopoly profit $\tilde{\pi}_N < \frac{1}{N}\tilde{\pi}_m$.

[5] Rosenberg, Solan and Vieille (2007) study a similar social learning model by incorporating private information in payoffs.

In addition to investing in R&D to discover the new product, each firm is also assumed to sell an established product. Unlike the new product which will be sold in a market in which the firms will eventually compete with each other, each firm's established product is sold in an isolated product market. Although other interpretations are possible, one could think of the firms as currently operating in independent local geographies but competing to discover a product that could be sold in a national or global market. At any instant in time, each firm's established market contains a measure one of potential customers, of which $(1-\alpha)$ percent are served and $\alpha$ percent have yet to be attracted to a firm's established product. These remaining consumers can be tapped by devoting resources to marketing the established product. We assume that each sale in a firm's established market generates a profit $s$.

We model R&D competition among firms as a two-armed bandit problem. A two-arm bandit model is especially useful for gaining insight into situations in which decision makers are both optimizing their actions and learning at the same time. The safe arm for each firm is its established market; the risky arm is the R&D project whose viability is unknown. At each instant $t$, a firm has only one unit of a resource, to be allocated between marketing its established product and investing in R&D to discover the new product. The scarce resource could be interpreted most tangibly as a monetary investment in R&D, but it could also be interpreted more abstractly as scarce managerial or organizational "attention" that must be allocated between nurturing an established product and discovering a new product.[6] If the firm allocates $k_t^i \in [0,1]$ to R&D, then its investment in marketing the established product is $\left(1-k_t^i\right)$, which allows the firm to attract $\left(1-k_t^i\right)\alpha$ new customers in its established market. This implies that the total net profit each firm obtains in the established market is $\left(1-k_t^i\right)\alpha s+(1-\alpha)s=\left(1-\alpha k_t^i\right)s$. If firm $i$ sets $k_t^i=0$, it then spends all its resources on marketing the established product, in which case it earns a steady stream of profit $s$ for every unit of time before any firm succeeds in developing the new product.

Although no firm knows whether the R&D investment is viable, firms share the same prior belief over the likelihood that the R&D project will eventually yield a new discovery. Let $\eta$ be a Bernoulli random variable such that $\eta=1$ implies the R&D project will eventually succeed almost surely and $\eta=0$ implies that the project is destined to fail.

[6] Literature on diversification often assumes limited managerial attention to which a firm has access (Bowen and Wiersema, 2005). A critical problem identified in this literature is how firm management allocates its limited attention between its core and non-core business when it comes to the decision on diversification.

Suppose $k_t^i \in [0,1]$ is allocated to the investment in R&D. Conditional on $\eta = 1$, firm $i$ may succeed in developing the new product with probability $\lambda k_t^i dt$ within the time interval $[t, t+dt)$. In other words, even though the R&D project is destined to succeed when $\eta = 1$, the timing of the success is uncertain, but can be accelerated in a stochastic sense by the level of the firm's investment.

Let $p_0$ be the common prior belief about the viability of the new R&D investment. In other words, $p_0 = \Pr(\eta = 1)$, meaning that each firm believes that with probability $p_0$ the new product R&D project is viable. Conditional on the project being viable, it is a matter of time before some firm succeeds in developing the product. On the other hand, with probability $1 - p_0$, the project is destined to fail. Each firm observes whether its competitor has succeeded or not. As time passes, and no firm has succeeded, each firm updates its belief about the viability of the R&D investment. Suppose the current date is $t$ and firms' belief about whether the project is viable is $p(t)$. According to Bayes rule, if no firm succeeds within the interval $[t, t+dt)$, their belief about the project being viable becomes

$$p(t+dt) = \frac{p(t)\left(1 - \lambda \sum_{i=1}^{N} k_t^i dt\right)}{1 - p(t) + p(t)\left(1 - \lambda \sum_{i=1}^{N} k_t^i dt\right)}. \tag{1}$$

Conditional on the new product being feasible, there is probability $\left(1 - \lambda \sum_{i=1}^{N} k_t^i dt\right)$ that no firm succeeds after the time has elapsed by $dt$.[7] Thus the numerator of equation (1) is the probability that there is no success within a time interval $[t, t+dt)$ when the project is indeed viable, while the denominator is the probability of observing no success within the time interval $[t, t+dt)$. It can be shown that

$$\frac{dp}{dt} = \lim_{dt \to 0} \frac{p(t+dt) - p(t)}{dt} = -\lambda \sum_{i=1}^{N} k_t^i p(t)(1 - p(t)). \tag{2}$$

This rate of belief updating is independent of its starting state, so we may rewrite it as

$$dp = -\lambda \sum_{i=1}^{N} k_t^i p(1-p)\, dt.$$

We assume that the discovery of the new product will affect each firm's established

[7] The probability of no success within time $dt$ for all firms $i$ is $e^{-\lambda \sum_{i=1}^{N} k_t^i dt}$, whose first order approximation is $1 - \lambda \sum_{i=1}^{N} k_t^i dt$.

market. Specifically, the arrival of the new product reduces the per unit profit of each firm's established product to $\beta s$, where $\beta \in [0,1]$. $\beta = 0$ implies that each firm's established market is completely eliminated by the new market, while $\beta = 1$ implies the old market is unaffected by the new market.

Let the discount rate be $r$, and define $\Pi_m = \int_0^T \tilde{\pi}_m e^{-rt} dt = \frac{\tilde{\pi}_m}{r}\left(1 - e^{-rT}\right)$ to be the discounted present value of monopoly profit from the new product during the patent protection periods. Hence, $\Pi_m$ is the prize for the winner of this R&D competition due to patent protection. Once the patent expires, any firm currently in the market is able to produce this new product, which results in a payoff of $\tilde{\pi}_N$ for each firm. Let $\pi_N = r \int_T^\infty \tilde{\pi}_N e^{-rt} dt = e^{-rT} \tilde{\pi}_N$ denote the profit flow that an oligopoly firm earns, discounted back to the date of breakthrough.

Having defined $\Pi_m$ and $\pi_N$, we can interpret our model as the one in which there is product market R&D spillover in the spirit of Dixit (1988)[8]. If there is no patent protection, i.e., $T = 0$, and thus $\Pi_m = 0$, then once one firm discovers the new product, all other firms immediately can sell the new product as well, and we have a complete spillover. If patent protection is infinite, i.e., $T = \infty$ and $\pi_N = 0$, then the new product is completely proprietary, and we have no spillover. A partial spillover, which is captured by the ratio $\xi = \frac{\pi_N}{r\Pi_m + \pi_N} \in (0,1)$, occurs when there is positive but finite patent protection. Note that there is a one-to-one correspondence between $\pi_N$ and the spillover ratio $\xi$ so that a large (small) value of $\pi_N$ is associated with a large (small) spillover ratio. This spillover is different from the technology spillover defined in traditional process R&D literature such as d'Aspremont and Jacquemin (1988). In our paper, each firm takes other firms' full investment in to account when updating its belief about the project viability, so we have implicitly assumed full technology spillover.

Note that the difference between $\pi_N$ and $(1-\beta)s$ can be interpreted as the externality as a result of post-patent competition, because $\pi_N$ is the discounted profit flow from the post-patent market in the new product and $(1-\beta)s$ is reduced profit flow from the established market. If $\pi_N > (1-\beta)s$, then the success of one firm in developing R&D generally lead to net improvement for all firms, and hence results in a positive externality; if $\pi_N < (1-\beta)s$, then we have a negative externality. For simplicity, we define

$$\theta = \pi_N - (1-\beta)s,$$

[8] In the language of Dixit, $\Pi_m$ is the appropriable benefit by the winner and $\pi_N$ is the spillover to other firms.

so the sign of $\theta$ coincides with the type of externality.

In this paper, we exclude the uninteresting case where R&D investment is strictly dominated. In other words, we assume a firm is willing to invest in R&D if its belief about the viability is $p_0 = 1$. This is tantamount to the following assumption:[9]

**Assumption 1** $\alpha s < \frac{\lambda[r\Pi_m + N\theta]}{r}$.

Our solution concept is Markov perfect equilibrium, and the belief $p$ is the only payoff relevant state. As a result, we derive the equilibrium strategies as a function of $p$.

Before concluding the presentation of the model, it is important to note that the assumption that the firm must allocate a scarce resource between marketing an established product and investing in a new product is not critical to the analysis (though it is, of course, important in interpreting the model as a two-arm bandit problem). Alternatively, we could have assumed that the firm can invest as little or as much as it wants in R&D up to some technologically determined limit (representing an extreme form of diminishing marginal returns). Under this interpretation $\alpha s$ is the instantaneous cost of the investment (which could be normalized to 1) and $-(1-\beta)s$ would represent the negative impact of the new discovery on the profitability of a firm's existing lines of business, so that $\pi_N - (1-\beta)s$ is then the post-patent instantaneous cash flow of a firm.

# 3 Equilibrium

In this section, we analyze the equilibrium under R&D competition. Once we understand an individual firm's equilibrium R&D investment strategy, we can quickly derive an R&D consortium's optimal investment decision, because in the later case, all firms act as if they were a single decision maker.

## 3.1 R&D Competition

Let $V_i^n(p)$ denote firm $i$'s value function when the common belief about the new product viability is $p$. We can write firm $i$'s Bellman equation as:

$$V_i^n(p) = \max_{k_t^i \in [0,1]} \left\{ \left(1 - \alpha k_t^i\right) s dt + \lambda p k_t^i dt \Pi_m + e^{-rdt} E\left(V_i^n(p+dp) \,|p, \tilde{k}\right) \right\},$$

[9]Note that if $p = 1$, each firm's expected payoff when investing $k = 1$ in R&D is $\int_0^\infty N\lambda e^{-N\lambda t}\left[\int_0^t e^{-r\tau}(1-\alpha)sd\tau + e^{-rt}\left(\frac{\Pi_m}{N} + \frac{\pi_N + \beta s}{r}\right)\right]d\tau$, which is strictly greater than $\frac{s}{r}$ (the payoff from the current market if no firm invests in R&D) if and only if Assumption 1 holds.

where $\tilde{k} = \left(k_t^1, k_t^2, ..., k_t^N\right)$. If firm $i$ spends $k_t^i$ on the R&D investment, it earns a profit of $\left(1 - \alpha k_t^i\right) sdt$ from its established market. With probability $\lambda p k_t^i dt$, firm $i$ may succeed in the R&D project, which brings a prize of $\Pi_m$ as a result of patent protection. Furthermore, the firm's continuation payoff is $E\left(V_i^n\left(p + dp\right)|p, \tilde{k}\right)$, which is a function of its updated belief. Two events may occur after time $dt$ elapses that are embedded in the continuation payoff. With probability $\lambda p \sum_{j=1}^N k_t^j dt$, one firm will succeed in the R&D project, reducing each firm's profit from the established market to $\beta s$, and giving each firm a payoff flow of $\pi_N$ in the post-patent oligopoly market for the new product. Hence the total discounted profit flow from the post patent competition is $\pi_N + \beta s = s + \theta$. On the other hand, with probability $1 - \lambda p \sum_{j=1}^N k_t^j dt$ no firm achieves a breakthrough, and a firm's value function becomes $V_i^n\left(p + dp\right)$. The expected continuation payoff is thus

$$E\left(V_i^n\left(p + dp\right)|p, \tilde{k}\right) = \lambda p \sum_{j=1}^N k_t^j dt \frac{s+\theta}{r} + \left(1 - \lambda p \sum_{j=1}^N k_t^j dt\right) V_i^n\left(p + dp\right).$$

Defining $K_t^{-i} = \sum_{j \neq i} k_t^j$, we can simplify the value function as

$$rV_i^n\left(p\right) = s + K_t^{-i} b\left(p, V_i^n\right) + \max_{k_t^i \in [0,1]} k_t^i \left(b\left(p, V_i^n\right) - c\left(p\right)\right), \tag{3}$$

where

$$b\left(p, V_i^n\right) = \lambda p \left(\frac{s+\theta}{r} - V_i^n\left(p\right) - \left(1-p\right) V_i^{n\prime}\left(p\right)\right), \tag{4}$$

$$c\left(p\right) = \alpha s - \lambda p \Pi_m. \tag{5}$$

Differentiating (3) with respect to $k_t^i$ gives us

$$g\left(p\right) = b\left(p, V_i^n\right) - c\left(p\right).$$

Because the first order condition does not contain $k_t^i$, we have

$$k_t^i = \begin{cases} 1 & \text{if } g\left(p\right) > 0 \\ \kappa \in [0,1] & \text{if } g\left(p\right) = 0 \\ 0 & \text{if } g\left(p\right) < 0 \end{cases}.$$

Notice that $g\left(p\right) > 0$ implies $b\left(p, V_i^n\right) > c\left(p\right)$. The term $b\left(p, V_i^n\right)$ is a firm's expected marginal benefit of investing in the R&D project. With probability $\lambda p$, the success of the R&D project will transform the market into a new oligopoly, leaving each firm a payoff flow of $s+\theta$. The difference between $\frac{s+\theta}{r}$ and $V_i^n(p)$ is the additional payoff achieved through this success while the term $(1-p)V_i^{n\prime}(p)$ is the negative effect on the value function if it does not succeed. On the other hand, $c(p)=\alpha s-\lambda p\Pi_m$ is the net marginal opportunity cost of R&D investment: $\alpha s$ is the R&D expense while $\lambda p\Pi_m$ is the expected one time payoff from this investment. If the marginal benefit $b(p,V_i^n)$ is strictly greater than the net opportunity cost $c(p)$, firm $i$ should invest all available resources in R&D. In contrast, if $b(p,V_i^n)$ is strictly less than $c(p)$, firm $i$ should not invest in R&D.

We can rewrite firm $i$'s optimal investment strategy as follows:

$$k_t^i=\begin{cases} 1 & \text{if } V_i^n>\frac{s}{r}+\frac{c(p)}{r}K_t^{-i} \\ \kappa\in[0,1] & \text{if } V_i^n=\frac{s}{r}+\frac{c(p)}{r}K_t^{-i} \\ 0 & \text{if } V_i^n<\frac{s}{r}+\frac{c(p)}{r}K_t^{-i} \end{cases}.$$

Firm $i$'$s$ optimal strategy is illustrated in Figure 1. If $K_t^{-i}>0$, then on the non-negative quadrant of the $(V_i^n,p)$ space, line $AC$ represents the linear function $V_i^n=\frac{s}{r}+K_t^{-i}\frac{c(p)}{r}$, which is a strictly decreasing function of $p$ and crosses line $BC$ at $p^m=\frac{\alpha s}{\lambda\Pi_m}$. In the area to the left of line $AC$, we must have $V_i^n<\frac{s}{r}+\frac{c(p)}{r}K_t^{-i}$, and thus firm $i$ sets $k_t^i=0$; in the area to the right of line $AC$, firm $i$ invests all available resources in the R&D project. A larger $K_t^{-i}$ will rotate the line $AC$ to the right pivoting at point $C$, implying firm $i$'$s$ non-investing region enlarges with its competitors' investment level. This illustrates that free riding incentive may be present in the noncooperative R&D competition. If $K_t^{-i}=0$, firm $i$ will invest $k_t^i=1$ if and only if $V_i^n>\frac{s}{r}$.

Given firm $i$'$s$ optimal investment strategy, we can derive the symmetric equilibrium of this noncooperative game of R&D competition. We will first derive the explicit solution of the value function. Indeed, there are three cases to consider: $\forall i=1,...,N$, $k_t^i=1$, $k_t^i=0$, or $k_t^i=\kappa\in[0,1]$. In the first case where $\forall i$ $k_t^i=1$ , we have

$$rV_i^n(p)=(1-\alpha)s+\lambda p\Pi_m+N\lambda p\left(\frac{s+\theta}{r}-V_i^n(p)-(1-p)V_i^{n\prime}(p)\right).$$

A general solution to this differential equation is

$$V_H^n(p)=\frac{s(1-\alpha)}{r}+\frac{N\lambda\left(\frac{\Pi_m}{N}+\frac{\alpha s+\theta}{r}\right)}{(r+N\lambda)}p+B_n^1(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}},$$

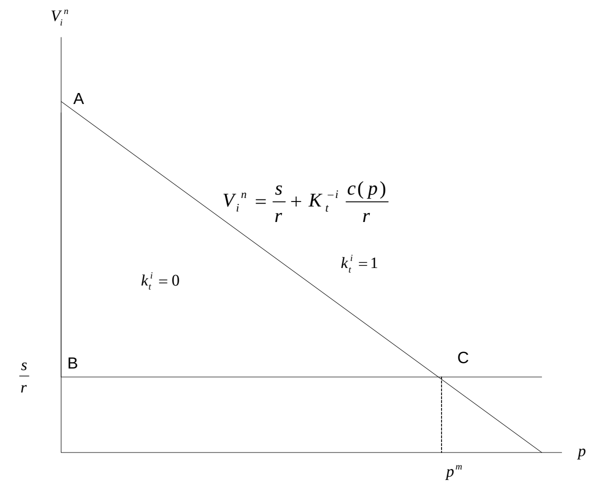


Figure 1: Firm $i$'s investment decision

where $B_n^1$ is a constant to be determined. As we are considering a symmetric equilibrium, we will henceforth omit the subscript $i$. $\frac{s(1-\alpha)}{r}$ is the payoff from its current market, $\frac{N\lambda\left(\frac{\Pi m}{N}+\frac{\alpha s+\theta}{r}\right)}{(r+N\lambda)}p$ is the additional expected discounted payoff from the success of R&D, and $B_n^1 (1-p) \left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}$ is the option value. The option value is related to experimentation and learning. If, in a symmetric equilibrium, all firms stop investing, they terminate the learning process as well, effectively killing the incentive to resume R&D at some point in the future. By investing in R&D in the next instant in time, firms keep open the option to terminate the project at some point in the future. This option has value, which is captured by the term $B_n^1 (1-p) \left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}$.

In the second case, each firm invests zero in R&D, i.e., $k_t^i = 0 \ \forall i$, so the solution to the differential equation is

$$V_L^n (p) = \frac{s}{r}.$$

In the last case with $k_t^i = \kappa \in [0,1] \ \forall i$, we know the first-order condition is $g(p) = 0$. In fact, there may be a range of beliefs under which one firm would find it beneficial to refrain from investing when the other firms invest all resources in R&D, but the initial firm would find it worthwhile to invest in R&D if its competitors refrain from investing. In such a case, there cannot be a symmetric equilibrium in which all firms fully invest or all firms refrain from investing. In a symmetric equilibrium, each firm is just indifferent over whether to invest or not, which implies the firm's value function $V_i^n(p)$ satisfies

$$g(p) = \lambda p \left( \frac{s+\theta}{r} - V_i^n(p) - (1-p) V_i^{n\prime}(p) \right) - (\alpha s - \lambda p \Pi_m) = 0. \tag{6}$$

Solving this differential equation gives us

$$V_M^n(p) = \frac{\lambda \left( \Pi_m + \frac{s+\theta}{r} \right) - \alpha s}{\lambda} - B_n^2 (1-p) + \frac{\alpha s (1-p)}{\lambda} \ln \frac{1-p}{p},$$

where $B_n^2$ is a constant to be determined. The value function $V_M^n(p)$ can be shown to be strictly convex in $p$.

Recall that we have defined $p^m = \frac{\alpha s}{\lambda \Pi_m}$ and $c(p^m) = 0$. Now let us define $p_n^{**} = \frac{r \alpha s}{\lambda (r \Pi_m + \theta)}$, which satisfies $r V_M^n(p_n^{**}) = s + r(N-1) c(p_n^{**})$. The following proposition generalizes Proposition 5.1 in Keller, Rady and Cripps (2005). Their restaurant hunting problem, translated into our context, is the special case with positive externality $(\theta > 0)$.

**Proposition 1** *(i) If $\theta > 0$, then the following R&D investment strategy constitutes a Markov Perfect equilibrium under R&D competition:*

$$k_t^i = \begin{cases} 1 & \text{if } p \geq p_n^{**} \\ \frac{r V_M^n(p) - s}{r(N-1)c(p)} & \text{if } p_n^{**} > p > p_n^* \\ 0 & \text{if } p \leq p_n^* \end{cases}, \quad i = 1, ..., N,$$

*and each firm's value function is*

$$V^n(p) = \begin{cases} V_H^n(p) = & \frac{s(1-\alpha)}{r} + \frac{N\lambda \left( \frac{\Pi_m}{N} + \frac{\alpha s + \theta}{r} \right)}{r + N\lambda} p + B_n^1 (1-p) \left( \frac{1-p}{p} \right)^{\frac{r}{N\lambda}} & \text{if } p \geq p_n^{**} \\ V_M^n(p) = & \frac{\lambda \left( \Pi_m + \frac{s+\theta}{r} \right) - \alpha s}{\lambda} - B_n^2 (1-p) + \frac{\alpha s (1-p)}{\lambda} \ln \frac{1-p}{p} & \text{if } p_n^* < p < p_n^{**} \\ V_L^n(p) = & \frac{s}{r} & \text{if } p \leq p_n^* \end{cases},$$

*where*

$$B_n^2 = \frac{r \Pi_m + \theta}{r} + \frac{\alpha s}{\lambda} \ln \frac{1 - p_n^*}{p_n^*},$$

*and*

$$B_n^1 = \frac{1}{1 - p_n^{**}} \left( \frac{1 - p_n^{**}}{p_n^{**}} \right)^{-\frac{r}{N\lambda}} \left[ V_M^n(p_n^{**}) - \frac{s(1-\alpha)}{r} - \frac{N\lambda \left( \frac{\Pi_m}{N} + \frac{\alpha s + \theta}{r} \right)}{r + N\lambda} p_n^{**} \right].$$

*(ii) If $\theta \leq 0$, the following R&D investment strategy constitutes the unique symmetric Markov Perfect equilibrium under R&D competition:*

$$k_t^i = \begin{cases} 1 & \textit{if } p \geq p_n^* \\ 0 & \textit{if } p < p_n^* \end{cases}, \; i = 1, ..., N,$$

*and each firm's value function is*

$$V^n(p) = \begin{cases} V_i^n(p) = \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{\Pi_m}{N} + \frac{\alpha s + \theta}{r}\right)}{r + N\lambda} p + B_n^3 (1-p) \left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}} & \textit{if } p \geq p_n^* \\ \frac{s}{r} & \textit{if } p < p_n^* \end{cases},$$

*where* $B_n^3 = \frac{1}{1-p_n^*} \left(\frac{1-p_n^*}{p_n^*}\right)^{-\frac{r}{N\lambda}} \frac{\alpha s}{r} \left[1 - \frac{r(r\Pi_m + N(\theta + \alpha s))}{(r+N\lambda)(r\Pi_m + \theta)}\right].$

**Proof.** See Appendix. ■

Figure 2 illustrates the value function. If there are positive externalities ($\theta > 0$), each firm's equilibrium strategy is divided into three parts. If the firms' common prior belief is $p_0 > p_n^{**}$, every firm will invest $k = 1$ in the R&D project. If no firm succeeds and the updated belief falls below $p_n^{**}$, firms will maintain a positive level of investment $k \in (0, 1)$. By doing so, a firm strictly improves its payoff if it succeeds, which results in a prize of $\Pi_m$. On the other hand, if other firms are maintaining enough investment, a firm might want to free ride on its competitors by refraining from investment. Due to an interaction between those two effects, in a symmetric equilibrium when the belief is between $p_n^*$ and $p_n^{**}$, every firm maintains a positive level of investment that is strictly less than 1.

On the other hand, if there are negative externalities ($\theta < 0$), the symmetric equilibrium is a bang-bang solution. If firms' posterior belief is greater than $p_n^*$, each firm will invest $k = 1$ in the R&D project, otherwise each firm will invest $k = 0$. Note that the partial investment does not happen in this case, because the incentive for a firm to free ride on the investments of others is eliminated. In fact, if a non-discovering firm suffers from the discovery of a new product by its competitors, its incentive to invest in R&D is magnified. As shown in the right panel of Figure 2, for $p \in [p_n^*, p_n^{**}]$, each firm's value function is strictly below its payoff if all firms stop investing in R&D and switch back to their current markets. However, if other firms stop investing, a firm has a strong incentive to continue R&D investment because winning the R&D competition will allow it to receive a patent protection. This incentive disappears when the belief reaches $p_n^*$, the belief level that induces the last firm to drop the investment in R&D.

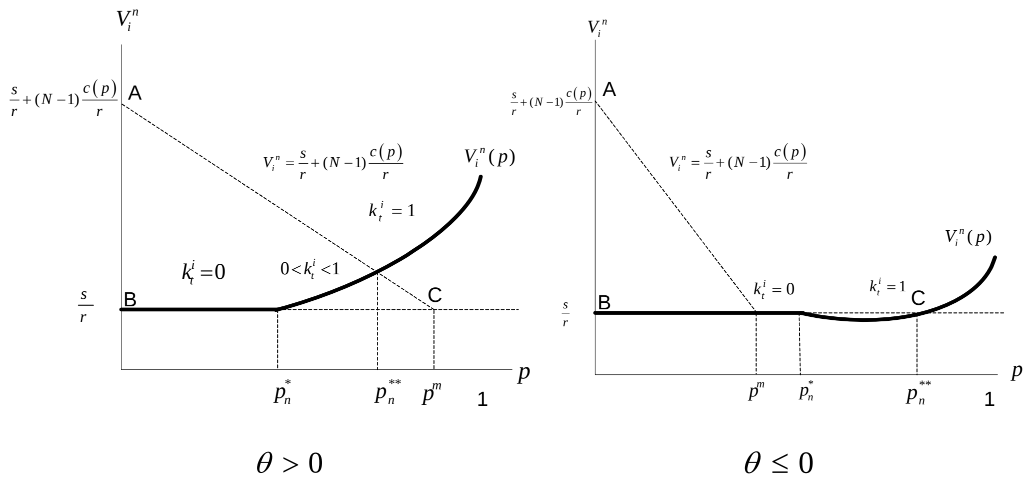


Figure 2: Value Function

The following corollary summarizes some comparative statics results regarding the minimum belief that induces positive R&D investment.

**Corollary 2** *$p_n^*$ is increasing in $s$ and $\alpha$ but decreasing in $\lambda$, $\beta$, $\tilde{\pi}_m$, $\tilde{\pi}_N$, and $T$.*

**Proof.** The proof is immediate after substituting the expression of $\Pi_m$ and $\pi_N$ in $p_n^*$, which results in

$$p_n^* = \frac{r\alpha s}{\lambda\left(\tilde{\pi}_m\left(1-e^{-rT}\right)+e^{-rT}\tilde{\pi}_N-(1-\beta)\,s\right)}.$$

■

To understand these comparative statics results, note first that a higher $s$ implies higher profit from each firm's established market, and thus increases the opportunity cost of R&D investment, causing the firm to drop the R&D project earlier.

To see why, $p_n^*$ is increasing in $\alpha$, the percentage of customers yet to be captured in the existing market, note that the more resources allocated to expanding the existing market, the higher the profit from the established market. As a result, higher $\alpha$ also increases the net marginal opportunity cost of investing in the risky R&D project.

Recall that $\lambda$ measures the likelihood of making a breakthrough in the R&D. The larger $\lambda$ is, the more likely it is that R&D will succeed conditional on being viable, thereby reducing a firm's reluctance in investing in R&D. Thus $p_n^*$ is decreasing in $\lambda$.

The parameter $\beta$ represents the direct impact of the new product on each firm's established market. A higher value of $\beta$ implies a smaller impact and lower $p_n^*$, implying firms are more willing to conduct R&D if the impact is smaller. If $\beta = 1$ the success of the new market has no impact on the old market, in which case the symmetric equilibrium involves possible free riding as shown in the left-hand panel in Figure 1. By contrast, if $\beta = 0$, then the new product market will completely replace the established market, which in turn will induce intensified competition over R&D, resulting in firms continuing investment in R&D even after its value is strictly lower than devoting resources to the established market only.

Recall that $\tilde{\pi}_m$ and $\tilde{\pi}_N$ are the monopoly and oligopoly payoffs from the success in an R&D project while $T$ measures the length of patent protection. A higher prize from winning the R&D project and the oligopoly payoff thereafter and longer patent protection period will induce firms to invest in an R&D project that is less likely to succeed.

Finally, the effect of the discount rate on $p_n^*$ is ambiguous. A higher discount rate has two effects. It increases the loss of profit from the current market; however, it also makes winning the R&D competition more attractive. Therefore the net impact will depend on which of these effects is stronger.

## 3.2 R&D Cooperation Through a Research Consortium

Given our analysis of R&D competition, we can easily derive the optimal decision rule of a research consortium. Firms in a research consortium act as if they were a single agent by pooling their resources and coordinating their research activities to maximize the consortium's total payoff. Let us define $K = \sum_{i=1}^{N} k_t^i$, the total investment in the R&D project by the research consortium at date $t$. If the research consortium succeeds in developing the new product, its expected discounted profit during the patent protection period is $\Pi_m$. Patent protection lasts a period of length $T$, during which the research consortium as a whole earns a monopoly profit flow $\tilde{\pi}_m$ from the new market. At the same time, each member firm also receives $\beta s$ from its old market. After the patent expires, each firm earns an oligopoly profit $\tilde{\pi}_N$ from the new market plus the profit from its current business, $\beta s$. Recall that we have defined $\pi_N = e^{-rT}\tilde{\pi}_N$, so each firm's expected payoff discounted back to the date of success of R&D is $\frac{\Pi_m}{N} + \frac{s+\theta}{r}$.

Because firms are identical, we let $k = \frac{K}{N}$ represent each firm's share of investment for the consortium. The Bellman equation for a representative firm in the research consortium is:

$$V^c(p) = \max_{k\in[0,1]} \left[(1-\alpha k)\, sdt + \lambda pNkdt\frac{\Pi_m}{N} + e^{-rdt}E\left(V^c(p+dp)\,|p, Nk\right)\right]. \tag{7}$$

If the research consortium asks each member firm to invest $k$ in the R&D project, a firm earns a profit of $(1-\alpha k)\, sdt$ from its established market. With probability $\lambda pNkdt$, the research consortium is able to develop the product within a time interval of $dt$, which gives a representative firm a prize of $\frac{\Pi_m}{N}$. Furthermore, the continuation payoff of the representative firm in the consortium is represented by $E\left(V^c(p+dp)\,|p, Nk\right)$, the expected value after the belief updating. Following similar steps as in the noncooperative case, we can simplify the Bellman equation as

$$rV^c(p) = s + \max_{k\in[0,1]} k\left[Nb\left(p, V^c\right) - c(p)\right],$$

where $b(p, V^c)$ and $c(p)$ have been defined in equation (4) and (5) respectively. $b(p, V^c)$can be viewed as the per firm benefit of the R&D investment for a representative firm and $c(p)$ is the net marginal opportunity cost of investing in R&D for a representative firm in the consortium. By investing each unit of resource in the R&D project, the firm sacrifices $\alpha s$ in profit but gains $\frac{\Pi_m}{N}$ if a breakthrough occurs in the next instant, which occurs with probability $\lambda Np$. Observing that the Bellman equation is linear in $k$, we know the research consortium's optimal investment decision is a bang-bang solution.[10] The firm will invest all available resources in R&D if $Nb(p, V^c) \geq c(p)$, otherwise it will not spend anything on R&D. The following proposition summarizes the research consortium's optimal R&D investment decision. Because the research consortium can internalize the externality, its behavior is the same as the cooperative solution presented in Proposition 3.1 of Keller, Rady and Cripps (2005).

**Proposition 3** *The research consortium's optimal R&D investment decision is that each member firm invests $k = 1$ if $p \geq p_c^* = \frac{r\alpha s}{N\lambda\left(\frac{r\Pi_m}{N}+\theta\right)}$, otherwise $k = 0$. The value function*

[10] To prove this result, we begin by assuming the contrary, which results in two thresholds. The first one $p_c^*$ is the belief level that the consortium starts making positive partial investment and the second one $p_c^{**}$ is the belief level that the consortium starts fully investing in R&D. It can be shown that $p_c^* = p_c^{**}$ because the consortium does not have a coordination problem. The proof is omitted but is available from the authors.

*for each member firm is*

$$V^c(p) = \begin{cases} \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{\Pi_m}{N}+\frac{\alpha s+\theta}{r}\right)}{(r+N\lambda)}p + B_c^*(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}} & \text{if } p \geq p_c^* \\ \frac{s}{r} & \text{if } p < p_c^* \end{cases},$$

*where* $B_c^* = \frac{N\lambda\alpha s}{r(r+N\lambda)}\left(\frac{1-p_c^*}{p_c^*}\right)^{-\frac{r}{N\lambda}}$.

**Proof.** See Appendix. ■

The value function is very intuitive. If the research consortium decides not to invest in this R&D project when its belief is $p < p_c^*$, each firm will devote its full resources to marketing its established product, thereby generating a steady stream of profit $s$, making its value $\frac{s}{r}$. However, if $p \geq p_c^*$, firms in the research consortium invest all their available resources in R&D. In this case, its value function consists of three terms. First, $\frac{s(1-\alpha)}{r}$ is the expected payoff from the current market. Second, $\frac{N\lambda\left(\frac{\Pi_m}{N}+\frac{\alpha s+\theta}{r}\right)}{r+N\lambda}p$ is the additional expected payoff for each firm if it succeeds. This additional expected payoff consists of four parts: $\frac{\Pi_m}{N}$ is a firm's share of the prize; $\alpha s$ is the additional profit the firm gets by redirecting back into its established market the resources it had been spending on R&D; $\frac{\pi_N}{r}$ is the present value of oligopoly profits in the post-patent market; and $-\frac{(1-\beta)s}{r}$ is the present value of the loss in profit in the established market due to the emergence of the new product. This additional payoff may arrive at a random time of $\tilde{\tau}$, so the expected discount rate is $E\left(e^{-r\tilde{\tau}}\right) = \int_0^\infty N\lambda e^{-N\lambda\tau}e^{-r\tau}d\tau = \frac{N\lambda}{r+N\lambda}$. The third term, $B_c^*(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}$, is the option value due to the fact that the research consortium retains the option to redirect its investment spending back to the safe arm (i.e., the established market) at any time. The lower the belief $p$, the higher the option value, reflecting that firms are more reluctant to invest in R&D when the belief of viability is low.

# 4 Cooperative versus Noncooperative Research

Recent empirical work by Becker and Dietz (2004) on the impact of R&D cooperation on R&D investment levels suggests that cooperation is associated with higher levels of R&D intensity and a greater likelihood of new product development. However, this association is across a large sample of manufacturing industries and does not address the possibility that the market structure of an industry or the effective level of patent protection in the industry could systematically alter the impact of R&D cooperation on R&D outcomes. In this section, we explore this question and show that both market structure and effective patent length affect how a move from R&D competition to R&D cooperation would influence the level of R&D investment and the probability of new product discovery.

Before we formally analyze the contrast between a research consortium's optimal investment strategy and the equilibrium strategies under R&D competition, let us briefly discuss the key differences between R&D cooperation and R&D competition. First, if firms are conducting research cooperatively, each receives only $\frac{1}{N}$th of the monopoly profit resulting from the patent protection, while if firms compete in R&D, the winner of the R&D competition appropriates the entire monopoly profit. Second, under research cooperation, firms, in effect, pool their investments, which means that the consortium's likelihood of discovering the new product is $N$ times as large as that of any individual firm under R&D competition. Thus, even though an individual firm in a consortium receives only $\frac{1}{N}$th of the prize, its chance of receiving that prize is $N$ times as large as an individual noncooperative firm. This means that the first two effects offset each other. Third, and most significantly, under noncooperative R&D, firms do not internalize the incremental impact of their investment on their competitors. This non-internalized impact may be positive or negative, depending on the post-patent market structure. This factor affects the marginal benefit $b(p, V)$ and as we will see, is the important determinant of the difference in equilibrium behavior under R&D cooperation and competition.

We can formalize the difference between R&D cooperation and R&D competition as follows. Recall that the Bellman equation for a representative firm in an $N$-firm research consortium is

$$rV^c(p) = s + \max_{k \in [0,1]} k \left[ Nb(p, V^c) - c(p) \right],$$

and the Bellman equation for an individual firm engaging in R&D competition is

$$rV^n(p) = s + K_t^{-i} b(p, V^n) + \max_{k^i \in [0,1]} k^i \left[ b(p, V^n) - c(p) \right],$$

where $c(p) = \alpha s - \lambda p \Pi_m$ is the instantaneous net marginal opportunity cost of investing in the risky R&D project. The opportunity cost is the same in both the cooperative and noncooperative cases. However, the difference is in the extent to which firms internalize the impact of their investment decisions on competitors, as reflected in the difference between the $Nb(p, V^c)$ term in the case of a research consortium and the $b(p, V^c)$ term in the case of research competition. This generates a different incentive in the different research structure and in turn leads to different R&D behavior, depending on market structure and the length of the patent protection.

## 4.1 Post-Discovery Market Structure

The post-discovery market structure is reflected in the parameters $\tilde{\pi}_m$ and $\tilde{\pi}_N$, where $\tilde{\pi}_m$ is the monopoly profit flow from the patent protection, and $\tilde{\pi}_N$ is the oligopoly profit after the patent expires. Recall that we have defined $\Pi_m$ to be the total monopoly asset value from the patent protection and $\pi_N$ to be the discounted profit flow from the oligopoly market.

Recall further that $p_c^*$ and $p_n^*$ are the minimum belief levels that induce positive investment in R&D under the cooperative and noncooperative regimes. Given a specific patent policy, the relationship between $p_c^*$ and $p_n^*$ as well as overall investment levels depends solely on the post-patent oligopoly market structure. This is shown in Proposition 4 and 5 below.

**Proposition 4** $p_c^* < p_n^*$ *if and only if* $\theta > 0$. *In other words, if post-patent oligopoly profits are sufficiently high, the research consortium will keep a project alive longer than noncooperative firms. If post-patent oligopoly profits are sufficiently low, the research consortium shuts down the research project sooner than noncooperative firms.*

**Proof.** This follows immediately from

$$p_c^* - p_n^* = -\frac{r\alpha s\,(N-1)\,\theta}{\lambda\,(r\Pi_m + N\theta)\,(r\Pi_m + \theta)}.$$

■

**Proposition 5** *Let* $k^c(p)$ *and* $k^n(p)$ *be individual firm's investment levels as a function of* $p$.

*(i) If* $\theta > 0$, *we have* $k^c(p) \geq k^n(p)$, *and the inequality is strict for* $p \in (p_c^*, p_n^{**})$, *i.e., the research consortium devotes at least as much of the scarce resource to R&D as do noncooperative firms and sometimes strictly more.*

*(ii)If* $\theta < 0$, *we have* $k^n(p) \geq k^c(p)$, *and the inequality is strict for* $p \in (p_n^*, p_c^*)$, *i.e., noncooperative firms devote at least as much of the scarce resource to R&D as does the research consortium and sometimes strictly more.*

**Proof.** If $\theta > 0$, we know $p_n^* > p_c^*$. In addition, if $p < p_c^*$, $k^c(p) = k^n(p) = 0$ but if $p_c^* \leq p < p_n^{**}$, $k^c(p) = 1 > k^n(p)$. For $p > p_n^{**}$, we have $k^c(p) = k^n(p) = 1$. Hence we have $k^c(p) \geq k^n(p)$ if $\theta > 0$, and the equality is strict for $p \in (p_c^*, p_n^{**})$.

If $\theta < 0$, we have $p_n^* < p_c^*$. There are three cases. First, if $p < p_n^*$, $k^c(p) = k^n(p) = 0$; if $p_n^* < p < p_c^*$, $k^n(p) = 1 > k^c(p) = 0$; and if $p_c^* < p$, $k^n(p) = k^c(p) = 1$. ■

As discussed in connection with Proposition 1, if $\theta > 0$, there is positive externality in that the discovery of new knowledge via R&D benefits every firm engaging in R&D, including those who lose the race to discover the new product. As discussed above, this creates a free-rider problem, which gives an individual firm a weaker incentive to compete in R&D because it can still benefit from the success of the other firms. Since free riding is not a problem in the research consortium, firms in a consortium have an incentive to invest in R&D beyond the belief point where noncooperative firms would not invest. In this respect, if the post-patent protection oligopoly market is sufficiently attractive so that $\theta > 0$, the formation of a research consortium would result in stronger incentive for R&D. If, by contrast, the post-patent market structure is sufficiently unattractive so that $\theta < 0$, the free-rider problem does not arise, and in fact, a firm has a strong incentive to avoid losing the R&D race. Thus if post-patent competition is expected to be intense, perhaps because of the inherent conditions of price competition, or because post-patent entry barriers are low, formation of an R&D consortium would actually dull firms' incentive to invest.

Figure 3 illustrates the results in Proposition 4 and 5, with the dotted line representing the investment of a representative firm in a research consortium while the solid line represents that of a noncooperative firm. If $\theta > 0$, the research consortium's optimal stopping belief is $p_c^*$, smaller than a noncooperative firm's threshold $p_n^*$. In this case, each firm invests $k_t = 0$ if $p < p_c^*$ and $k_t = 1$ if $p \geq p_c^*$. By contrast, under R&D competition, a firm will not make positive investment until $p \geq p_n^*$. As shown in the graph, a firm in a research consortium always invests no less than a firm engaged in R&D competition for any posterior belief $p$, and each firm in the research consortium invests more when the posterior belief is in the range $[p_c^*, p_n^{**}]$. The research consortium not only keeps the project alive longer, as shown in Proposition 4, but also invests no less and sometimes strictly more than firms engaged in R&D competition.

On the other hand, if $\theta < 0$, i.e., there are negative externalities, then noncooperative firms will continue to invest in R&D until the posterior belief falls to $p_n^*$. In contrast, a research consortium will stop investing at a higher threshold $p_c^* > p_n^*$. As a result, noncooperative firms invest no less than cooperative firms for $p > p_c^*$ or $p < p_n^*$ but invest

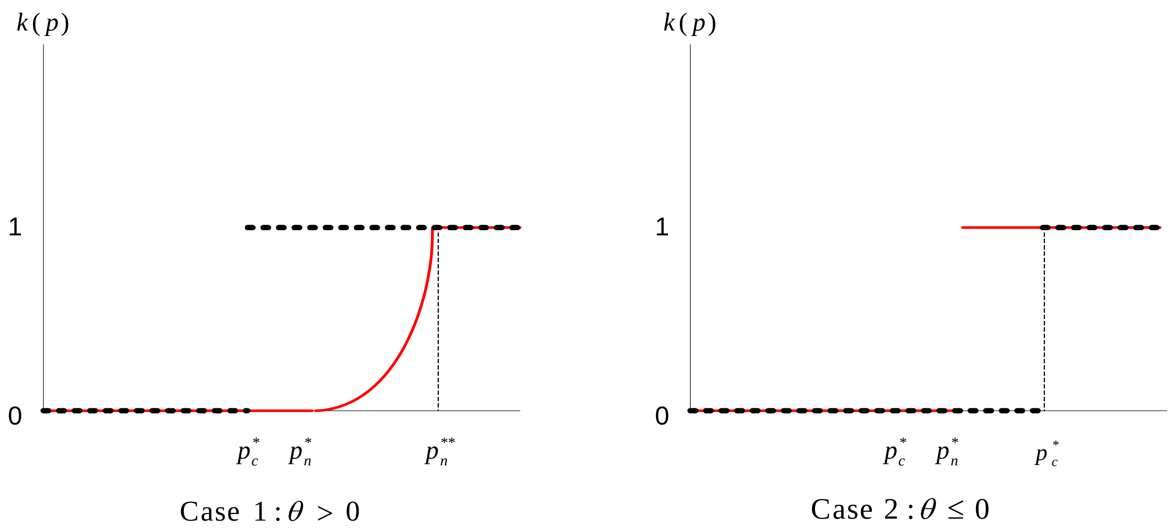


Figure 3: Investment Flow in Cooperative vs. Noncooperative Research

more than cooperative firms for $p \in [p_n^*, p_c^*]$, as shown in the right panel of Figure 3.

Proposition 4 and 5 pertain to the investment strategies followed by firms in equilibrium under each configuration of research. However, we can also compare the outcomes of R&D activity in two ways: *ex ante* R&D investment and the likelihood of discovery of the new product, both of which have received extensive attention in the R&D literature. Unlike previous models, firms are not certain about the viability of the R&D project. However, they have a prior belief $p_0$ that the project is viable. Hence, the unconditional expected investment in R&D is the sum of the expected investment when the project is not viable and the project is viable, which can be written as follows:

$$E[K(p_0)] = p_0 \int_0^\infty K(p(t)) f(t)\, dt + (1-p_0) \int_0^\infty K(p(t))\, dt, \tag{8}$$

where $f(t)$ is the density function over the "arrival" time of success and $K(p(t))$ is the R&D investment level at time $t$ when the belief is $p(t)$. The following Lemma compares the *ex ante* investment in a project destined to fail, which has an *ex ante* probability of $1 - p_0$.

**Lemma 6** *(i) If* $\theta < 0$, *then noncooperative firms invest more than the research consortium for project that is destined to fail.*

(*ii*)*If* $\theta > 0$, *then the research consortium invests more than the noncooperative firms for project that is destined to fail.*

**Proof.** According to Lemma 3.1 of Keller, Rady and Cripps (2005), the fact that $dp = -\lambda K_t p(1-p)\,dt$ implies the total investment the total investment $K(p)$ when firms stop investing when the belief falls below $p$ is

$$K(p) = \int_0^\infty K_t dt = \int_{p_0}^{p} -\frac{1}{\lambda}\frac{1}{p(1-p)}\frac{dp}{dt}dt = \frac{1}{\lambda}\left[\ln\frac{1-p}{p} - \ln\frac{1-p_0}{p_0}\right].$$

As a result,

$$K(p_n^*) - K(p_c^*) = \frac{1}{\lambda}\left[\ln\frac{1-p_n^*}{p_n^*} - \ln\frac{1-p_c^*}{p_c^*}\right] > 0,$$

if and only if $p_n^* < p_c^*$, which is true if and only if $\theta < 0$. ■

Intuitively, when $\theta < 0$, an individual firm sees the new market as a danger to the current market and the only way out is to win the patent protection. The fear of market competition and the desire to win patent protection gives a noncooperative firm greater incentives to invest in R&D. As a result, if the R&D project turns out to be nonviable, noncooperative firms waste more resources than the cooperative firm. On the other hand, if $\theta > 0$, cooperative firms waste more resources than noncooperative firms. In this case, the free-rider problem that arises under R&D competition works to reduce waste.

With this result, we can further compare the *ex ante* expected investment between the cooperative and noncooperative research.

**Proposition 7** (*a*)*If* $\theta < 0$, *total expected investment by noncooperative firms exceeds total expected investment by cooperative firms.*

(*b*)*If* $\theta > 0$, *total expected investment by cooperative firms exceeds total expected investment by noncooperative firms.*

**Proof.** See Appendix. ■

Proposition 7 can be thought of as analogous to d'Aspremont and Jacquemin's (1988) result that cooperative R&D leads to higher investment only if the spillover effect exceeds 0.5. The difference, however, is the key role played by the spillover coming from the product market rather than the spillover from the innovation process as in d'Aspremont and Jacquemin (1988). In their paper, the spillover means a firm's cost can be lowered as a result of other firm's R&D investment, which is often called technology spillover. In our paper, we already assume full technology spillover as a firm updates its belief about project viability when other firms make R&D investment. In contrast, as Proposition 7 shows, higher R&D investment only occurs when there is also significant product market spillover. This is because when the underlying R&D project exhibits "if" and "when" uncertainty, an R&D consortium may terminate the project earlier when there is no significant product market spillover ($\theta < 0$).

The following result compares the *ex ante* likelihood of discovery under each organizational mode:

**Proposition 8** *(i) If $\theta > 0$, the ex ante likelihood of a new product is higher from a research consortium than from firms engaging in R&D competition.*

*(ii) If $\theta < 0$, the ex ante likelihood of a new product is higher from firms engaging in R&D competition than from a research consortium.*

**Proof.** Suppose $\hat{t}$ is the date of success. Let $\hat{t}_n^*$ and $\hat{t}_c^*$ denote the date at which noncooperative and cooperative firms stop investing respectively. That is, $p\left(\hat{t}_n^*\right) = p_n^*$ and $p\left(\hat{t}_c^*\right) = p_c^*$. To compare the *ex ante* likelihood of new product, it suffices to compare the likelihood of no new product conditional on its being viable, which is defined as $\Pr\left(T > \hat{t}_j^*\right)$, $j = n, c$. In particular, according to Bayes rule, we have[11]

$$\begin{aligned} \ln \Pr\left(T > \hat{t}_j^*\right) &= \int_0^{\hat{t}_j^*} -\lambda \sum_{i=1}^{N} k(t)\, dt \qquad (9) \\ &= \int_{p_0}^{p_j^*} -\lambda \sum_{i=1}^{N} k(t) \frac{dt}{-\lambda \sum_{i=1}^{N} k(t)\, p(1-p)\, dt} dp \\ &= \ln \frac{p_j^*}{1-p_j^*} - \ln \frac{p_0}{1-p_0}. \end{aligned}$$

We know that $h(x) = \ln \frac{x}{1-x}$ is increasing in $x$ for $x \in (0,1)$. Hence, if $\theta > 0$, we must

[11] To understand Equation (9), suppose the current date is $\tau$ and each firm invests $k(\tau)$, then the probability of no success within $[\tau, \tau + \Delta t)$ conditional on no success before $\tau$ is $\Pr(T > \tau + \Delta t | T > \tau) = e^{-\lambda \sum_{i=1}^{N} k(\tau)\Delta t}$. Furthermore, we have

$$\begin{aligned} \Pr(T > \tau) &= \Pr(T > \tau | T > \tau - \Delta t) \Pr(T > \tau - \Delta t | T > \tau - 2\Delta t) \cdot \ldots \cdot \Pr(T > \Delta t | T > 0) \cdot \Pr(T > 0) \\ &= \Pi_{s=0}^{\tau} e^{-\lambda \sum_{i=1}^{N} k(s)\Delta t}, \end{aligned}$$

which gives the probability of no success until time $\tau$. Hence, taking the logarithm of both sides and let $\Delta t \to 0$, we have Equation (9).

have $p_c^* < p_n^*$, which then implies $\Pr\left(T > \hat{t}_c^*\right) < \Pr\left(T > t_n^*\right)$. This then implies that the likelihood of success is higher for the cooperative R&D. On the other hand, if $\theta < 0$, we must have $p_c^* > p_n^*$, which then implies the opposite result. ■

Proposition 8 implies that when post-patent competition is sufficiently intense, a research consortium results in a lower likelihood of new product discovery than is the case under R&D competition. The winner-take-all nature of R&D competition that arises when $\theta < 0$ tends to work in favor of new product discovery. When $\theta > 0$, the likelihood of new product development is higher when firms form a research consortium. This is due to two effects: first, a research consortium continues to invest for a longer time than noncooperative firms; second, a research consortium invests more intensively than noncooperative firms, because noncooperative firms will reduce their investment level below the maximum level once their common posterior belief falls below $p_n^{**}$ while a research consortium will continue to invest at the maximum level until the posterior belief falls below $p_c^*$, which is less than $p_n^{**}$.

## 4.2 Patent Policy

In this section, we consider the effect of patent length $T$ on cooperative and noncooperative R&D investments. The effective patent length $T$ could reflect the impact of public policy or in industries such as aerospace where new products may not be patentable, the period length $T$ could represent the time it takes a competitor to imitate a new product breakthrough. The longer the patent protection period, the greater the prize for the winner of the R&D competition, although the distribution of this prize differs across the two research setups. The research consortium shares the monopoly profit resulting from the patent while a noncooperative winning firm retains the entire monopoly payoff for itself.

Until now, we had a fixed patent length $T$, which allows us to define $\pi_N = e^{-rT}\tilde{\pi}_N$, where $\tilde{\pi}_N$ is the oligopoly flow profit if firms succeeded in discovering the new product. Previous discussion indicates that the relationship between $\pi_N$ and $(1-\beta)\,s$ is critical. Recall that $\theta = \pi_N - (1-\beta)\,s > (<)\,0$ implies the R&D project exhibits positive (negative) externalities. To study the effect of patent policy represented by patent length $T$, we fix $\tilde{\pi}_N$, implying that $\pi_N$ (and thus $\theta$) varies with patent length $T$.

**Proposition 9** *(i) If $\tilde{\pi}_N > (1-\beta)\,s$, then $p_n^* > p_c^*$ if and only if $T < \frac{1}{r}\ln\frac{\tilde{\pi}_N}{(1-\beta)s}$, i.e., for the case of sufficiently large post-patent profit, there is a wider zone of investment under* *cooperative research than under noncooperative research if and only if the patent length is sufficiently small.*

*(ii) If* $\tilde{\pi}_N < (1-\beta)\, s$ *and* $T > \frac{1}{r} \ln \frac{\tilde{\pi}_m - N\tilde{\pi}_N}{\tilde{\pi}_m - N(1-\beta)s}$, *then* $p_n^* < p_c^*$, *i.e. for the case of sufficiently small post-patent profits, there is a wider zone of investment under noncooperative research than under cooperative research if and only if the patent length is sufficiently large.*

**Proof.** See Appendix. ■

Even if the resulting oligopoly market makes every firm better off $(\tilde{\pi}_N > (1-\beta)\, s)$, it is not clear whether a research consortium is the best configuration in pursuing difficult R&D projects. Indeed, if the patent period $T$ is long enough, a firm engaged in R&D competition has a stronger incentive to invest because its dropout threshold $p_n^*$ is lower. Furthermore, if there are negative externalities $(\tilde{\pi}_N < (1-\beta)\, s)$, an individual firm has a stronger incentive to invest unless the patent period $T$ is significantly higher than required by the previous case.

Patent policy may affect firms' incentive in choosing research configurations. Even if the success of an R&D project improves each firm's profit, i.e., $\tilde{\pi}_N > (1-\beta)\, s$, varying patent protection period $T$ may result in a different outcome in the different research configurations, as shown in the following corollary.

**Corollary 10** *Assume* $\tilde{\pi}_N > (1-\beta)\, s$. *As* $T$ *increases, then* $\pi_N$ *decreases. If* $\pi_N$ *decreases from the point at which* $\pi_N > (1-\beta)\, s$ *to the point at which* $\pi_N < (1-\beta)\, s$, *then increased patent protection will cause a switch from a regime in which there is greater expected R&D investment from cooperative than noncooperative R&D to a regime in which there is greater expected R&D investment flow from noncooperative than from cooperative R&D.*

### 4.3 Empirical Implications

The principal empirical implication of the results in Section 4.1 and 4.2 is that tests of the impact of R&D cooperation on R&D investment intensity and outcomes should take into account market structure conditions and the effective length of patent protection. In industries in which post-patent competition is likely to be weak and/or the patent length is short, a research consortium would be expected to increase the intensity of R&D efforts relative to R&D competition, all else being equal. However, if post-patent competition is intense and/or the patent length is long, a research consortium would be associated with a lower level of R&D intensity. Of course, the incentives to form research consortia (a subject beyond the scope of this paper) are endogenous. To the extent that these incentives depend systematically on market structure conditions, as we would expect they might, this "selection effect" would need to be taken into account as well.

## 5 Value and Welfare Comparisons

In this section, we will study the value and welfare implications of research cooperation. We begin with a comparison of a firm's value function under R&D cooperation and competition.

The following proposition characterizes the relationship of value functions in two cases depending on whether the post-patent competition improves firms' profitability. In both cases, firms are always better off if they form a research consortium, although for different reasons. If $\theta > 0$, the free-rider incentive dominates for a low enough belief level, which leads to a lower investment level than a research consortium, lengthening the expected time to discover the new product. Firms are indeed worse off due to the free riding behavior. However, if $\theta < 0$, competition incentive causes firms to overinvest even if their value function falls below its safe arm alternative, because each firm hopes to win the R&D competition so that they can enjoy a certain period of patent protection. Firms are worse off because of this overinvestment behavior.

**Proposition 11** *(a) If $\theta < 0$, then $V^c(p) \geq V^n(p)$ and the inequality is strict if $p > p_n^*$.*

*(b) If $\theta > 0$, then $V^c(p) \geq V^n(p)$ and the inequality is strict if $p > p_c^*$.*

**Proof.** See Appendix. ■

Let us now consider a social planner's problem. Suppose the social planner does not know whether the project is viable or not but it can form a research consortium and decide its investment level. Because the social welfare includes both consumer and producer surplus, we need to introduce notation for consumer surplus in the established and new product market. Let consumer surplus per unit sold in each firm's established market be denoted by $cs_o$. For the new product, let $cs_m$ and $cs_N$ denote the consumer surplus in the monopoly and oligopoly markets, respectively. The social welfare flow from a firm's established market is thus $w_o = s + cs_o$. Similarly, the social welfare flows from the new product are $\tilde{w}_m = \tilde{\pi}_m + cs_m$ and $\tilde{w}_N = \tilde{\pi}_N + cs_N$, for the monopoly and oligopoly market, respectively. Furthermore, let us define $W_m = \int_0^T \tilde{w}_m e^{-rt} dt$ and $w_N = e^{-rT}\tilde{w}_N$ similar to our definition of $\Pi_m$ and $\pi_N$. Accordingly, we also have $CS_m = \int_0^T cs_m e^{-rt} dt$ and

$CS_N = e^{-rT} cs_N$. A social planner's goal is to maximize total social welfare, so its value function is

$$SW^c(p) = \max_{K \in [0,N]} \left[ \left(1 - \frac{K}{N}\alpha\right) w_o dt + \lambda p K dt \frac{W_m}{N} + e^{-rdt} E\left(SW^c(p + dp|p, K)\right) \right].$$

If the social planner allocates $K$ units of resource to the R&D project, the total surplus flow from the current market for each member firm in the consortium is $\left(1 - \frac{K}{N}\alpha\right) w_o$. With probability $\lambda p K dt$, the social planner will succeed in developing the new product, and receive a welfare boost $W_m$ during the patent protection period. In addition, its continuation value is represented by $SW^c(p + dp|p, K)$. With some routine algebra, we can rewrite the Bellman equation as

$$rSW^c(p) = \max_{K \in [0,N]} \left[ \left(1 - \frac{K}{N}\alpha\right) w_o + \lambda p \frac{K}{N} W_m + \lambda p K \left[ \frac{w_N + \beta w_o}{r} - SW^c(p) - (1-p) SW^{c\prime}(p) \right] \right].$$

Following a similar approach as in Section 3.2, we can solve the social planner's optimal investment threshold in terms of a belief cutoff:

$$p^*_{sc} = \frac{r\alpha w_o}{\lambda\left(rW_m + Nw_N - N(1-\beta) w_o\right)}.$$

In other words, the social planner will allocate all resource to R&D investment as long as its belief of viability is greater than $p^*_{sc}$.

**Proposition 12** *$p^*_{sc} < p^*_c$ if and only if $\frac{cs_o}{s} < \frac{rCS_m + N\,CS_N}{r\Pi_m + N\pi_N}$. In other words, the social planner will drop the project later than a research consortium if the flow ratio between consumer surplus with respect to profit in the new product market is greater than the established market.*

**Proof.** Note that

$$\begin{aligned} p^*_{sc} - p^*_c &= \frac{r\alpha w_o}{\lambda\left(rW_m + Nw_N - N(1-\beta) w_o\right)} - \frac{r\alpha s}{\lambda\left(r\Pi_m + N\pi_N - N(1-\beta) s\right)} \\ &= \frac{r\alpha\left(cs\left(r\Pi_m + N\pi_N\right) - s\left(rCS_m + NCS_N\right)\right)}{\lambda\left(rW_m + Nw_N - N(1-\beta) w_o\right)\left(r\Pi_m + N\pi_N - N(1-\beta) s\right)}. \end{aligned}$$

Hence, we have

$$p^*_{sc} - p^*_c > 0 \Longleftrightarrow \frac{cs_o}{s} > \frac{rCS_m + NCS_N}{r\Pi_m + N\pi_N}.$$

$\blacksquare$

Note that $\frac{cs_o}{s}$ and $\frac{rCS_m+NCS_N}{r\Pi_m+N\pi_N}$ are consumer surplus to profit ratios. If this ratio is higher for the new market than it is for a firm's established market, (i.e., $\frac{cs_o}{s} < \frac{rCS_m+NCS_N}{r\Pi_m+N\pi_N}$), then $p^*_{sc} < p^*_c$, which implies that the social planner will cease investment in the project later than the research consortium would. This implies that the research consortium does not maximize social welfare. Nevertheless, if $\theta > 0$, then $p^*_{sc} < p^*_c < p^*_n$, so the research consortium generates a larger expected social welfare than noncooperative research. On the other hand, if $\theta < 0$, we must have $p^*_c > p^*_n$, and thus in this case, noncooperative research could result in higher social welfare than a research consortium. The next corollary addresses this possibility:

**Proposition 13** *Define* $D_{sc} = rW_m + Nw_N - N(1-\beta)w_o$ *and* $D_n = r\Pi_m + \pi_N - (1-\beta)s$. *Further, suppose Assumption 1 holds and* $\frac{cs_o}{s} < \frac{rCS_m+NCS_N}{r\Pi_m+N\pi_N}$.

*(i) If* $\theta > 0$, $p^*_{sc} < p^*_c < p^*_n$, *i.e., the research consortium terminates the project closer to the socially optimal stopping threshold than noncooperative research, and thus formation of a research consortium improves social welfare relative to noncooperative R&D.*

*(ii) If* $\theta < 0$ *and* $\frac{cs_o}{s} < \min\left\{\frac{D_{sc}-D_n}{D_n}, \frac{rCS_m+NCS_N}{r\Pi_m+N\pi_N}\right\}$, *then* $p^*_{sc} < p^*_n < p^*_c$, *i.e., noncooperative research firm's optimal stopping threshold is closer to the socially optimal threshold than the research consortium's, and thus formation of a research consortium would reduce social welfare.*

Recall that when $\theta > 0$, the free-rider problem is present under R&D competition, and as a result, an R&D consortium will result in stronger investment incentives. Proposition 13 shows that in this case, the formation of an R&D consortium will also improve social welfare (provided the assumptions of the proposition hold). However, as the proposition shows, there are conditions under which a research consortium may actually reduce welfare. These align with the case in which $\theta < 0$, i.e., when R&D competition imposes a net harm on non-discovering firms and R&D competition resembles a winner-take-all race.

Figure 4 illustrates the optimal stopping thresholds chosen by social planner $(p^*_{sc})$, research consortium $(p^*_c)$, and a noncooperative firm $(p^*_n)$ as a function of $\beta$. Recall that $\beta$ measures the impact of a new product market on the established market, where smaller $\beta$ implies larger negative impact on firm's profitability. $p^*_{sc}(\beta)$, $p^*_c(\beta)$, and $p^*_n(\beta)$ are represented by lines $CD$, $AB$, and $EF$ respectively.[12] Note that for $\beta > 0.24$, we have $p^*_n >$

[12] We assume the following parameter values: $r = 0.1$, $\alpha = 0.3$, $s = 1$, $N = 3$, $\tilde{\pi}_m = 5$, $\tilde{\pi}_N = 1.25$, $T = 5$, $\lambda = 0.03$, $cs_o = 0.8$, $cs_m = 1$, $cs_N = 2$.

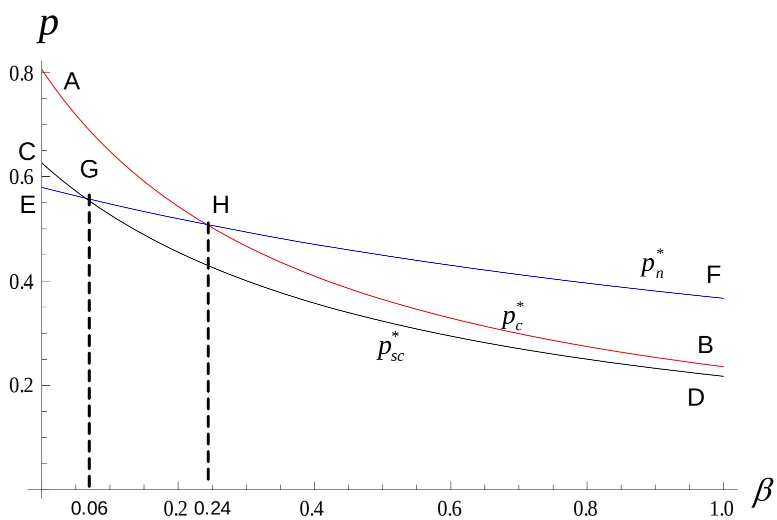


Figure 4: Welfare Comparison

$p_c^* > p_{sc}^*$, so a research consortium represents a welfare improvement over noncooperative R&D. However, for $\beta \in (0.06, 0.24)$, we have $p_{sc}^* < p_n^* < p_c^*$, which implies that formation of a research consortium reduces social welfare relative to noncooperative R&D. Finally, for $\beta < 0.06$, we have $p_c^* > p_{sc}^* > p_n^*$, and the welfare effect is ambiguous.

# 6 Conclusion

This paper studies the trade-offs between R&D cooperation and R&D competition when firms face "if" and "when" uncertainty about the R&D project. Using a continuous time, two-armed bandit model, we derive the optimal decision rule of a research consortium and a symmetric Markov perfect equilibrium when firms are conducting research noncooperatively. Post-patent market structure plays an important role in determining the differences between R&D competition and R&D cooperation. If post-patent oligopoly profits are sufficiently large and/or the length of the patent period is sufficiently short, then R&D competition will give rise to a free-rider problem. In this case, for any given set of beliefs about the viability of the R&D project, firms invest at least as much, and sometimes more, under R&D cooperation than they do under R&D competition. Furthermore, overall *ex ante* undiscounted investment is higher under R&D cooperation, and R&D cooperation results in a higher level of social welfare. These results are analogous to the results in the traditional literature on R&D cooperation. However, free-rider incentives do not always arise. If post-patent competition is sufficiently intense and/or the length of the patent period is long, R&D competition resembles a winner-take- all race, and R&D cooperation will result in lower investment and a lower likelihood of discovering the new product. Furthermore, R&D cooperation does not necessarily improve social welfare. These results suggest that empirical research on the impact of R&D cooperation should take into account the market structure that firms will face once new products are discovered.

A number of extensions of this research are possible. We have not considered incentives for endogenous formation of research consortia. To do so, we would need to explore the equilibrium that arises in mixed cases in which some, but not all, firms participate in a consortium. We have not considered the possibility of partial overlap of the R&D process. We essentially assume that firms are fully aware of the investment levels and the R&D outcomes of their competitors. To extend the model to include partial information spillover would require that we model the asymmetric effect of belief updating as a result of a firm's own and its competitors investment levels. This would involve multiple state variables.

# 7 Appendix

## 7.1 Proof of Proposition 1

**Proof.** Part $(i)$ corresponds to the case with $\theta > 0$ that follows Keller, Rady and Cripps (2005) and part $(ii)$ corresponds to the opposite case. For completeness, we include proofs of both parts.

$(i)$ Define the line $h(p) = \frac{s}{r} + K_t^{-i}\frac{c(p)}{r}$. For all $K_t^{-i} \in (0, N-1]$, we know $h(p)$ is downward sloping and crosses the horizontal line $v = \frac{s}{r}$ at $p^m = \frac{\alpha s}{\lambda \Pi_m}$. If $K_t^{-i} = 0$, then it overlaps the horizontal line $v = \frac{s}{r}$. We know firm $i$ will invest $k_t^i = 0$ in the region that is below line $h(p)$ and invest $k_t^i = 1$ in the region that is to the right of line $h(p)$.

Consider the value function $V_M^n(p)$ that solves the differential equation $(6)$. The value matching condition implies that $V_M^n(p)$ connects $v = \frac{s}{r}$ at point $p_n^*$ :

$$V_M^n(p_n^*) = \frac{s}{r}. \tag{10}$$

The strict convexity of $V_M^n(p)$ implies that for it to be a part of an equilibrium value function, it has to reach level $v = \frac{s}{r}$ from the right of $p_n^*$ :

$$V_M^{n\prime}\left(p_n^*\right) = 0. \tag{11}$$

Solving those two equations gives

$$\begin{aligned} p_n^* &= \frac{r\alpha s}{\lambda\left(r\Pi_m + \theta\right)}, \\ B_n^2 &= \frac{\left(r\Pi_m + \theta\right)}{r} + \frac{\alpha s}{\lambda}\ln\frac{1 - p_n^*}{p_n^*}. \end{aligned}$$

Furthermore for all $p > p_n^*$, we must have $V_M^{n\prime}(p) > 0$ as implied by the strict convexity. Define

$$k^n(p) = \frac{V_M^n(p) - \frac{s}{r}}{(N-1)\,c(p)} = \frac{rV_M^n(p) - s}{r\,(N-1)\,c(p)}.$$

$k^n(p)$ gives the investment level of its competitor at belief $p$ that makes firm $i$ indifferent between investing and not investing. Note that $V_M^n(p)$ is increasing and $c(p)$ goes to 0 as $p \to p^m = \frac{\alpha s}{\lambda\Pi_m}$, which implies $k^n(p)$ is increasing and goes to $\infty$ as $p \to p^m$. Because $\pi_N > (1-\beta)\,s$, we must have $p_n^* < p^m$. Consequently, there exists $p_n^* \leq p_n^{**} < p^m = \frac{\alpha s}{\lambda\Pi_m}$ such that $k^n(p_n^{**}) = 1$, which is the point on the line $h(p)$ with $V_H^n(p_n^{**}) = V_M^n(p_n^{**})$. Hence, we can solve $B_n^1$, which is $B_n^1 = \frac{1}{p_n^{**}}\left(\frac{1-p_n^{**}}{p_n^{**}}\right)^{-\frac{r+N\lambda}{N\lambda}}\left[V_M^n(p_n^{**}) - \frac{s(1-\alpha)}{r} - \frac{N\lambda\left(\frac{r\Pi_m}{N} + \alpha s + \theta\right)}{r(r+N\lambda)}p_n^{**}\right]$.

$(ii)$ Consider the case with $\theta \leq 0$. We first show that it is a symmetric equilibrium where all firms stop investing in R&D when their common belief $p$ is below $p_n^*$, then we show that each firm will invest all of their resources in R&D until their belief reaches $p_n^*$.

Suppose $p \leq p_n^*$. In this case, if all other firms have stopped investing in R&D, firm $i's$ value function must satisfy

$$rV_i^n(p) = s + \lambda p\left(\frac{s+\theta}{r} - V_i^n(p) - (1-p)\,V_i^{n\prime}(p)\right) - \left(\alpha s - \lambda p\Pi_m\right),$$

if it continues investing in R&D, and

$$rV_i^n(p) = s$$

if it stops investing in R&D. Using value matching and smooth pasting condition, we have

$$p_n^* = \frac{r\alpha s}{\lambda\left(r\Pi_m + \theta\right)},$$

which implies stopping R&D investment is also optimal for firm 1 if others have quit R&D investment. This implies that if $p \leq p_n^*$, it is an equilibrium for all firms to stop investing in R&D.

Now consider the case with $p \geq p_n^*$. If all firms invest $k = 1$ in the R&D project, then firm $i's$ value function must satisfy

$$rV_i^n(p) = s + N\lambda p\left(\frac{s+\theta}{r} - V_i^n(p) - (1-p)V_i^{n\prime}(p)\right) - (\alpha s - \lambda p \Pi_m), \tag{12}$$

which has a general solution

$$V_i^n(p) = \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{r\Pi_m}{N} + \alpha s + \theta\right)}{r(r+N\lambda)}p + B_n^3(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}.$$

To determine the constant $B_n^3$, we impose the value matching condition $V_i^n(p_n^*) = \frac{s}{r}$, which implies

$$\begin{aligned} B_n^3 &= \frac{1}{1-p_n^*}\left(\frac{1-p_n^*}{p_n^*}\right)^{-\frac{r}{N\lambda}} \frac{\alpha s}{r}\left[1 - \frac{r(r\Pi_m + N\theta + N\alpha s)}{(r+N\lambda)(r\Pi_m + \theta)}\right] \\ &> \frac{1}{1-p_n^*}\left(\frac{1-p_n^*}{p_n^*}\right)^{-\frac{r}{N\lambda}} \frac{\alpha s}{r}\left[-\frac{(N-1)\theta}{(r\Pi_m+\theta)}\right] \\ &\geq 0. \end{aligned}$$

The first inequality follows from Assumption 1 and the second inequality follows from $\theta \leq 0$. This implies $V_i^n(p)$ is strictly convex. Substituting $V_i^n(p_n^*) = \frac{s}{r}$ into $(12)$, we have

$$V_i^{n\prime}(p_n^*) = \frac{\alpha s}{N\lambda p_n^*(1-p_n^*)}\left[\frac{(N-1)\theta}{r\Pi_m + \theta}\right] \leq 0.$$

This implies that $V_i^n(p)$ is a convex function and is decreasing at $p_n^*$. It remains to show that investing $k = 1$ is indeed optimal for firm $i$ if other firms are also investing $k = 1$ in the R&D. We know that if $V_i^n > \frac{s}{r} + \frac{c(p)}{r}K_t^{-i}$, firm $i$ invests $k = 1$. Given the strict convexity of $V_i^n$, it suffices to show the case with $\theta < 0$, otherwise $\theta = 0$ implies $p_n^* = p^m$ and $V_i^n(p_n^*) = \frac{s}{r} > \frac{s}{r} + \frac{c(p)}{r}K_t^{-i}$ since $c(p) = \alpha s - \lambda p \Pi_m \leq 0$ for all $p \geq p^m$.

Suppose $\theta < 0$, then we need to show

$$V_i^n(p) \geq \frac{s}{r} + \frac{(N-1)(\alpha s - \lambda p \Pi_m)}{r} = h(p) \text{ for all } p \geq p_n^*.$$

To show that the above inequality holds, we show that $V_i^n(p)$ is above another curve which is increasing and lies to the right of the line $h(p)$. Define $M(p,C) = \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{r\Pi_m}{N}+\alpha s+\theta\right)}{r(r+N\lambda)} p + C(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}$. Note that $V_i^n(p) = M\left(p, B_n^3\right)$. In addition, $M(1,C) = V_i^n(1) = \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{r\Pi_m}{N}+\alpha s+\theta\right)}{r(r+N\lambda)}$ for all $C > 0$. Graphically, $M(p,C)$ represents a family of convex functions that only intersect at point $(1, V_i^n(1))$. Note that the family $M(p,C)$ can be ordered using its minimum. Let $(p(\kappa), C(\kappa))$ satisfy

$$\begin{aligned} M(p(\kappa), C(\kappa)) &= \kappa \\ \frac{\partial M(p(\kappa), C(\kappa))}{\partial p} &= 0. \end{aligned}$$

That is, $M(p,C)$ is ranked by its minimum $\kappa$. Then we have

$$p(\kappa) = \frac{r\alpha s + r(r\kappa - s)}{\lambda\left(r\Pi_m + N(s+\theta-r\kappa)\right)},$$

which in turn gives $\kappa$ as a function of $p$:

$$\kappa(p) = \frac{\lambda p\left(\Pi_m + \frac{N(s+\theta)}{r}\right) + s(1-\alpha)}{(r+\lambda p N)}.$$

First of all, because $p(\kappa)$ is increasing in $\kappa$ and $p\left(\frac{s}{r}\right) = p_c^* > p_n^*$, we must have $\kappa(p_n^*) < \frac{s}{r}$. Furthermore, we know

$$k(p_n^*) - h(p_n^*) = -\frac{(n-1)\, n\alpha^2 s^2\theta}{r\left(r\Pi_m+\theta\right)\left(r\Pi_m+\theta+N\alpha s\right)} > 0.$$

Let $\underline{V}(p,C) = M(p,C)$ be the curve that satisfies $\underline{V}\left(p_n^*, C(\kappa(p_n^*))\right) = \kappa(p_n^*)$. The fact that $V_i^n(p_n^*) = \frac{s}{r}$ implies $V_i^n(p)$ lies above $\underline{V}(p,c)$, which in turn implies $V_i^n(p) > h(p)$ for all $p \geq p_n^*$ because $h(p)$ is decreasing.

Now we show that this symmetric equilibrium is unique. Note that a firm will not invest in R&D for $p = 0$. Continuity implies there is an interval of $p$ containing 0 such that a firm's optimal strategy is not to invest in R&D. Similarly, a firm will invest $k = 1$ for $p$ that is close to or equal to 1. The uniqueness then follows because the interval of $p$ such that $k(p) = 0$ for each firm is $[0, p_n^*]$ as shown above. ■

## 7.2 Proof of Proposition 3

**Proof.** If $k = 1$, we have the following equation as the value function

$$rV^c(p) = s + N\left[\lambda p\left(\frac{s+\theta}{r} - V^c(p) - (1-p)V^{c\prime}(p)\right) - \frac{\alpha s - \lambda \Pi_m p}{N}\right].$$

The solution for this first order differential equation is

$$V_H^c(p) = \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{\Pi_m}{N} + \frac{\alpha s+\theta}{r}\right)}{r+N\lambda}p + B_c^*(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}.$$

If $k = 0$, the solution to the differential equation is

$$V_L^c(p) = \frac{s}{r}.$$

The cutoff belief $p_c^*$ and the constant $B_c^*$ are the solutions to the following system of equations

$$V_H^c(p_c^*) = V_L^c(p_c^*). \tag{13}$$
$$V_H^{c\prime}(p_c^*) = V_L^{c\prime}(p_c^*). \tag{14}$$

Equation (13) is the value matching condition while Equation (14) is the smooth pasting condition. Substituting the expression for $V_H^c(p)$ and $V_L^c(p)$, we have

$$p_c^* = \frac{r\alpha s}{N\lambda\left(\frac{r\Pi_m}{N} + \theta\right)}$$
$$B_c^* = \frac{N\lambda\alpha s}{r(r+N\lambda)}\left(\frac{1-p_c^*}{p_c^*}\right)^{-\frac{r}{N\lambda}}.$$

■

## 7.3 Proof of Proposition 7

**Proof.** Lemma 6 compares the expected investment when the project is destined to fail. The proof is complete if we can show the same result in terms of expected investment when the project is viable. Conditional on the project being viable, we let $K_v(p)$ denote the total expected investment when the belief is $p$. We can write $K_v(p)$ recursively

$$K_v(p) = \sum_{i=1}^{n} k_i(p)\,dt + \left(1 - \lambda \sum_{i=1}^{n} k_i(p)\,dt\right) K_v(p + dp),$$

which can be simplified to

$$0 = 1 - \lambda K_v(p) - \lambda p(1-p) K_v'(p),$$

for any belief $p$ such that $k_i(p) > 0$. A general solution to this differential equation is

$$K_v(p) = \frac{1}{p}\left(\frac{1}{\lambda} - D(1-p)\right),$$

where $D$ is a constant to be determined. Note that firms will stop investing when the belief falls below $p_j^*$, $j = c, n$. Hence, we must have

$$K_v\left(p_j^*\right) = 0,\ \ j = c, n.$$

As a result, we have

$$\begin{aligned} D_c^* &= \frac{1}{\lambda(1-p_c^*)} \\ D_n^* &= \frac{1}{\lambda(1-p_n^*)} \end{aligned}$$

and $D_c^* > D_n^*$ if and only if $p_c^* > p_n^*$. Finally, to compare the *ex ante* investment, we have

$$K_v^c(p_0) - K_v^n(p_0) = \frac{1-p_0}{p_0}\left(D_n^* - D_c^*\right).$$

Because if $\theta > (<)0$, then $p_c^* < (>)p_n^*$, and thus $K_v^c(p_0) > K_v^n(p_n)$ if and only if $\theta > 0$. This result combined with Lemma 6 completes the proof. ■

## 7.4 Proof of Proposition 9

**Proof.** Recall that $\Pi_m(T) = \int_0^T \tilde{\pi}_m e^{-rt}dt = \frac{\tilde{\pi}_m}{r}\left(1 - e^{-rT}\right)$ and $\pi_N(T) = e^{-rT}\tilde{\pi}_N$. Hence

$$p_n^* - p_c^* = \frac{r\alpha s(N-1)\left(e^{-rT}\tilde{\pi}_N - (1-\beta)s\right)}{\lambda\left(\tilde{\pi}_m(1-e^{-rT}) + Ne^{-rT}\tilde{\pi}_N - N(1-\beta)s\right)\left(\tilde{\pi}_m(1-e^{-rT}) + e^{-rT}\tilde{\pi}_N - (1-\beta)s\right)}$$

If $\tilde{\pi}_N > (1-\beta)s$, then we have

$$\tilde{\pi}_m\left(1-e^{-rT}\right)+e^{-rT}\tilde{\pi}_N-(1-\beta)s=\tilde{\pi}_m-(1-\beta)s-(\tilde{\pi}_m-\tilde{\pi}_N)e^{-rT}>\tilde{\pi}_N-(1-\beta)s>0,$$

and

$$\tilde{\pi}_m\left(1-e^{-rT}\right)+Ne^{-rT}\tilde{\pi}_N-N(1-\beta)s=\tilde{\pi}_m-N(1-\beta)s-(\tilde{\pi}_m-N\tilde{\pi}_N)e^{-rT}>N(\tilde{\pi}_N-(1-\beta)s)>0,$$

which implies $p_n^* > p_c^*$ if and only if

$$e^{-rT}\tilde{\pi}_N-(1-\beta)s>0\Leftrightarrow T<\frac{1}{r}\ln\frac{\tilde{\pi}_N}{(1-\beta)s}.$$

If $\tilde{\pi}_N < (1-\beta)s$, then we must have

$$e^{-rT}\tilde{\pi}_N<(1-\beta)s.$$

Note that for $p_n^*$ and $p_c^*$ to be well defined, we need

$$\tilde{\pi}_m\left(1-e^{-rT}\right)+e^{-rT}\tilde{\pi}_N-(1-\beta)s>0\Leftrightarrow e^{-rT}<\frac{\tilde{\pi}_m-(1-\beta)s}{\tilde{\pi}_m-\tilde{\pi}_N}, \tag{15}$$

and

$$\tilde{\pi}_m\left(1-e^{-rT}\right)+Ne^{-rT}\tilde{\pi}_N-N(1-\beta)s>0\Leftrightarrow e^{-rT}<\frac{\tilde{\pi}_m-N(1-\beta)s}{\tilde{\pi}_m-N\tilde{\pi}_N}<\frac{\tilde{\pi}_m-(1-\beta)s}{\tilde{\pi}_m-\tilde{\pi}_N}. \tag{16}$$

Hence if $T > \frac{1}{r}\ln\frac{\tilde{\pi}_m-N\tilde{\pi}_N}{\tilde{\pi}_m-N(1-\beta)s}$, then both inequalities (15) and (16) are satisfied. Because $\tilde{\pi}_N < (1-\beta)s$, we must have $e^{-rT}\tilde{\pi}_N < (1-\beta)s$, which then implies $p_c^* > p_n^*$. ■

## 7.5 Proof of Proposition 11

**Proof.** $(i)$ If $\theta < 0$, we must have $p_n^* < p_c^*$. Define $M(p,C) = \frac{s(1-\alpha)}{r} + \frac{N\lambda\left(\frac{r\Pi_m}{N}+\alpha s+\theta\right)}{r(r+N\lambda)}p + C(1-p)\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}$ for $p\in[0,1]$. If $C>0$, then $M(p,C)$ represents a loci of convex functions that share the same value at $p=1$. In addition, $M(p,C_1)\neq M(p,C_2)$ for all $p\in(0,1)$ with $C_1\neq C_2$. Note that $V^n(p) = M\left(p,B_n^1\right)$ for $p\geq p_n^*$ and $V^c(p) = M(p,B_c^*)$

for $p \geq p_c^*$. As a result, if $V^n(p)$ and $V^c(p)$ intersects at $p \in [p_c^*, 1)$, we must have $B_n^3 = B_c^*$ and thus $M\left(p, B_n^3\right) = M\left(p, B_c^*\right)$ for all $p \in [0,1]$, which is not true for the following reasons. First of all, $V^c(p) = M\left(p, B_c^*\right)$ is convex and achieves its minimum $\frac{s}{r}$ at $p = p_c^*$. On the other hand, $V^n(p) = M\left(p, B_n^1\right)$ is strictly convex and $V^{n\prime}(p_n^*) < 0$ and $V^n(1) > \frac{s}{r}$ imply there exists a unique $\bar{p}_n > p_n^*$ such that $V^n(\bar{p}_n^*) = \frac{s}{r}$ and $V^{n\prime}(\bar{p}_n^*) > 0$. Hence, there exists a range of $p$ such that $V^n(p) = M\left(p, B_n^1\right) < \frac{s}{r}$, which contradicts the fact $M(p, B^*) \geq \frac{s}{r}$ for all $p \in (0,1)$. Hence we must have $B_n^1 \neq B_c^*$. We now claim $V^n(p_c^*) < V^c(p_c^*)$. Suppose not. Then we must have $V^n(p_c^*) = M\left(p_c^*, B_n^1\right) \geq M\left(p_c^*, B_c^*\right) = V^c(p_c^*) = \frac{s}{r}$. In addition, $V^n(p_n^*) = M\left(p_n^*, B_n^1\right) = \frac{s}{r} < M\left(p_n^*, B_c^*\right) = V^c(p_n^*)$. Hence there exists a $p \in (p_n^*, p_c^*)$ such that $M\left(p, B_c^*\right) = M\left(p, B_n^1\right)$, which implies $B_n^1 = B_c^*$, a contradiction. Hence we must have $V^n(p_c^*) < V^c(p_c^*)$ and thus $V^n(p) < V^c(p)$ for all $p > p_c^*$. This also implies $\bar{p}_n > p_c^*$. Hence for $p \in (p_n^*, \bar{p}_n)$, we have $V^n(p) < \frac{s}{r} \leq V^c(p)$. This completes the proof for part $(i)$.

$(ii)$ If $\theta > 0$, we must have $p_n^{**} > p_n^* > p_c^*$. Note that for $p \geq p_n^{**}$, both the noncooperative and cooperative firm's value function share the similar functional form with $V^n(p) = M\left(p, B_n^1\right)$ and $V^c(p) = M\left(p, B_c^*\right)$.Because $M\left(p, B_n^1\right)$ and $M\left(p, B_c^*\right)$ cannot intersect, either $M\left(p, B_n^1\right) > M\left(p, B_c^*\right)$ or $M\left(p, B_n^1\right) < M\left(p, B_c^*\right)$ for $p \in (0,1)$. Note that $V^c(p) = M\left(p, B_c^*\right)$ for $p \geq p_c^*$, is the optimal value function for a research consortium in which every firm invests $k = 1$ until belief is lower to threshold $p_c^*$. $V^n(p) = M\left(p, B_n^1\right)$ is the value function for noncooperative research when each of them is investing $k = 1$. We must have $M\left(p, B_n^1\right) \leq M\left(p, B_c^*\right)$, otherwise it violates the optimality of the research consortium's decision because in this case the research consortium can always mimic the noncooperative firm's investment decision. This implies $V^c(p) > V^n(p)$ for $p \in (p_n^{**}, 1)$. For $p \in (p_n^*, p_n^{**})$, we know $V^n(p) = V_M^n(p)$. In addition, we know $V_M^n(p_n^*) = \frac{s}{r} < V^c(p_n^*)$. If $V_M^n(p)$ intersects $V^c(p)$ at some $p \in (p_n^*, p_n^{**})$, then we can find $p'$ satisfying $V_M^{n\prime}(p') > V^{c\prime}(p')$. $p'$ exists because $V_M^n(p)$ has to cross $V^c(p)$ from below first if they ever intersect. However, we know

$$\frac{V_M^{n\prime\prime}(p)}{V^{c\prime\prime}(p)} = \frac{\frac{\alpha s}{p^2(1-p)\lambda}}{\frac{\alpha s}{(1-p)p^2 N\lambda}\frac{\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}}{\left(\frac{1-p_c^*}{p_c^*}\right)^{\frac{r}{N\lambda}}}} = N\frac{\left(\frac{1-p_c^*}{p_c^*}\right)^{\frac{r}{N\lambda}}}{\left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}} > 1.$$

The inequality follows from the fact that $f(p) = \left(\frac{1-p}{p}\right)^{\frac{r}{N\lambda}}$ is a decreasing function and $p \geq p_n^* > p_c^*$. This implies $V_M^{n\prime}(p) > V^{c\prime}(p)$ for $p \geq p'$, which is a contradiction because we know $V_M^n(p_n^{**}) = V_H^n(p_n^{**}) < V^c(p_n^{**})$. Hence we must have $V_M^n(p) < V^c(p)$ for all

$p \in (p_n^*, p_n^{**})$. Finally, for all $p \in (p_c^*, p_n^*)$, we must have $V^c(p) > V^n(p) = \frac{s}{r}$, which completes the proof. ■

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