# Capacity Preemption and Leadership Contest in a Market with Uncertainty

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**Abstract**

This paper introduces a continuous-time game to study two *ex ante* identical firms' incentives in capacity preemption. Each firm can choose small or large capacity and investment timing to enter a new industry whose demand grows until an unknown maturity date and then declines until it disappears. Previous literature usually predicts that the Stackelberg leader, whether endogenously or exogenously determined, is better off by building a larger capacity than its rival. In contrast, this paper proves that in most cases the first mover's equilibrium strategy is to enter with a smaller capacity than the follower. If the first mover had chosen the larger capacity, its follower could, and in fact would use a smaller plant to force it out of the market through a leadership contest. The large leader lacks incentive to fight for the market because the small firm can make a credible commitment to stay with its higher option value of waiting to exit.

## 1 Introduction

Industrial organization theorists have pinpointed many circumstances under which a first mover advantage can be expected. Through investing in a larger capacity ahead of its competitor, a firm often achieves higher profit than its follower.[1] This advantage is so appealing that when one firm is competing with another to enter a market with demand uncertainty, it is willing to sacrifice its flexibility in exchange for Stackelberg leadership. (Maggi, 1996)

In the real business world, however, we rarely observe leading firms engaging in large capacity preemption in markets with uncertainty. On the contrary, there are plenty of cases in which a firm

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[1] See Gal-Or (1985) . Tirole (1988) also has a nice discussion of Stackelberg leadership.

either failed to seize or deliberately gave up the opportunity of large capacity preemption. For example, in the titanium dioxide industry, Du Pont decided not to expand its capacity to preempt Ker McGee, its major competitor, citing market uncertainty.[2] A recent example is in the hot spot industry: T-mobile managed to enter the market first with a capacity significantly smaller than what its competitor–Cometa Networks–had announced. In turn, Cometa Networks, a joint venture initiated by Intel, IBM, and AT&T, attempted to contest T-mobile's leadership by building a nationwide hot spot network, which is three times larger than T-mobile network. Nevertheless, T-mobile survived the contest, forcing Cometa Networks to exit the market. Looking back, Cometa Networks blamed its investors for their lack of support while the investors responded by citing the uncertain future of the hot spot service.[3]

The gap between the theoretical predictions and real business practices prompts the question of why a first entrant would refrain from preempting its competitor with a large capacity while its advantage seems so compelling in theory. In this paper, we tell a story that reconciles this gap. Our story starts with the observation that the literature has focused on the cases in which uncertainty either does not exist or will be resolved immediately after the entry of the leader.[4] Little is known about the case where firms have to begin operation in a market with evolving uncertainty—a case that is more likely to occur in reality. Indeed in many new product industries, demand grows as the product becomes more widely accepted. The product life cycle, however, is often uncertain: it may continue to expand or it may switch to decline or crash at some point for many reasons, for example, the discovery of an unpleasant side effect, or the development of a superior product. Firms have no knowledge of how long the market growth will last, and yet they still have to decide when to enter and what capacity to invest.

To model this situation, we introduce a continuous-time setup to study firms' entry timing and capacity choice in a new market with evolving uncertainty. Surprisingly, in most cases the leader would rather enter the market with a smaller capacity than its follower. In fact, in order to seize the market leadership the leader not only needs to move ahead of its competitor, but also needs to credibly commit to stay in the market. However, investing in a larger capacity than its follower in a premature market exposes a serious vulnerability of the leader—rather than delaying the entry of the follower, it invites its

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[2] See Ghemawat (1984). This has become a classical case to teach the advantage of capacity preemption to MBA students in many U.S. business schools but the irony is that Du Pont in fact chose not to preempt.

[3] Investors worried that hot spot service would lose its most important customers–business travelers–once cellular companies rolled out a new network based on Third-Generation(3G) technology. This technology could deliver hi-speed internet service via cellular network and thus had a much larger coverage than hotspot service. See "Chill hits Hot Spots." Wall Street Journal, March 18, 2004.

[4] See, for example, Sadanand and Sadanand (1996), van Damme and Hurkens (1999), and Hirokawa and Sasaki (2001).

follower to start a hostile leadership contest, resulting in a war of attrition in which the small follower wins in most cases. The strategic advantage that helps the small firm to win the leadership contest against a large firm comes from its higher option value giving it more incentive to stay throughout the fight. To understand the source of this option value, we first notice that the large firm has a disadvantage in the declining industry: it will have to exit the market earlier than its smaller competitor (Ghemawat and Nalebuff, 1985). However, the market considered in this paper is currently expanding, except that neither firm knows when it will stop growing and switch to decline. In other words, the small firm will not be able to materialize its advantage in the declining industry until the market switches, which happens at a random date. Nevertheless, it turns out that this potential benefit arriving at an unknown future time gives the small firm a commitment power in the fight for leadership: the small firm has more incentive to continue its operation when challenging the large competitor's leadership. By staying in the market, it retains its option to exit in the future, a flexibility that it maintains to avoid losing its advantage from the declining industry. Indeed, the option value based on this flexibility becomes a source of strategic advantage: when competing with a large firm, the smaller firm has less to lose before investing so it can move earlier; on the other hand, it has more to lose if it has already entered the market, thus intending to stay longer. Consequently, when two firms are competing for market leadership, the leader prefers to invest in small capacity.

The present paper is the first one to study endogenous Stackelberg leadership in a continuous-time setup with evolving uncertainty. Previous papers have focused on a discrete setup, usually in two stages, largely based on the extended game with observable delay (Hamilton and Slutzky (1990) , Amir(1995)), a complete information game that potential entrants are required to announce and commit to their entry timing before they compete in capacities. Sadanand and Sadanand (1996) extend this model to incorporate demand uncertainty but the two entrants' capacities are exogenously determined. Maggi (1996) is the first one to allow firms to choose capacity and entry timing simultaneously.

This paper contributes to the literature on endogenous Stackelberg leadership in two aspects. First, it proves that maintaining flexibility to avoid bad market outcome can generate a strategic advantage, in contrast to the previous literature suggesting that firms tend to sacrifice flexibility in exchange for commitment advantage. The difference comes from the uncertainty setup. Previous papers limit the negative effect from adverse market outcomes. For example, Maggi (1996) assumes that the lower bound of the demand is large enough that a Cournot duopoly will never abandon the market in the second stage. Hirokawa and Sasaki (2001) assume price is always above cost. Those assumptions shelter the leader from losses due to unexpected bad market outcome and strengthen a firm's incentive to move aggressively in competing for Stackelberg leadership. In contrast, by focusing on the potential downside of the market, this paper complements previous literature through modeling uncertainty as an unknown market switching date whose arrival could inflict a loss on the incumbents and through showing that maintaining flexibility generates a strategic advantage: small capacity preemption strengthens the leader's commitment power to stay in the market when its leadership is being challenged by the follower.

The second innovative feature of this paper is to allow the follower to contest the first mover's leadership. Previous models implicitly assume that large capacity preemption is feasible immediately, and the leadership competition ends right after the first mover's entry. This makes the leadership competition a lottery game—whoever wins the lottery to move first will be able to enjoy the first mover advantage. In contrast, the present paper assumes that leadership competition does not end when one firm enters—the other firm may start a leadership contest. Immediately after the entry of the leader, the follower decides whether to challenge the first mover's position through a leadership contest. The possibility of leadership challenge changes a firm's incentive to pursue large capacity preemption—in most cases, the first mover prefers to enter with a small capacity to preempt its competitor. Furthermore, allowing the follower to contest the leadership softens market competition, reducing the loss of flexibility due to the competition, because the first entrant is able to enter closer to the date when the full option value is preserved. In addition, the whole industry are better off due to the softening of the competition.

This paper is closely related to the line of literature on preemption starting with the classic paper by Fudenberg and Tirole (1985) , which focuses on the timing of technology adoption. This paper extends this framework in two directions. First, firms can choose different capacities in addition to entry timing. Second, the follower may challenge the first mover through a leadership contest.

Finally, it is worth mentioning that the real options game introduced in this paper is a simplified stylized version, in contrast to those papers based on Geometric Brownian Motions, such as Grenadier (1996). The simple setup avoids stochastic calculus and yet still allows us to determine the option value and its related strategic advantage. Ruiz-Aliseda (2003) uses a similar uncertainty setup to study the entry and exit decision of two firms with different opportunity costs. His paper focuses on the effect of asset specificity on the credibility of preemptive actions, whereas my paper stresses the effect of capacity choices on firms' strategic entry timing in competition for market leadership.

This paper is divided into five sections. We start by laying out the model in Section 2 and discuss a benchmark monopoly case in section 3. Section 4 solves the duopoly game of capacity preemption and section 5 concludes the paper with some discussions.

## 2 The Model

Consider a continuous-time game with two *ex ante* identical firms, contemplating entry into a new industry that is currently expanding but may switch to decline at an unknown date. Each firm selects its own capacity level and entry timing. Once a firm decides to enter the market, it can operate immediately after making an investment that costs $\kappa$ per unit of capacity. Continuing operation requires a cost flow $c$ per unit of capacity, which can be avoided if the firm exits the market. Upon exiting, if the market is still expanding, a firm is able to recover $\gamma$ per unit of capacity, where $\gamma < \kappa$. Clearly, if $\gamma \geq 0$, the investment is at least partially sunk; if $\gamma < 0$, then it is considered as an exit cost. A firm's payoff from the declining market will be specified in Assumption 2 below. Reentry is not allowed.

In this paper, we analyze this continuous time game as the limit of discrete time game as the length of period goes to zero.[5] This approach has been adopted in the study of many other continuous time games of entry.[6] One common issue is the coordination when players are making simultaneous moves. Some papers assume firms moving sequentially (e.g. Maskin and Tirole, 1987), others allow *ex ante* simultaneous move but eliminate it *ex post*, such as Dutta, Lach and Rustichini (1995) . As we do not want to award any firm *ex ante* advantage in the order of their actions, we adopt the second approach:[7]

**Assumption 1** *If both firms attempt to enter the market at the same date* $t$, *then only one firm succeeds. In this case, firm 1 enters with probability* $p \in (0, 1)$ .

Specifically, we illustrate the timing of the game in Figure 1. Immediately after firm $i's$ entry at $t_1$, firm $j$ needs to make a decision: if firm $j$ decides to challenge firm $i's$ leadership, it enters right away

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[5] See Simon and Stinchcombe (1986) for a study of this approach.

[6] For example, Fudenberg and Tirole (1985) and Londregan (1990)

[7] This assumption might be the result of some institutional features; for example, to make the investment, the firms need to obtain a license, but the business licensing office can award one licence at a time. If two firms show up simultaneously, one of them will be randomly chosen to receive licence first.

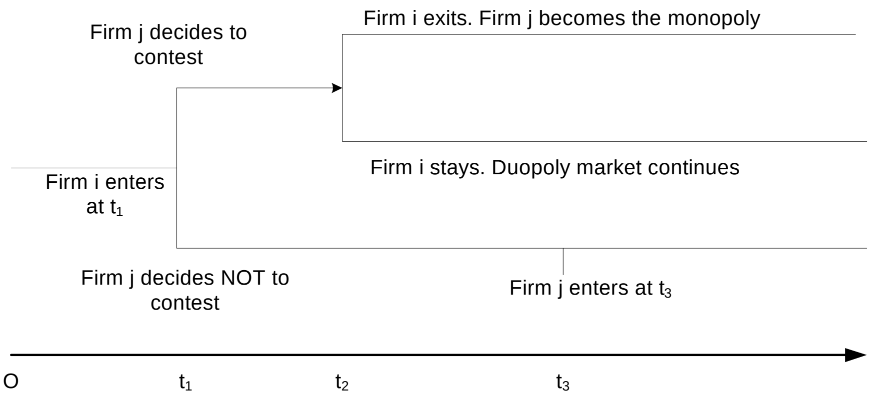

Figure 1: The timing of the game

to start a leadership contest that may cause firm $i$ to exit the market. However, if firm $j$ decides not to contest, it may enter at any time later, say $t_3$, and turns the market into a duopoly.

Our model can be viewed as an extension of Fudenberg and Tirole's (1985) technology adoption timging game by incorporating two new features: first, in addition to choosing entry timing, each firm can select its own capacity levels; second, the follower is allowed to contest the first mover's leadership. We model the second feature in a stylized way by limiting the window of opportunity for leadership contest to the date when the leader enters. Although it is not uncommon that a first mover can solidify its leadership quickly when left unchallenged for some period, restricting the window of challenge to the leader's entry date is for technical rather than empirical reasons—it allows us to consider only one war of attrition subgame initiated by the follower, which is sufficient for our purpose. Without this assumption, the follower can start a leadership contest at any time after the first mover's entry, making the model intractible because the optimal timing to contest is not always well defined given the noncompactness of the contesting region.

To enter the market, each firm chooses a capacity $q$ from a finite set of plant sizes. For simplicity, we assume there are two capacity levels: small $(S)$ and big $(B)$ .[8] Let $Q = q_1 + q_2$ denote the total capacity active in the market. Following Ghemawat and Nalebuff (1985) , we assume firms always produce at full capacity and sell all their outputs to the market. In other words, we model those industries wherein firms would rather let the price decrease than operating at less than full capacity, as in the case of auto

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[8] Analysis of multiple capacity level is too complicated analytically but can be done through numerical analysis. See Wu(2005) for a numerical analysis of multiple capacity case.

makers.[9] Following other real options models such as those presented in Dixit and Pindyck (1994) , we model the demand function as follows:

$$P_g(t, Q) = Y(t) D_g(Q), \ g = c, n. \tag{1}$$

Here, the subscript $g$ represents whether there is on-going leadership contest. $D_c(Q)$ represents the demand function in the presence of leadership contest while $D_n(Q)$ represents the normal no contest case, both of which are decreasing functions of $Q$. We deliberately differentiate the demand function in those two situations to account for the possibility that the market price is significantly lower when firms are competing neck-to-neck in the leadership contest. In other words, we assume $D_c(q) \leq D_n(q)$ for all $q \geq 0$. In addition, we also assume $D_n(2B) = 0 < D_n(S + B)$, implying the market is not large enough for two large firms. The assumption of $D_n(2B) = 0$ is for convenience, without which we need to discuss more cases that generate little additional insight.[10]

The other factor that affects the market price is the random demand shock represented by $Y(t)$. We assume $Y$ starts at $y_0 > 0$ and grows exponentially over time at the rate of $\alpha$. $Y(t)$ may stop increasing at a random date $\tilde{\tau}$ that is distributed exponentially with parameter $\lambda$. Once $\tilde{\tau}$ is realized, the market switches to a declining industry. As the demand continues to decline, firms will have to exit the market ultimately, although each firm prefers its competitor to leave the market first. The subgame after the market switches is a game of exit from the declining industry, which has been studied extensively. In a seminal paper, Ghemawat and Nalebuff (1985) prove that the firm with smaller capacity can outlast the larger firm in the declining industry. The following proposition is a restatement of their main findings:[11]

**Proposition 1** *(Ghemawat and Nalebuff, 1985) Suppose there are two active firms when the decline begins and the duopoly market price is no higher than marginal cost c . If $q_i > q_j$, firm i will exit the market immediately, leaving firm j ($j \neq i$) to monopolize the market until its monopoly exit date. The result is reversed if $q_i < q_j$. If $q_i = q_j$, there exists a symmetric mixed strategy equilibrium with both*

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[9] See "Ford learns to bend with the wind." *Financial Times*, February 14. Also, due to the uncertainty of the market, firms might be stuck with their labor and raw material supply contract, which makes capacity shrinkage extremely difficult.

[10] We briefly discuss the implications if $D(2B) > 0$ at the end of Section 4.1.

[11] For a proof, see Proposition 1 in Ghemawat and Nalebuff (1985).

*firms earning zero expected payoff. In summary, firm i's continuation payoff in the declining stage is*

$$W(q_i|q_j) = \begin{cases} W^m(q_i) & \text{if } q_i < q_j \text{ or } q_j = 0 \\ 0 & \text{if } q_i \geq q_j \end{cases},$$

*where $W^m(q_i)$ is firm i's monopoly payoff in the declining industry.*

The essence of this result is that the smaller firm is able to survive its larger competitor in a declining industry, because the smaller firm can stay longer than its larger competitor while still being profitable. Consequently, the smaller firm always wins the war of attrition in a subgame perfect equilibrium.[12] To use this result in our model, we also assume the declining industry starts with a sudden market switching which makes it unprofitable for a market with one small and one big firm, so the firm with the larger capacity has to exit immediately.[13] In addition, we let $m$ denote the average continuation payoff in the declining stage.[14] Furthermore, we make the following assumption

**Assumption 2** $\gamma < m < \kappa$.

This assumption has two implications. First, a monopoly firm will not exit the growing industry because the average payoff from the declining industry is better than salvage value received when the market is growing. Second, no firm will enter the declining industry because the payoff from the declining industry is not enough to cover the investment cost.

Finally, we let $r$ represent the discount rate and assume that $\lambda + r > \alpha$ in order to guarantee integrability when calculating firms' expected payoff.[15]

In this paper, the solution concept is pure strategy Markov perfect equilibrium. At each date $t$, a pure strategy of firm $i$, $\sigma_i$, is a function of payoff relevant state. The history at $t$ is $h_t = \left\{q_i^t, q_j^t, g_t, \omega_t\right\}$,

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[12] Here is the intuition: the larger firm will definitely exit the market when the current date is its monopoly exit date, because after this date, even though its competitor is not in the market, it cannot make any positive profit. However, at this date, the smaller firm is still profitable once the larger firm exits, so the smaller firm will stay through the larger firm's monopoly exit date while the large firm will exit. Realizing this, the larger firm will exit the market immediately once the war of attrition breaks out, because it has no chance of winning in a subgame perfect equilibrium.

[13] If the war of attrition does not happen immediately after the market switches, both firms will be able to earn a duopoly payoff until the large firm exits. To make the model tractable, we normalize this duopoly payoff to zero. In addition, an earlier version of this paper (available upon request from the author), contains a non-reduced form analysis, which is much more complicated.

[14] We implicitly assume the average continuation payoff display constant return to scale, i.e. $\frac{W^m(S)}{S} = \frac{W^m(B)}{B} = m$.

[15] Note that $\int_t^\infty \lambda e^{-\lambda\tau}\left(\int_t^\tau \left(y_0 e^{\alpha_1 s} D_g(q) - c\right) q e^{-rs}\right) d\tau$ is the expected payoff for a firm in the growing stage if it invests at date $t$, which is integrable if $\lambda + r - \alpha_1 > 0$.

where $t$ is the current date. Also, $q_i^t$ denotes firm $i's$ capacity at date $t$; $g_t \in \{c, n\}$ denotes whether there is on-going war on leadership and $\omega_t = \{0, 1\}$ denotes whether or not the market growth has stopped or not. As firms are *ex ante* identical, we analyze the equilibrium strategies in terms of leader and follower without referring to their identity explicitly. Assumption 1 guarantees that the leader and follower will be identified *ex post*. In other words, we need to specify each firm's entry timing depending on whether its competitor has entered and whether it is possible to contest the leader if preempted or whether to fight for the market if there is ongoing leadership contest.

Before solving the model, let us define some notations. In general we use $\bar{t}$ and $\underline{t}$ to denote the leader's and follower's entry dates respectively. For example, $\bar{t}_{\mu|\nu}$ denotes the entry time for a *leader* with capacity $\mu$ if its *follower* will be induced to choose capacity $\nu$, and $\underline{t}_{\nu|\mu}$ represents the entry date of a *follower* with capacity $\nu$ conditional on the *leader* picking capacity $\mu$. Similarly, the monopoly entry date for a firm with capacity $\mu$ is denoted as $t^m(\mu)$. Occasionally we use *firm* $q$ ($q \in \{S, B\}$) to denote the *firm with capacity* $q$ when there is no ambiguity. Most proofs are presented in the Appendix.

# 3 Monopoly

As a benchmark, we first consider a firm owning a monopoly right to invest in this new market. Let the current date be $t_0$. If the firm chooses capacity $\mu$ to enter at date $t \geq t_0$, then its expected payoff is[16]

$$M(t, \mu|t_0) = \int_t^\infty f(\tau|t_0) \left[ \int_t^\tau (P_n(s, \mu) - c)\, \mu e^{-rs} ds + e^{-r\tau} W(\mu|0) - \kappa \mu e^{-rt} \right] d\tau. \tag{2}$$

$f(\tau|t_0) = \lambda e^{-\lambda(\tau - t_0)}$ is the density function conditional on the date having reached $t_0$. The integrand of the monopoly's expected payoff consists of three parts: $\int_t^\tau (P_n(s, \mu) - c)\, \mu e^{-rs} ds$ is the expected discounted profit before the market switches; $W(\mu|0)$ is its continuation payoff in the declining industry, and $\kappa\mu$ is the total investment amount incurred before entering this market. The following lemma gives a monopoly firm's optimal entry decision:

**Lemma 1** *(i)A firm with capacity $\mu$ will enter the market at* $t = \max\{t_0, t^m(\mu)\}$, *where* $t^m(\mu) =$

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[16] In this paper, all payoffs are discounted back to date 0.

$\frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(\mu)}$. *In addition,* $t^m(\mu)$ *satisfies the following condition:*

$$P_n\left(t^m(\mu),\mu\right)-c=r\kappa+\lambda(\kappa-m) \tag{3}$$

*(ii)* $t^m(\mu)$ *is increasing in* $\mu$.

The monopoly's entry decision is a rule of marginal cost equal to marginal revenue. The left hand side of Equation (3) is the marginal cost of waiting to invest, which is the instantaneous profit foregone per unit of capacity if staying out of the market. The right hand side represents the marginal value of waiting, which consists of two parts: for each unit of capacity, $r\kappa$ is the investment cost saved by delaying entry and $\lambda(\kappa-m)$ is the marginal option value of waiting. To see why the second part is the option value, observe that for each instant, there is probability $\lambda$ that the market switches to decline, in which case the firm's continuation payoff is $W(\mu|0)$. According to Assumption 2, $m<\kappa$ implies that the firm will incur a loss if the market switches right after its entry. This loss may be avoided if it waits one more instant rather than immediately investing, because once the firm observes the market switch, it will not invest. Consequently, this potential loss avoided multiplied by $\lambda$, the probability of market switching, is exactly the option value of waiting. We name it marginal option value of waiting because it is measured marginally.

Assuming $t_0<t^m(\mu)$ and substituting $t^m(\mu)$ into $M(t,\mu|t_0)$, we have

$$M^*(\mu|t_0)=e^{\lambda t_0}\left(\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(\mu)}\right)^{-\frac{\lambda+r}{\alpha}}\left(\frac{\alpha\left(c+(\lambda+r)\kappa-\lambda m\right)}{(\lambda+r-\alpha)(\lambda+r)}\right)\mu$$

In fact, the market opening at date 0 implies that the monopoly firm is able to enter the market at its monopoly date for sure. Therefore, we can derive the monopoly's optimal entry capacity choice.

**Proposition 2** *If Assumption 2 holds, the monopoly firm chooses capacity* $\mu^*$ *to enter the market at* $t^m(\mu^*)=\frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(\mu^*)}$. *In particular,* $\mu^*=S$ *if* $\alpha\leq\bar{\alpha}$; *otherwise* $\mu^*=B$, *where* $\bar{\alpha}=\frac{(\lambda+r)(\ln D_n(S)-\ln D_n(B))}{\ln B-\ln S}$.

The result is very intuitive. The monopoly invests in large capacity for a fast growing market, otherwise it selects small capacity. No matter which size it chooses, it takes into account the flexibility by incorporating an option value in its entry timing choice. In fact, if we assume demand is linear and allow the firm to choose any positive capacity level, we will have a closed form solution to the monopoly's optimal entry capacity.

**Corollary 1** *In addition to Assumption 2, we also assume* $D_n(Q) = a - bQ$*. If the firm is free to choose any capacity* $\mu \in [0, \infty)$*, then its optimal capacity is* $\mu^* = \frac{a\alpha}{b(\lambda+r+\alpha)}$ *and its optimal entry date is* $t^m(\mu^*) = \frac{1}{\alpha} \ln \frac{(c+(\lambda+r)\kappa-\lambda m)(\lambda+r+\alpha)}{y_0 a(\lambda+r)}$*. Furthermore,* $\mu^*$ *is increasing in* $\alpha$ *but decreasing in* $\lambda$*.*

In this case, the faster the market grows, the larger the capacity chosen by the monopoly. In addition, higher $\lambda$ implies the market is more likely to switch in the next instant, making the monopoly choose a smaller capacity. Finally, the option value is proportional to the difference between investment cost $\kappa$ and the continuation payoff $m$. If $\kappa$ is fixed, smaller $m$ implies the larger the option value, causing the monopoly firm to further delay its entry.

# 4 Duopoly

The previous section analyzes a monopoly's entry strategy, especially the effect of marginal option value on a firm's entry timing. In this section, we will study the effect of this option value on the competition for market leadership. Because two potential entrants are *ex ante* identical, either firm may become the first mover. To find out the equilibrium, we start with the follower. When observing its competitor's entry before itself, a firm has two options: one is to accept its position as a Stackelberg follower; the other is to start a leadership contest to force out the leader. The second option is feasible if the market is still premature: the entry of a new firm causes the market price to drop below cost, making both firms unprofitable. As the market price may take a long time to recover to a profitable level, the leader may find it optimal to exit the market. Of course, the leader will take the possibility of leadership contest into account when picking its capacity and entry timing; it should not enter the market if it would be forced out by the second mover that result in a loss for the leader. In the following discussion, we start with the assumption that the follower takes the leader's position as *fait accompli* and derive its optimal entry strategy. Later when discussing a leader's strategy, we will check whether the follower can indeed contest the first mover's leadership.

### 4.1 The Follower

Suppose the current date is $t_0$ and one firm has already entered the market with capacity $\mu$. If the noncontesting follower decides to invest in a plant with capacity $\nu$ at date $t \geq t_0$, its expected payoff is

$$V_f\left(t, \nu|\mu, t_0\right) = \int_t^\infty f\left(\tau|t_0\right)\left(\int_t^\tau \left[P_n\left(s, \mu+\nu\right) - c\right]\nu e^{-rs}ds + e^{-r\tau}W\left(\nu|\mu\right) - e^{-rt}\kappa\nu\right)d\tau. \tag{4}$$

The integrand of $V_f\left(t, \nu|\mu, t_0\right)$ consists of three parts: the first item, $\int_t^\tau \left[P_n\left(s, \mu+\nu\right) - c\right]\nu e^{-rs}ds$, is the expected discounted profit from the duopoly growing market until the maturity date. The second item, $W\left(\nu|\mu\right)$ is the continuation payoff associated with the declining stage, and the third item $\kappa\nu$ is the sunk investment cost, all of which are discounted to date 0. If the follower chooses not to contest the first entrant's leadership, then its optimal entry timing is derived in the following proposition.

**Proposition 3** *Let $t_0$ be the current date and suppose that the leader has entered the market with capacity $\mu$. If the follower with capacity $\nu$ does not start the leadership contest, its optimal entry date is $t^f = \max\left\{t_0, \underline{t}_{\nu|\mu}\right\}$, where $\underline{t}_{\nu|\mu}$ satisfies*

$$P_n\left(\underline{t}_{\nu|\mu}, \mu+\nu\right) - c = r\kappa + \lambda\left(-\frac{W\left(\nu|\mu\right) - \kappa\nu}{\nu}\right). \tag{5}$$

The follower's entry decision rule is similar to that of the monopoly firm discussed in the previous section. The left hand side of equation (5) is the marginal cost of waiting to invest—the instantaneous profit per unit of capacity foregone if not entered. The right hand side represents the marginal value of waiting to invest, which consists of two parts. The first item $r\kappa$ is the per unit investment cost saved by delaying entry into the market, and the second part is the marginal option value of waiting.

**Corollary 2** *Suppose the duopoly market has one small and one big firm and the leader has entered before $\underline{t}_{S|B}$. If the leader chooses a big capacity, then the small follower will enter at $\underline{t}_{S|B}$, earlier than the big follower's entry date $\underline{t}_{B|S}$ when the leader has chosen a small capacity. That is $\underline{t}_{S|B} < \underline{t}_{B|S}$.*

**Proof.** This is immediate by comparing $\underline{t}_{S|B}$ and $\underline{t}_{B|S}$ by noting that $W\left(S|B\right) > W\left(B|S\right) = 0$. ■

When the follower chooses smaller capacity, it can enter the market earlier than if it chooses the larger capacity to enter the market. This implies that if the resulting duopoly market structure is the same, large capacity preemption might achieve just the opposite by inviting the follower to move into the market earlier, even if it simply enters as a Stackelberg follower with no intention to contest the first entrant's leadership. This is because the smaller firm has a lower option value of waiting and thus has more incentive to invest earlier.

Recall that $t^m(\mu)$ is the monopoly's entry date when its capacity is $\mu$. In the next subsection, we will show that the leader will never enter later than its monopoly entry date $t^m(\mu)$ if there exists a first mover advantage. Assuming the leader enters on or before $t^m(\mu)$, the following corollary shows that a noncontesting follower with capacity $\nu$ will be able to enter at its optimal entry date $\underline{t}_{\nu|\mu}$.

**Corollary 3** *Suppose Assumption 2 holds, then the noncontesting follower's optimal entry date* $t^f$ *satisfies* $t^f > t^m(\mu)$.

As a result of Corollary 3, we can substitute the follower's optimal entry date into its payoff function and have $V_f\left(\underline{t}_{\nu|\mu}, \nu|\mu, t_0\right) = V_f\left(\underline{t}_{\nu|\mu}, \nu|\mu, t_0\right) = e^{\lambda t_0} F^*(\nu|\mu)$, where

$$F^*(\nu|\mu) = \left(\frac{c + (\lambda + r)\kappa - \lambda\frac{W(\nu|\mu)}{\nu}}{y_0 D_n(\mu + \nu)}\right)^{-\frac{\lambda+r}{\alpha}} \frac{\alpha\left(c + (\lambda + r)\kappa - \lambda\frac{W(\nu|\mu)}{\nu}\right)}{(\lambda + r - \alpha)(\lambda + r)}\nu. \qquad (6)$$

Given the follower's optimal entry timing, we can characterize its optimal capacity choice.

**Proposition 4** *Let* $\alpha_S = \frac{(\lambda+r)(\ln D_n(2S) - \ln D_n(S+B))}{\ln B - \ln S}$. *Given Assumption 1-2 and the leader has already invested before the current date* $t_0 < t^m(B)$.

*(i) If* $\alpha \leq \alpha_S$, *then the follower always choose capacity* $S$.

*(ii) If* $\alpha > \alpha_S$, *the follower chooses capacity* $S$ *if the leader invests in capacity* $B$, *and vice versa.*

If the leader decides to start with a small plant, the follower can choose capacity freely. In this case, it depends on how fast the market is growing. If it is expanding faster than $\alpha_S$, the follower will invest later with a large capacity, otherwise, it will invest in small capacity for the fear of losing the opportunity to invest. If the leader enters with capacity $B$, the follower will be forced to become a small competitor $S$, because according to the assumption that $D_n(2B) = 0$, the market is not large enough for two large firms. If we relax this assumption by allowing $D_n(2B) > 0$, then there exists $\alpha_B$ such that if $\alpha > \alpha_B$ the follower will invest in capacity $B$ even if the leader has invested in large capacity. Allowing this case will simply double the cases that need to be analyzed in the next section but will generate few additional insights.

## 4.2 The Leader

Previous section studies the follower's optimal entry timing and capacity decision, provided it does not contest the first mover's leadership. In this section, we consider the leader's entry strategy. Because our main question is whether the leader can credibly preempt the follower with a larger capacity when competing for endogenous Stackelberg leadership, we will focus on the case with $\alpha > \alpha_S$. In this case, the large capacity preemption, if successful, will force the follower to become a small competitor, as predicted by previous literature on endogenous Stackelberg leadership. In other words, this case provides the basis to compare our results with those in the previous literature. We will first discuss the leadership contest and then move on to analyze the leader's optimal entry strategy.

### 4.2.1 Leadership Contest

Once a firm sees its competitor entering the market, it either accepts its position as the follower or starts a leadership contest to challenge the first mover's position. However, as the second mover, it will not contest the first mover's leadership if it has no hope of winning, otherwise if it declared war on leadership immediately after the first mover's entry but failed to force out the leader, it would have entered suboptimally. Clearly, if two firms are of the same capacity, they will suffer from the same stream of loss during the leadership contest, and thus no firm has a clear incentive to give up.[17] Therefore, to force out the first mover, a firm has to choose a capacity level different from its competitor.

Finding itself in a leadership contest, a firm decides whether to continue its operation by comparing two incentives. On the one hand, as the market keeps growing and no crash happens, a firm's profit will become positive after some time, which makes sense for a firm to keep staying in the market. On the other hand, it also understands the risk that recovery might never come because the market might crash before reaching the profitable level. In this respect, exiting may be optimal to avoid further loss. One firm's exit is certainly good news for the other, because the latter will start making positive profits immediately after the former's exit as the market price rises back to the monopoly level. In this respect, each firm wishes its competitor to exit the market first.

More specifically, suppose firm 1 enters at date $t_0$ with capacity $q_1$ and firm 2 decides to contest the leadership by entering immediately with capacity $q_2 \neq q_1$. Immediately, the price drops from the

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[17] We are only interested in the pure strategy equilibrium in this paper, and it is unrealistic to assume firms use mixed strategy that result in building a plant with random capacity levels.

monopoly level in the normal market, $P_n(t_0, q_1)$, to a duopoly level in a contest market, $P_c(t_0, q_1 + q_2)$. In this case, each firm needs to consider its optimal exit strategy at every date if its competitor has not given up. If its competitor has exited the market, the remaining firm has no incentive to exit as the price will rise to monopoly level. The problem that firm $i$ faces is when to exit the market if its competitor has not exited. Let $t_i$ be its chosen exit date, then $t_i = t_0$ implies an immediate exit while $t_i = \infty$ implies staying in the market until its crash. We can write down firm $i's$ payoff function conditional on firm $j$ having no incentive to leave the market before $t_i$:

$$\begin{aligned} & V_c(q_i, t_i|q_j, t_0) \\ = & \int_{t_0}^{t_i} f(\tau|t_0)\left(\int_{t_0}^{\tau}(y_0 e^{\alpha s} D_c(q_1+q_2) - c)\, q_i e^{-rs} ds + e^{-r\tau} W(q_i|q_j)\right) d\tau \\ & + \int_{t_i}^{\infty} f(\tau|t_0)\left(\int_{t_0}^{t_i}(y_0 e^{\alpha s} D_c(q_1+q_2) - c)\, q_i e^{-rs} ds + e^{-rt_i}\gamma q_i\right) d\tau. \end{aligned} \tag{7}$$

Each firm's investment is sunk when the war of attrition starts, so it does not enter its payoff function regarding its exit decisions. The first integral is firm $i's$ expected payoff if the market switches before firm $i's$ planned exit date $t_i$: it earns a discounted stream of profits until date $\tau$ and a continuation payoff $W(q_i|q_j)$ after date $\tau$. If the market switches after $t_i$, it exits the market at $t_i$, and obtains a salvage value of $\gamma q_i$. Substituting $t_i = t_0$ into Equation (7), we have $V_c(q_i, t_0|q_j, t_0) = e^{-rt_0}\gamma q_i$, which is the expected payoff if the firm exits immediately at $t_0$. If $t_i^c = \infty$, firm $i$ decides not to exit at all. If no firm exits the market, its payoff is

$$V_c(q_i, \infty|q_j, t_0) = \int_{t_0}^{\infty} f(\tau|t_0)\left(\int_{t_0}^{\tau}(y_0 e^{\alpha s} D_c(q_1+q_2) - c)\, q_i e^{-rs} ds + e^{-r\tau} W(q_i|q_j)\right) d\tau.$$

The following proposition presents the subgame equilibrium of this leadership contest between a small firm and a large firm.

**Proposition 5** *Suppose a leadership contest starts at date $t_0$ between firm 1 with capacity $S$ and firm 2 with capacity $B$. If $t_0 < t_B^c$, then firm 2 exits the market immediately while firm 1 stays, where $t_B^c$ satisfies $V_c(B, t_B^c|S, t_0) = V_c(B, \infty|S, t_0)$ and is increasing in $\gamma$.*

In the proof of Proposition 5, $V_c(q_i, t_i|q_j, t_0)$ is shown to be strictly quasi-convex in $t_i$, minimized

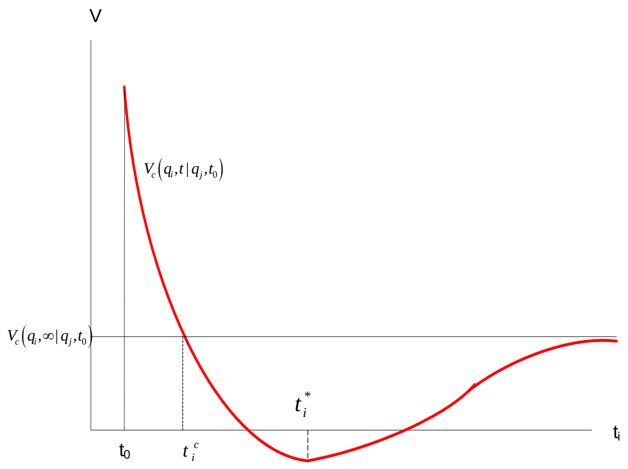

Figure 2: Duopoly payoff function in leadership contest

at $t_i^*$, as illustrated in Figure 2. Hence, this is a war of attrition with eventual continuation.[18] In other words, for each firm $i$, there exists $t_i^c$ such that at $t > t_i^c$, the firm has a dominant strategy to stay. Assume firm $i$ has capacity $q_i$. Let $t_i^c$ satisfies $V_c\left(q_i, t_i^c|q_j, t_0\right) = V_c\left(q_i, \infty|q_j, t_0\right)$. As shown in Figure 2, $t_i^c$ is the intersection between the payoff function $V_c\left(q_i, t_i|q_j, t_0\right)$ and the horizontal line $V = V_c\left(q_i, \infty|q_j, t_0\right)$. The strict quasi-convexity of $V_c\left(q_i, t_i|q_j, t_0\right)$ implies that if $t \geq t_i^c$, firm $i$ should not exit the market as its payoff is lower than that from not exiting until the market crashes. Hence firm $i's$ exit region is $[t_0, t_i^c)$. In the proof we also show that $t_i^c$ is increasing in $\gamma$. That is, the higher the salvage value, the more likely the firm is going to exit the market as its exit region is increasing in $\gamma$.

Without loss of generality, Proposition 5 assumes firm 1 has capacity $S$ and firm 2 has capacity $B$. In the proof, we show that $t_1^c < t_2^c$, implying the small firm has a shorter exit region. Whenever the war of attrition breaks out, the small firm will stay while its large competitor will leave the market. To understand this result, suppose the current date is $t_1^c$. The small firm has a dominant strategy to stay after this date while the larger firm still has an incentive to exit because delaying exit will further reduce its payoff. As a result, the large firm will exit the market while the smaller firm will stay. Furthermore, if the large firm is willing to exit at $t_1^c$, it should have exited earlier, because its payoff function is decreasing for $t < t_i^*$. Consequently, the large firm exits the market immediately after the war of attrition breaks out, leaving the small firm to monopolize the market.

To understand the source of this advantage, notice that the upper bound of $t_i^c$ is $t_i^*$, where $t_i^*$ is the

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[18] See Fudenberg and Tirole (1991), page122∼125, for a discussion of war of attrition game with eventual continuation.

date when the minimum of $V_c\left(q_i, t_i | q_j, t_0\right)$ is reached. If the date has passed $t_i^*$, further delaying exit will improve a firm's payoff, thus a firm has no incentive to exit for $t > t_i^*$. In the proof we show that

$$t_1^* = \frac{1}{\alpha} \ln \frac{c + r\gamma - \lambda \left(\frac{W(S|B)}{S} - \gamma\right)}{y_0 D_c\left(S + B\right)} < \frac{1}{\alpha} \ln \frac{c + r\gamma - \lambda \left(\frac{W(B|S)}{B} - \gamma\right)}{y_0 D_c\left(S + B\right)} = t_2^*$$

A quick comparison of $t_1^*$ and $t_2^*$ reveals the advantage of firm 1, the small firm. The only difference between $t_1^*$ and $t_2^*$ is one term on the numerator, $\lambda\left(\frac{W(S|B)}{S} - \gamma\right)$ for the small firm and $\lambda\left(\frac{W(B|S)}{B} - \gamma\right)$ for the large firm. Those two terms are exactly the marginal option value for a firm to delay its exit from the market. To see why, suppose the market is growing at the moment. $W\left(\nu|\mu\right)$ is the continuation payoff for a firm with capacity $\nu$ after the market switches. By staying in the market during the war of attrition, the firm can avoid losing $W\left(\nu|\mu\right) - \gamma\nu$ when the market suddenly switches to decline. $\left(W\left(\nu|\mu\right) - \gamma\nu\right)$ multiplied by $\lambda$, the probability of market switching, is exactly the expected potential losses avoided by choosing not to exit the market, or in a word, its option value to delay exit. According to Proposition 1, $W\left(S|B\right) > W\left(B|S\right) = 0$, which gives $t_1^* < t_2^*$. That is, a smaller firm has higher option value and thus reduces its willingness to exit the market.

Proposition 5 supplements the result presented in Ghemawat and Nalebuff (1985) by extending it to a growing industry by showing the small firm's advantage lies not only in the declining industry but also in the growing industry. In their model, the advantage of the small firm in the declining market comes from the fact that the small firm may earn some positive profit even after the market becomes unprofitable for the large firm. In our model, the advantage of the small firm in the growing market comes from its higher option value—it wants to wait so that it can avoid losing its advantage in the declining stage when the market switches. It is this option value that gives the small firm higher commitment power to fight for the market.

We have assumed that $D_c\left(q\right) \leq D_n\left(q\right)$ for all $q \geq 0$ to account for the likely significant price drop due to intensified competition in a leadership contest. The larger the price drop, the more likely the follower will start a leadership contest, because the lower the price in a contesting market, the wider the exit region, and the more likely the large leader could be forced out by the small follower.

### 4.2.2 Equilibrium in Capacity Preemption

The previous section shows that if the first mover expects to lose the leadership contest, it will exit the market immediately after the follower's entry. However, the leader would not enter the market if it expected to lose the leadership contest. In this section, we will first find out the potential equilibrium outcomes when no leadership contest occurs, and then we will analyze whether these outcomes are indeed a result of subgame perfect equilibrium when leadership contest is allowed.

In order to compare the leader and follower's payoffs, we start with defining their payoffs as functions of the leader's entry date. Let $t_0$ be the current date and $F(t,\nu|\mu,t_0)$ be the expected payoff for a follower with capacity $\nu$ when the *leader* enters at $t$ with capacity $\mu$. As discussed in section 4.1, the optimal entry date for a noncontesting follower with capacity $\nu$ is $\underline{t}_{\nu|\mu}$ if the leader has already entered with capacity $\mu$ before $\underline{t}_{\nu|\mu}$. Otherwise the second mover will follow the leader to enter the market immediately. This implies

$$F(t,\nu|\mu,t_0)=\begin{cases} V_f\left(\underline{t}_{\nu|\mu},\nu|\mu,t_0\right) & \text{if } t\le \underline{t}_{\nu|\mu} \\ V_f(t,\nu|\mu,t_0) & \text{if } t>\underline{t}_{\nu|\mu} \end{cases},$$

where $V_f(\cdot)$ has been defined by equation $(4)$. Define $\pi(t_1,t_2,x,y)=\int_{t_1}^{t_2}(P_n(s,x+y)-c)\,xe^{-rs}ds$ to be the expected discounted profit that a firm with capacity $x$ earns from date $t_1$ to $t_2$ when its competitor's capacity is $y$ in a noncontesting market. In particular, if $y=0$, then the firm with capacity $x$ is a monopoly. If the leader decides to enter at $t\ge t_0$, its expected payoff is

$$L(t,\mu|\nu,t_0)=\begin{cases} \begin{array}{l} \int_t^{\underline{t}_{\nu|\mu}} f(\tau|t_0)\left[\pi(t,\tau,\mu,0)-e^{-rt}\kappa\mu+e^{-r\tau}W(\mu|0)\right]d\tau \\ +\int_{\underline{t}_{\nu|\mu}}^{\infty} f(\tau|t_0)\left[\begin{array}{c}\pi\left(t,\underline{t}_{\nu|\mu},\mu,0\right)+\pi\left(\underline{t}_{\nu|\mu},\tau,\mu,\nu\right)\\ -e^{-rt}\kappa\mu+e^{-r\tau}W(\mu|\nu)\end{array}\right]d\tau \end{array} & if\ t<\underline{t}_{\nu|\mu} \\ \int_t^{\infty} f(\tau|t_0)\left[\pi(t,\tau,\mu,\nu)-e^{-rt}\kappa\mu+e^{-r\tau}W(\mu|\nu)\right]d\tau & if\ t\ge \underline{t}_{\nu|\mu} \end{cases}. \tag{8}$$

Figure 3 illustrates the components of the leader's payoff defined in Equation $(8)$. In the first case, the leader enters the market before the market maturity date $\tau$ is realized but the market switches before the follower's optimal entry date, *i.e.*, $\tau<\underline{t}_{\nu|\mu}$. In this case, the leader will monopolize the whole market and receive a payoff as the sum of two parts: a discounted profit stream, $\pi(t,\tau,\mu,0)$, before the market switches, and a discounted monopoly continuation payoff $W(\mu|0)$ in the declining stage.

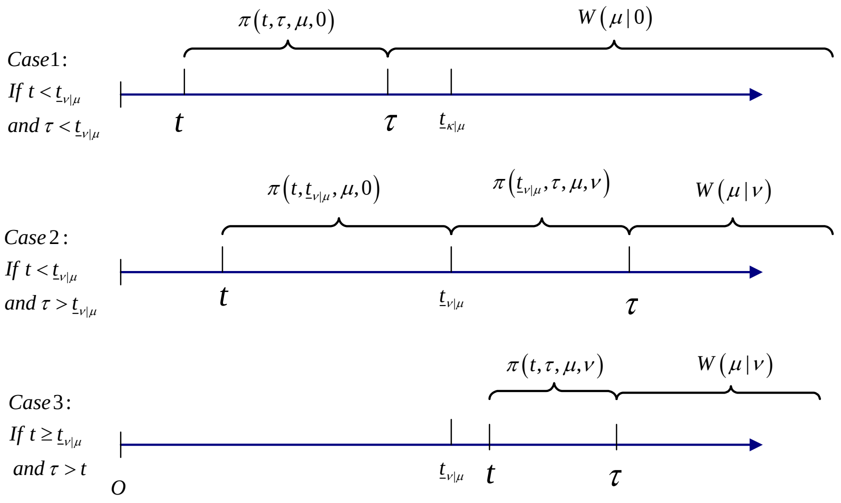

Figure 3: Leader's Payoff

In the second case where the market matures later than $\underline{t}_{\nu|\mu}$, the leader's monopoly position ends at $\underline{t}_{\nu|\mu}$ due to the follower's entry. Its payoff before date $\underline{t}_{\nu|\mu}$ is $\pi\left(t,\underline{t}_{\nu|\mu},\mu,0\right)$ while its payoff after $\underline{t}_{\nu|\mu}$ consists of two parts: $\pi\left(\underline{t}_{\nu|\mu},\tau,\mu,\nu\right)$ is the duopoly profit before the market switches, while $W\left(\mu|\nu\right)$ is its continuation payoff after the market switches. This case is characterized in the middle of Figure 3. Finally, in the third case the leader chooses $t\geq\underline{t}_{\nu|\mu}$ and the follower enters the market immediately with capacity $\nu$. In this case, the market becomes a duopoly right after the leader's entry.

If the leader enters at $t<\underline{t}_{\nu|\mu}$, we can rewrite the leader's payoff function as

$$L\left(t,\mu|\nu,t_0\right)=M\left(t,\mu|t_0\right)-A\left(\mu|\nu,t_0\right),$$

where $A\left(\mu|\nu,t_0\right)=\int_{\underline{t}_{\nu|\mu}}^{\infty}f\left(\tau|t_0\right)\left[\int_{\underline{t}_{\nu|\mu}}^{\tau}y_0e^{\alpha s}\left[D_n\left(\mu\right)-D_n\left(\mu+\nu\right)\right]\mu e^{-rs}ds+e^{-r\tau}\left[W\left(\mu|0\right)-W\left(\mu|\nu\right)\right]\right]d\tau$. Recall that $M\left(t,\mu|t_0\right)$ is the expected monopoly profit for a firm with capacity $\mu$ to enter at date $t$, which is strictly quasi-concave over $t$ and maximized at $t^m\left(\mu\right)$. In addition, observe that $A\left(\mu|\nu,t_0\right)$ is independent of the leader's entry date $t$. Therefore, $L\left(t,\mu|\nu,t_0\right)$ is also strictly quasi-concave over $t$ on the interval $\left[0,\underline{t}_{\nu|\mu}\right]$, reaching its local maximum at $t^m\left(\mu\right)$. This local maximum is also the global maximum if it is also greater than the leader's maximum payoff when it enters after $t\geq\underline{t}_{\nu|\mu}$. In the latter case, the market becomes a duopoly right after the leader's entry. Its payoff in the duopoly market has been derived in Section 4.1.[19] If the maximum duopoly payoff is smaller than $L\left(t^m\left(\mu\right),\mu|\nu,t_0\right)$

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[19] When a follower with capacity $\nu$ follows the leader with capacity $\mu$ to enter the market immediately, the leader's payoff

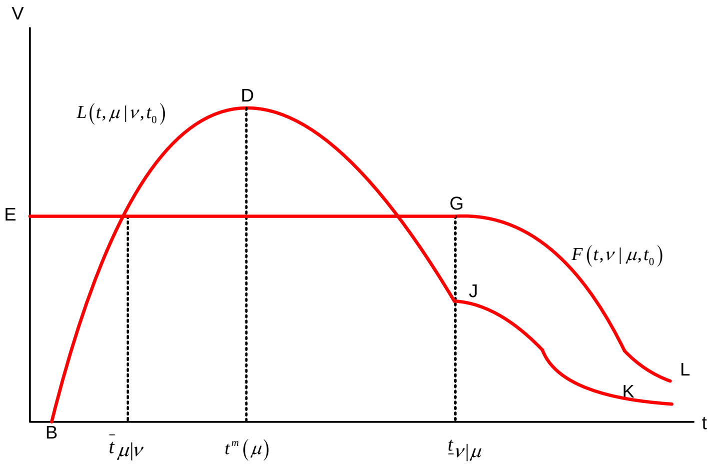

Figure 4: Leader and follower's payoff function

when it enters at $t^m(\mu)$, then the leader prefers to enter alone first.

Figure 4 illustrates the leader and the follower's payoff functions when diffusion outcome is possible. Without loss of generality, assume firm 1 is the leader with capacity $\mu$ and firm 2 is the follower with capacity $\nu$. Firm $1's$ (the leader's) payoff function $L(t, \mu|\nu, t_0)$ is represented by curve $BDJK$, which consists of two parts connected at $\underline{t}_{\nu|\mu}$. Note that the part with $t < \underline{t}_{\nu|\mu}$ is maximized at $t^m(\mu)$, the monopoly entry date of a firm with capacity $\mu$. The second part starts at $\underline{t}_{\nu|\mu}$, which is the optimal entry date for a follower with capacity $\nu$. On the other hand, firm $2's$ (the follower's) payoff function $F(t, \nu|\mu, t_0)$ is represented by curve $EGL$. It is flat for $t \leq \underline{t}_{\nu|\mu}$ because the follower's payoff is not a function of the leader's entry date if the leader enters before $\underline{t}_{\nu|\mu}$. However, if the leader enters at $t > \underline{t}_{\nu|\mu}$, the follower will enter immediately after the leader. In this case, $F(t, \nu|\mu, t_0)$ is decreasing for $t > \underline{t}_{\nu|\mu}$.

Given the shape of the leader and the follower's payoff function, we can analyze the leader's optimal entry strategy. We will proceed in two steps. First, we let the firms choose only the entry timing by fixing the leader capacity and not allowing the follower to contest the leadership. After we analyze this game of timing, we will move on to study the full game by relaxing two restrictions—allowing the leader to choose its capacity freely and the follower to contest the leadership.

Now assume whoever enters first chooses capacity $\mu$, leaving the other firm to select the follower's optimal capacity according to Proposition 4, say $\nu$. In this case, because the capacities are predetermined,

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is the same as if it were the "follower" of its competitor.

both firms are competing in entry timing only, similar to the preemption game studied by Fudenberg and Tirole (1985) . The following lemma characterizes the diffusion equilibrium in this subgame.

**Lemma 2** *Suppose Assumptions 1-2 hold and no leadership contest is allowed. If the leader must choose capacity* $\mu$*, inducing a noncontesting follower to choose capacity* $\nu$ *and there exists* $\bar{t}_{\mu|\nu} < t^m(\mu)$ *such that* $L\left(\bar{t}_{\mu|\nu}, \mu|\nu, 0\right) = F\left(\bar{t}_{\mu|\nu}, \nu|\mu, 0\right)$ *, then there exists a unique equilibrium outcome where the leader enters at* $\bar{t}_{\mu|\nu}$ *and the follower enters at* $\underline{t}_{\nu|\mu}$*.*

Lemma 2 defines the equilibrium in terms of the leader and the follower without specifying their identity because two firms are *ex ante* identical. This is the rent equalization equilibrium: the leader enters at the date when its payoff is equal to that of the follower.[20] As shown in Figure 4, no firm is willing to enter earlier than $\bar{t}_{\mu|\nu}$, because it could earn higher payoff as a follower even if it were preempted before $\bar{t}_{\mu|\nu}$. On the other hand, the first mover could not have entered after $\bar{t}_{\mu|\nu}$, because its competitor can move slightly ahead to preempt it. As a result, the only pure strategy equilibrium is that the leader enters at $\bar{t}_{\mu|\nu}$ and the follower enters at $\underline{t}_{\nu|\mu}$. Finally, the condition $\bar{t}_{\mu|\nu} < t^m(\mu)$ implies $L(t^m(\mu), \mu|\nu, 0) > F(t^m(\mu), \nu|\mu, 0)$, which is necessary for the diffusion equilibrium, otherwise firms have no incentive to become the first mover.

With this result, we can move on to solve our general capacity preemption game, which includes two additional strategic choices: first, the leader chooses its own capacity; second, the follower can contest the first entrant's leadership.

Let $\bar{t}(t_0)$ be the leader's entry date that satisfies the rent equalization condition $L(\bar{t}(t_0), \mu|\nu, t_0) = F(\bar{t}(t_0), \nu|\mu, t_0)$ . In the proof of Lemma 2 we show that $\bar{t}(t_0) = \bar{t}_{\mu|\nu}$ for all $t_0 < \bar{t}_{\mu|\nu}$. In other words, $\bar{t}$ is independent of $t_0$. This is because the expected payoffs for both the leader and the follower at date $t_0$ are their expected payoffs at date 0 multiplied by $e^{\lambda t_0}$, thus the rent equalization date is the same for all $t_0 < \bar{t}_{\mu|\nu}$. This implies that we can solve the leader's optimal entry timing based on firms' payoff function at date 0. To save some space, we abuse the notation by omitting the current date in the leader and the follower's payoff functions in the rest of discussions. In other words, we drop $t_0$ by writing $L(t, \mu|\nu, t_0)$ as $L(t, \mu|\nu)$ and $F(t, \nu|\mu, t_0)$ as $F(t, \nu|\mu)$ .

We define $L^*(\mu|\nu) = L(t^m(\mu), \mu|\nu)$ as the maximum payoff if the leader enters at $t < \underline{t}_{\nu|\mu}$. This is the payoff level at point $D$ in Figure 4. In addition, recall that we have defined $F^*(\nu|\mu)$ , the

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[20] Fudenberg and Tirole (1985) first propose the concept of rent equalization equilibrium. They prove a version in terms of the technology adoption timing.

noncontesting follower's maximum payoff, in Equation (6), which is the payoff level at point $G$ in Figure 4. If $L^*(\mu|\nu) \leq F^*(\nu|\mu)$, no firm has an incentive to preempt its competitor because its payoff as a leader is always smaller than that of the follower. Since our main question is how firms compete for the Stackelberg advantage, we maintain the following assumption throughout the rest of the paper:[21]

**Assumption 3** $L^*(\mu|\nu) > F^*(\nu|\mu)$ : *A first mover advantage exists.*

Given $\alpha > \alpha_S$, we know the leader and the follower will choose different capacities: one firm selects $S$ and the other will select $B$ and vice versa. There are two cases to consider depending on the relationship between $L^*(B|S)$ and $L^*(S|B)$.

Let us start with the case $L^*(B|S) > L^*(S|B)$. In this case, the maximum payoff for a large leader is greater than that of a small leader, thus the leader has an incentive to pursue large capacity preemption. Again, there are two subcases depending on the follower's payoffs. In the first subcase, $F^*(B|S) > F^*(S|B)$ implies that the follower prefers to be preempted by a small rather than a large leader. The second subcase is when the inequality sign is reversed. The first subcase is more interesting because the leader and the follower's incentives are in conflict: while the leader prefers to enter with large rather than small capacity, the follower prefers the opposite. This relationship between the leader and the follower's capacities is prevalent in previous literature on capacity competition, so we will focus on this subcase first and study the opposite case later.

Because there are two possible market configurations—a large leader with a small follower or a small leader with a large follower, we can draw their payoff functions on the same figure. As illustrated in Figure 5, we assume that the curve of $L(t, S|B)$ and $L(t, B|S)$ intersect at point $X$.[22] Even if this intersection point exists, the exact location of $X$ is ambiguous. Specifically, if $L(t, B|S)$ and $L(t, S|B)$ intersects at $t^*$, define $L_X = L(t^*, B|S) = L(t^*, S|B)$. Depending on the location of $t^*$, we need to consider three cases: $L_X \geq F^*(B|S) > F^*(S|B)$, $F^*(B|S) > L_X > F^*(S|B)$, and $F^*(B|S) > F^*(S|B) \geq L_X$. In fact, the last case also includes a situation when $L(t, S|B)$ and $L(t, B|S)$ never intersects at the positive quadrant, as will be clear in the discussions below. The first and the second cases are illustrated in Figure 5.

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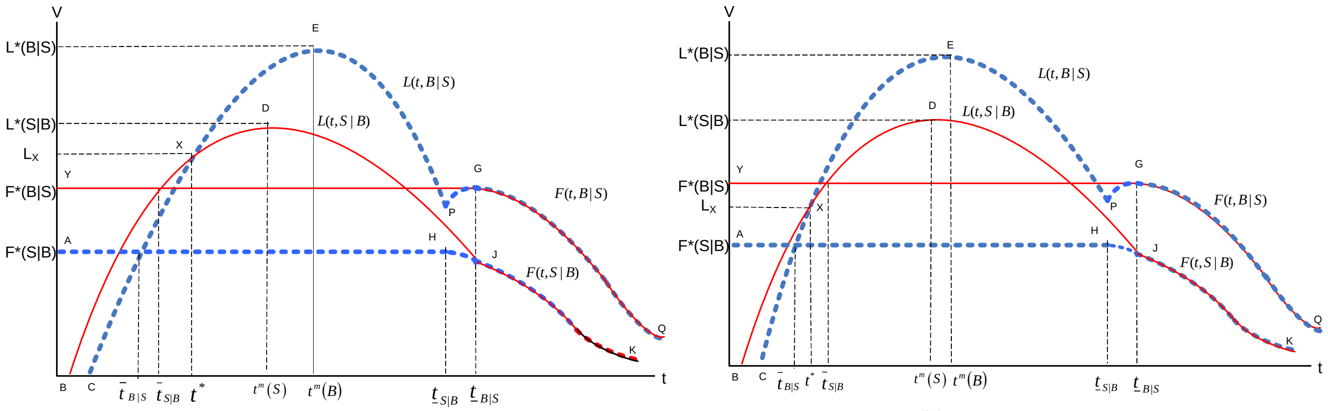
[21] This assumption assumes two inequalities simultanesouly hold: $L^*(L|S) > F^*(S|L)$ and $L^*(S|L) > F^*(L|S)$. The case that only one inequality holds is possible but provides little additional insight and is thus omitted.

[22] See Lemma 3 in the Appendix, which lays out the set of conditions for existence and uniqueness of the intersection point $X$.



(a) $L_X \geq F^*(B|S) > F^*(S|B)$

(b) $F^*(B|S) > L_X > F^*(S|B)$

Figure 5: Leader and follower's payoff functions

### Case 1: $L_X \geq F^*(B|S) > F^*(S|B)$

This case is illustrated in Panel $(a)$ of Figure 5. In this case, the leader's optimal entry capacity is $S$ if $t \leq t^*$; $B$ if $t > t^*$. Observe that entering with capacity $B$ is not feasible because no firm will wait until $t^*$ to enter the market otherwise its competitor will preempt it by entering earlier than $t^*$ with capacity $S$. As a result, the only feasible entry capacity for the leader is $S$, leaving the follower to invest in capacity $B$ later. According to Lemma 2, the leader will enter at $\bar{t}_{S|B}$ with capacity $S$ while the follower will wait until $\underline{t}_{B|S}$ to enter with capacity $B$. In addition, once the leader enters with capacity $S$, the follower has no incentive to contest the leadership as implied by Proposition 5.

### Case 2: $F^*(B|S) > L_X > F^*(S|B)$

The second case that $F^*(B|S) > L_X > F^*(S|B)$ is illustrated in Panel $(b)$ of Figure 5. In this case, no firm would enter with capacity $B$ before $t^*$, because its payoff is strictly lower than that of small capacity. This effectively removes the rent equalization outcome as an equilibrium with leader entering at $\bar{t}_{B|S}$ with capacity $B$ and the follower entering at $\underline{t}_{S|B}$ with capacity $S$, because the leader can deviate to capacity $S$ for higher expected payoff. However, for $t > t^*$, the leader is better off by preempting its competitor with large capacity. But if the leader enters the market with capacity $B$, the follower may want to start a leadership contest to force out the large leader. However, according to Proposition 5, it will succeed if and only if the large leader enters at a date $t < t_B^c$. $t_B^c$ is derived in the proof of Proposition 5. $t_B^c$ is not shown in Figure 5 because its exact location depends on parameter values.

However, depending on the relationship between $t_B^c$, $t^*$ and $\bar{t}_{S|B}$, there are three cases to consider.

If $t_B^c \leq t^*$, there is no pure strategy equilibrium. To understand this, note that the leader will not enter the market before $t^*$. Once the date passes $t^*$, the leader prefers large capacity without worrying about being forced out as $t_B^c < t^*$. To see this, let us assume $p = \frac{1}{2}$ so that no firm has any advantage *ex ante*. Competition seems to induce both firms to enter the market at $t^*$, leaving each firm an expected payoff of $\frac{1}{2}\left(L_X + F^*\left(S|B\right)\right).$ However, continuity of the payoff function over $t$ implies that one firm can deviate by entering slightly earlier with capacity $S$ and achieve a payoff close to $L_X$ but still strictly greater than $\frac{1}{2}\left(L_X + F^*\left(S|B\right)\right).$ On the other hand, entering with capacity $S$ first before $t^*$ is not optimal because the firm would rather become the follower with capacity $B$ to earn a higher payoff. In a word, there is no pure strategy equilibrium in this case.

Now suppose $t^* < t_B^c < \bar{t}_{S|B}$. Let us maintain the assumption that $p = \frac{1}{2}$. First of all, no firm is willing to enter at $t \in (t^*, t_B^c)$ with capacity $B$, because the second mover can challenge the first mover's leadership, in which case the follower will succeed and earn a monopoly payoff $M\left(t, S\right),$ a payoff that is strictly higher than that of a Stackelberg follower because $M\left(t, S\right) > L\left(t, S|B\right) > F\left(t, S|B\right)$. That is, large capacity preemption is not credible until date $t_B^c$. However, both firms are willing to wait until $t_B^c$ to compete for the position of the large leader only if $\frac{1}{2}\left(L\left(t_B^c, B|S\right) + F\left(S|B\right)\right) \geq L\left(t_B^c, S|B\right)$, otherwise, the incentive to preempt prompts one firm to move ahead of the other in order to avoid the lottery of the leadership. This inequality implies that firms are better off participating in the leadership lottery at $t_B^c$ rather than preempting a competitor by entering earlier as a smaller firm. Note that in this equilibrium the *ex post* leader earns strictly higher payoff than its follower.

In the third case, if $\bar{t}_{S|B} < t_B^c$, then no firm is willing to enter as a large firm before $\bar{t}_{S|B}$, because the follower will enter with a small firm to force it out of the market. In addition, the small leader and the large follower's payoffs are equalized when the leader enters at $\bar{t}_{S|B}$. As a result, the leader has no incentive to deviate and the follower does not want to contest, thus it is an equilibrium is for the leader to enter at $\bar{t}_{S|B}$ with capacity $S$ and the follower enters at $\underline{t}_{B|S}$ with capacity $B$.

**Case 3:** $F^*\left(B|S\right) > F^*\left(S|B\right) \geq L_X$

This case is illustrated in Figure 6. First, we claim that if $\bar{t}_{B|S} > t_B^c$, it is an equilibrium for the leader to enter at $\bar{t}_{B|S}$ with capacity $B$ and the follower to enter at $\underline{t}_{S|B}$ with capacity $S$. This rent equalization

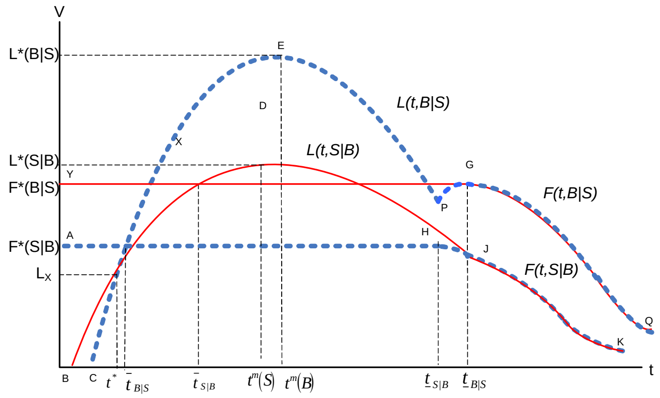


Figure 6: $F^*(B|S) > F^*(S|B) \geq L_X$

equilibrium is the result of two observations. First, no firm is willing to enter earlier than $\bar{t}_{B|S}$. Second, once a firm enters at $\bar{t}_{B|S}$ with capacity $B$, the follower cannot contest its leadership as $\bar{t}_{B|S} > t^c_B$.

However, if $\bar{t}_{B|S} < t^c_B \leq \bar{t}_{S|B}$, the leader could not enter with capacity $B$ at $\bar{t}_{B|S}$ if $M\left(\bar{t}_{B|S}, S\right) > F\left(\bar{t}_{B|S}, S|B\right)$ because the follower will simply force out the leader so that it can monopolize the rest of the market. Therefore, if $M\left(\bar{t}_{B|S}, S\right) \leq F\left(\bar{t}_{B|S}, S|B\right)$, it is an equilibrium for the leader to enter with capacity $B$ and the follower to enter with capacity $S$ with their payoff equalized. This condition removes the follower's incentive to challenge as its payoff is smaller even if it could force out the leader by investing in capacity $S$.

Finally, if $\bar{t}_{S|B} < t^c_B$, then there exist another equilibrium: the leader enters at $\bar{t}_{S|B}$ with capacity $S$ and the follower enters at a $\underline{t}_{B|S}$ with capacity $B$. If one firm tries to enter with capacity $B$ at $t < \bar{t}_{S|B}$, it will be forced out by the follower with capacity $S$. In this equilibrium both the leader and the follower are earning higher payoff than the equilibrium in which the leader enters with capacity $B$ at $\bar{t}_{B|S}$ and the follower enters with capacity $S$ at $\underline{t}_{S|B}$. This implies that the possibility of a leadership contest in fact improves industry profit as both firms are becoming more cautious.

In summary, we have the following proposition:

**Proposition 6** *Let Assumption 1-3 hold. Suppose* $\alpha > \alpha_S$ *and there exists* $t^* < t^m(S)$ *such that* $L(t^*, B|S) = L(t^*, S|B) = L_X$. *It is an equilibrium that*

$(i)$ *the leader enters at* $S$ *at* $\bar{t}_{S|B}$ *and the follower enters at* $\underline{t}_{B|S}$ *if one of the following condition is satisfied*

$(a)$ $L^*(B|S) \leq L^*(S|B)$

*(b)* $L^*(B|S) > L^*(S|B)$ *and either* $L_X \geq F^*(B|S) > F^*(S|B)$ *or* $\bar{t}_{S|B} < t_B^c$;

*(ii) the leader enters with capacity* $B$ *at* $\bar{t}_{B|S}$ *and the follower enters with capacity* $S$ *at* $\underline{t}_{S|B}$ *if* $L^*(B|S) > L^*(S|B)$, $F^*(B|S) > F^*(S|B) \geq L_X$ *and either* $\bar{t}_{B|S} \geq t_B^c$ *or* $M\left(\bar{t}_{B|S}, S\right) \leq F\left(\bar{t}_{B|S}, S|B\right)$.

*(iii) if* $p = \frac{1}{2}$, *the leader enjoys a first mover advantage if the following conditions are satisfied:* $L^*(B|S) > L^*(S|B)$ *and* $F(B|S) > L_X > F(S|B)$ , $t^* < t_B^c < \bar{t}_{S|B}$ *and* $\frac{1}{2}\left(L\left(t_B^c, B|S\right) + F(S|B)\right) \geq L\left(t_B^c, S|B\right)$.

Proposition 6 identifies three types of equilibrium outcomes: parts $(i)$ and $(ii)$ are rent equalization equilibria with firms choosing different capacities while part $(iii)$ is the large capacity preemption outcome. Those equilibria are jointly determined by two types of conditions: feasibility and credibility. The conditions on the relationship between $L^*(B|S)$ and $L^*(S|B)$ determines whether the large capacity preemption is feasible while the conditions on the location of $t^*$ determines when it becomes feasible. The comparison between the rent equalization date with $t_B^c$ determines whether the large capacity preemption is credible while comparison of the monopoly payoff to the follower payoff determines the incentive to contest. Each equilibrium is a result of a combination of those conditions.

To our surprise, in almost every case we have discussed, the leader preempts its follower with a small rather than a large capacity. This is in stark contrast with the predictions by the previous literature on endogenous Stackelberg leadership. In the first case with $L^*(B|S) \leq L^*(S|B)$, large capacity preemption is simply not feasible: because the small leader's payoff is strictly greater than large leader for all feasible entry timing for the leader. It is no surprise that the first entrant will enter with a small capacity. However, in the second case with $L^*(B|S) > L^*(S|B)$, large capacity preemption is indeed feasible at least after some date. The surprising result is that the leader still wants to enter small in equilibrium. This is because large capacity preemption is not credible. Had it entered first with a large plant, it would have been forced out by the follower using a leadership contest. The small follower is able to win the war of attrition against its large leader because it has a higher option value that delays its exit. This option value comes from its advantage from the declining industry, in which it also has more incentive to stay as shown by Ghemawat and Nalebuff (1985).

On the other hand, the follower does not always force out the large leader whenever it is feasible. For example, if $F^*(B|S) > F^*(S|B) > L_X$, the leader is able to preempt its follower with large capacity without the possibility of being contested by its follower if the monopoly payoff resulting from forcing out the leader is no larger than the payoff as a follower. Thus it is not because the follower cannot contest the leadership, but because the follower cannot improve its payoff through a leadership contest and thus has no incentive to start the war of attrition. Furthermore, in this case, if $\bar{t}_{S|B} < t^c_B$, there is another symmetric equilibrium where no firm enters until $\bar{t}_{S|B}$. One firm succeeds in investing in a small plant and leaves its competitor to build a large plant later. This equilibrium outcome is better than the large leader small follower outcome because both firms are able to earn a higher payoff. In this sense, allowing the follower to contest the market leadership in fact improves industry profit and reduces the loss of option value due to competition. The large firm can fully appreciates the option value while the small leader suffers less because it can enter closer to the date that fully incorporates the option value.

Given those results, a natural question is when large capacity preemption becomes an equilibrium outcome with the leader earning higher payoff, the result predicted by the previous literature on Stackelberg leadership. The only case in which the leader can enjoy the benefit of large capacity preemption is when the multiple conditions of part (*iii*) in Proposition 6 are satisfied. In this case, no firm is willing to enter until $t^c_B$, at which date the large capacity preemption is immediately feasible and credible. Both firms will attempt to enter the market simultaneously as the leader, and whoever wins the leadership will be able to earn higher payoff than its follower. This result coincides with previous literature: if large capacity preemption is feasible and credible at the beginning of the market, the first entrant is able to enjoy first mover advantage, earning higher payoff than its follower.

This result reveals two critical conditions implicitly assumed in the previous literature on Endogenous leadership. First, large capacity preemption has to be the immediately feasible; Secondly, large capacity preemption has to be credible in terms of defending its position from leadership contest. Previous papers based on a two-stage model implicitly assume the immediate feasibility of large capacity preemption. This is necessary but not sufficient. The condition of credible large capacity preemption is trivially satisfied in the previous papers because they remove the possibility of leadership contest. However, as shown in this paper, those two conditions turn out to be critical in generating first mover advantage. In reality, large capacity preemption is so rare because either the market is very premature or it has no commitment power to survive the leadership contest.

Finally, we briefly discuss the remaining case with $F^*(S|B) > F^*(B|S)$, the case that a noncontesting follower prefers to be preempted by a large leader. This case could happen if the continuation payoff from the declining industry is large enough that the follower prefers to invest in small capacity.[23]

**Proposition 7** *Given Assumption 1-3. Suppose* $\alpha > \alpha_S$, $L^*(B|S) > L^*(S|B)$ *but* $F^*(S|B) > F^*(B|S)$, *and there exists* $t^* < t^m(S)$ *such that* $L(t^*, B|S) = L(t^*, S|B) = L_X$. *It is an equilibrium that*

*(i) the leader enters at* $\bar{t}_{S|B}$ *with capacity* $S$ *and the follower enters at* $\underline{t}_{B|S}$ *with capacity* $B$ *if one of the following conditions are satisfied:*

*(a)* $L_X \geq F^*(B|S)$

*(b)* $F^*(B|S) > L_X$ *and* $\bar{t}_{S|B} < t^c_B$;

*(ii) if* $F^*(S|B) > L_X > F^*(B|S)$, *then the leader enters at* $\bar{t}_{B|S}$ *with capacity* $B$ *and the follower enters at* $\underline{t}_{S|B}$ *provided* $\bar{t}_{B|S} \geq t^c_B$ *or* $L\left(\bar{t}_{B|S}, B|S\right) > M\left(\bar{t}_{B|S}, S\right)$;

*(iii) if* $p = \frac{1}{2}$, *then the leader enjoys first mover advantage only if the following conditions are satisfied:* $L_X < F^*(S|B)$, $t^c_B < \bar{t}_{S|B}$, $\frac{1}{2}\left(L\left(t^c_B, B|S\right) + F^*(S|B)\right) > L\left(t^c_B, S|B\right)$ *and* $L(t, B|S) < M(t, S)$ *for all* $t \in \left[\bar{t}_{B|S}, t^c_B\right]$.

Even if the noncontesting follower prefers to be preempted by a large firm, large capacity preemption will not benefit from this preference because competition for entry timing drives away the potential advantage. However, the leader does benefit from this case because it does not need to worry about the potential leadership contest, as the leader earns no more than the follower.

# 5 Conclusion

This paper sheds light on why firms lack incentive to pursue large capacity preemption in a market with uncertainty. In particular, unlike previous literature that claims preemption destroys the option value, we are able to show that the desire for flexibility and the associated option value itself might change a firm's incentive to preempt. In particular, the option value generates a strategic advantage for the firm with smaller capacity, giving it more incentive to preempt its competitor and higher commitment power to fight for the leadership in the presence of a leadership contest. The paper employs a pretty stylized model with two feasible capacities, but we believe the main conceptual result still holds when additional

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[23] The proof is lengthy but follows a strategy similar to that of of Proposition 6 by examining different cases. It is omitted but is available upon request from the author.

capacity levels are allowed. Above all, we only need to check two conditions to determine a firm's entry capacity: whether or not the large capacity preemption is feasible and credible. A credibility check relies on two conditions: whether or not the leader's capacity level is so large that it can be forced out by the follower and whether the follower has the incentive to force the leader out of the market.

This paper contributes to the burgeoning literature on industry dynamics with uncertainty. Although the model is pretty simple and stylistic, it leaves many unanswered questions that are worth pursuing. First, firms might update their information about the future of the market as they continue their operation, which might change their incentive to exit when challenged by its competitors. Therefore, an interesting question is whether the learning effect works to strengthen or reduce a firm's incentive to pursue large capacity preemption. Second, although we are focusing on capacity preemption in this paper, the model can be extended to study other related questions that generate additional payoff for the first mover, such as R&D competition.

# 6 Appendix

## 6.1 Proof of Lemma 1

**Proof.** Differentiating $M\left(t, \mu | t_0\right)$ with respect to $t$ and setting it equal to zero and rearranging the terms, we have

$$P\left(t^m\left(\mu\right), \mu\right) - c = r\kappa + \lambda\left(\kappa - m\right), \tag{9}$$

where $P\left(t^m\left(\mu\right), \mu\right) = y_0 e^{\alpha t^m(\mu)} D_n\left(\mu\right)$ as defined in Section 2. In addition, solving equation (9) gives $t^m\left(\mu\right) = \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(\mu)}$. It is routine to check that the second order condition is also satisfied. The strict quasi-concavity of $M\left(t, \mu | t_0\right)$ implies the monopoly firm will wait until $t^m\left(\mu\right)$ to enter the market if $t_0 < t^m\left(\mu\right)$, otherwise it will invest immediately. Part $(ii)$ follows from the fact that $D_n\left(\mu\right)$ is decreasing in $\mu$. ■

## 6.2 Proof of Proposition 2

**Proof.** We have $M^*\left(\mu | t_0\right) = e^{\lambda t_0}\mu\left(\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(\mu)}\right)^{-\frac{\lambda+r}{\alpha}}\left(\frac{\alpha(c+(\lambda+r)\kappa-\lambda m)}{(\lambda+r-\alpha)(\lambda+r)}\right)$ after substituting $t^m\left(\mu\right)$ into equation $(2)$. Hence, the monopoly will invest in capacity $B$ if and only if $D_n\left(S\right)^{\frac{\lambda+r}{\alpha}} S < D_n\left(B\right)^{\frac{\lambda+r}{\alpha}} B$, which is equivalent to $\alpha > \frac{(\lambda+r)(\ln D_n(S)-\ln D_n(B))}{\ln B-\ln S} = \bar{\alpha}$. ■

## 6.3 Proof of Corollary 1

**Proof.** From the proof of Proposition 2, we have

$$M^*\left(\mu|t_0\right) = e^{\lambda t_0}\alpha y_0^{\frac{\lambda+r}{\alpha}}\left(c+(\lambda+r)\kappa-\lambda m\right)^{1-\frac{\lambda+r}{\alpha}} D_n\left(\mu\right)^{\frac{\lambda+r}{\alpha}}\mu \tag{10}$$

Substituting $D_n(\mu) = a - b\mu$ in to equation $(10)$, differentiating it with respect to $\mu$ and setting it equal to zero, we have $\mu = \frac{a\alpha}{b(\lambda+r+\alpha)}$. It is routine to check the second order condition. Substituting the expression of $\mu$ back into the expression for $t^m(\mu)$, we have $t^m(\mu) = \frac{1}{\alpha}\ln\frac{(c+(\lambda+r)\kappa-\lambda m)(\lambda+r+\alpha)}{y_0 a(\lambda+r)}$. ■

## 6.4 Proof of Proposition 3

**Proof.** Differentiating (4) with respect to $t$, setting it equal to zero, and rearranging the terms, we have

$$P_n\left(t,\mu+\nu\right) - c = r\kappa + \lambda\left(-\frac{W\left(\nu|\mu\right)-\kappa\nu}{\nu}\right) \tag{11}$$

Substituting $P_n(t,\mu+\nu) = y_0 e^{\alpha t} D_n(\mu+\nu)$ into $(11)$, we have $\underline{t}_{\nu|\mu} = \frac{1}{\alpha}\ln\frac{c+r\kappa-\lambda\left(-\frac{W(\nu|\mu)-\kappa\nu}{\nu}\right)}{y_0 D_n(\mu+\nu)}$. Note that $\underline{t}_{\nu|\mu}$ is not a function of $t_0$. Therefore, if $t_0 \le \underline{t}_{\nu|\mu}$, the follower will enter at date $\underline{t}_{\nu|\mu}$, otherwise, it will enter immediately at $t_0$. ■

## 6.5 Proof of Corollary 3

**Proof.** If the leader chooses capacity $S$, we also have

$$\begin{aligned}
t^f\left(S|S\right) &= \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa}{y_0 D_n(2S)} > \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(S)} = t^m(S), \\
t^f\left(B|S\right) &= \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa}{y_0 D_n(S+B)} > \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(S)} = t^m(S).
\end{aligned}$$

If the leader chooses capacity $B$, we have

$$\begin{aligned}
t^f\left(S|B\right) &= \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(S+B)} > \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(B)} = t^m(B), \\
t^f\left(B|B\right) &= \infty > \frac{1}{\alpha}\ln\frac{c+(\lambda+r)\kappa-\lambda m}{y_0 D_n(B)} = t^m(B).
\end{aligned}$$

The second inequality follows from $D_n(2B) < 0$, so a follower with capacity $B$ will never enter the market when the leader is $B$. ■

## 6.6 Proof of Proposition 4

**Proof.** Substituting $\underline{t}_{\nu|\mu}$ into Equation (4), we have

$$V_f\left(\underline{t}_{\nu|\mu}, \nu|\mu, t_0\right) = e^{\lambda t_0}\left(\frac{c + (\lambda + r)\kappa - \lambda\frac{W(\nu|\mu)}{\nu}}{y_0 D_n(\mu+\nu)}\right)^{-\frac{\lambda+r}{\alpha}} \frac{\alpha\left(c + (\lambda + r)\kappa - \lambda\frac{W(\nu|\mu)}{\nu}\right)}{(\lambda + r - \alpha)(\lambda + r)}\nu.$$

First, let us consider the case with leader choosing $S$. We have $\frac{V_f\left(\underline{t}_{S|S}, S|S, t_0\right)}{V_f\left(\underline{t}_{B|S}, B|S, t_0\right)} = \left(\frac{D_n(S+B)}{D_n(2S)}\right)^{-\frac{\lambda+r}{\alpha}}\frac{S}{B}$, where have used the condition that $W(S|S) = W(B|S) = 0$. Consequently, $V_f\left(\underline{t}_{S|S}, S|S, t_0\right) \geq V_f\left(\underline{t}_{B|S}, B|S, t_0\right)$ if and only if $\left(\frac{D_n(S+B)}{D_n(2S)}\right)^{-\frac{\lambda+r}{\alpha}}\frac{S}{B} \geq 1$, which is equivalent to $\alpha \leq \frac{(\lambda+r)(\ln D_n(2S) - \ln D_n(S+B))}{\ln B - \ln S} = \alpha_S$.

If the leader chooses capacity $B$, because $D_n(2B) < 0$, the follower always chooses capacity $S$. Therefore, if $\alpha \leq \alpha_S$, the follower always invests in small capacity $S$ regardless of the leader's capacity choice. On the other hand, if $\alpha > \alpha_S$, the follower will choose $S$ if the leader is $B$ but will choose $B$ if the leader invests in $S$. ■

## 6.7 Proof of Proposition 5

**Proof.** Differentiating Equation (7) with respect to $t_i^c$, we have

$$\frac{\partial V_c}{\partial t_i} = e^{-(\lambda+r)t_i + \lambda t_0} q_i\left(\lambda\frac{W(q_i|q_j)}{q_i} - (\lambda + r)\gamma + y_0 e^{\alpha t_i} D_c(q_1 + q_2) - c\right)$$

Setting the first order condition to zero, we have $t_i^* = \frac{1}{\alpha}\ln\frac{c + r\gamma - \lambda\left(\frac{W\left(q_i|q_j\right)}{q_i} - \gamma\right)}{y_0 D_c(q_1+q_2)}$. Furthermore, $V_c$ is strictly quasi-convex, because its second order condition at $t_i^*$ is $\frac{\partial^2 V_i}{\partial\left(t_i^*\right)^2} = e^{-(\lambda+r)t_i^c + \lambda t_0} q_i \alpha e^{\alpha t_i^*} D_c(q_1 + q_2) > 0$. The quasi-convexity of $V_c$ implies that the firm either exits immediately or does not exit at all. If the firm exits immediately, its expected payoff is $V_c(q_i, t_0|q_j, t_0) = e^{-rt_0}\gamma q_i$. If the firm does not exit the market, its expected payoff is $V_c(q_i, \infty|q_j, t_0) = \int_{t_0}^{\infty} f(\tau|t_0)\left(\int_{t_0}^{\tau}(y_0 e^{\alpha s} D_c(q_1 + q_2) - c) q_i e^{-rs} ds + e^{-r\tau} W(q_i|q_j)\right) d\tau$. This implies that there exists $t_i^c$ such that for all $t < t_i^c \leq t_i^*$, $V_c(q_i, t_0|q_j, t_0) \geq V_c(q_i, \infty|q_j, t_0)$. That is, immediate exit is optimal. Therefore, $t_i^c$ solves $V_c(q_i, t_i^c|q_j, t_0) - V_c(q_i, \infty|q_j, t_0) = 0$.

Define $G(t_i,\gamma) = V_c(q_i, t_i^c|q_j, t_0) - V_c(q_i, \infty|q_j, t_0) = 0$. We have $\frac{\partial t_i^c}{\partial \gamma} = -\frac{\frac{\partial G}{\partial \gamma}}{\frac{\partial G}{\partial t_i}} > 0$, implying that $t_i^c$ is increasing in $\gamma$. Furthermore, observe that the solution $t_i^c$ is not a function of $t_0$. Because firm 1's capacity is $S$ and firm $2's$ capacity is $B$. Let us refer them as firm $S$ and firm $B$ respectively. We claim $t_S^c < t_B^c$. Suppose not, we have $t_S^c \geq t_B^c$, then we have

$$\begin{aligned} 0 &\leq \frac{V_c(S, t_B^c|B, t_0) - V_c(S, \infty|B, t_0)}{S} - \frac{V_c(B, t_B^c|S, t_0) - V_c(B, \infty|S, t_0)}{B} \\ &= \int_{t_B^c}^{\infty} f(\tau|t_0)\, e^{-r\tau} \left( \frac{W(B|S)}{B} - \frac{W(S|B)}{S} \right) d\tau < 0, \end{aligned}$$

which is a contradiction. As a result, firm $1's$ exit region is $[t_0, t_S^c]$ while firm 2's exit region is $[t_0, t_B^c]$. If $t_0 < t_B^c$, then firm 1 with capacity $S$ has a shorter exit region, so in a subgame perfect equilibrium, firm 1 stays and firm 2 exits the market immediately. ■

## 6.8 Proof of Lemma 2

**Proof.** Note that $\bar{t}_{\mu|\nu} < t^m(\mu)$ and it satisfies $L\left(\bar{t}_{\mu|\nu}, \mu|\nu, 0\right) = F\left(\bar{t}_{\mu|\nu}, \nu|\mu, 0\right)$. First of all, the leader and follower's payoff functions at date $t_0$ are

$$\begin{aligned} L(t, \mu|\nu, t_0) &= e^{\lambda t_0} \left[ e^{-(\lambda+r)t} \left( \frac{y_0 D_n(\mu)}{\lambda + r - \alpha} e^{\alpha t} - \frac{c + \kappa(\lambda + r) - \lambda m}{\lambda + r} \right) \mu - \bar{A}(\mu|\nu) \right], \\ F(t, \nu|\mu, t_0) &= e^{\lambda t_0} F^*(\nu|\mu), \end{aligned}$$

where $\bar{A}(\mu|\nu) = \int_{\underline{t}_{\nu|\mu}}^{\infty} \lambda e^{-\lambda\tau} \left[ \int_{\underline{t}_{\nu|\mu}}^{\tau} y_0 e^{\alpha s} \left[ D_n(\mu) - D_n(\mu + \nu) \right] \mu e^{-rs} ds + e^{-r\tau} \left[ W(\mu|0) - W(\mu|\nu) \right] \right] d\tau$.

$$L(t, \mu|\nu, t_0) - F(t, \nu|\mu, t_0) = e^{\lambda t_0} \left[ e^{-(\lambda+r)t} \left[ \frac{y_0 D_n(\mu)}{\lambda + r - \alpha} e^{\alpha t} - \frac{c + \kappa(\lambda + r) - \lambda m}{\lambda + r} \right] \mu - \bar{A}(\mu|\nu) - F^*(\nu|\mu) \right].$$

Therefore $L\left(\bar{t}_{\mu|\nu}, \mu|\nu, 0\right) = F\left(\bar{t}_{\mu|\nu}, \mu|\nu, 0\right)$ implies that $L\left(\bar{t}_{\mu|\nu}, \mu|\nu, t_0\right) = F\left(\bar{t}_{\mu|\nu}, \mu|\nu, t_0\right)$ for all $t_0 < \bar{t}_{\mu|\nu}$. In addition, $\bar{t}_{\mu|\nu} < t^m(\mu)$ implies that $L(t^m(\mu), \mu|\nu, t_0) > F(t^m(\mu), \nu|\mu, t_0)$ for all $t_0 < t^m(\mu)$, as shown in Figure 4.

According to our assumption, whoever becomes the leader has to choose capacity $\mu$, inducing a noncontesting follower to choose capacity $\nu$, so there is no leadership contest. As a result, firms' strategy is a timing decision: a function from the current state to an action set of $\{0, 1\}$. In other words, firms are competing on entry timing only. Define firm $1's$ strategy as follows: for all $t < \bar{t}_{\mu|\nu}$, do not enter the market. Enter the market at $t^m(\mu) \geq t \geq \bar{t}_{\mu|\nu}$ with capacity $\mu$ only if its competitor has not entered, otherwise wait until date $\underline{t}_{\nu|\mu}$ to enter as a follower with capacity $\nu$. For $t > t^m(\mu)$, enter the market with capacity $\mu$ as long as $t \in T^L = \{\tau : L(\tau, \mu|\nu, t_0) \geq F(\tau, \nu|\mu, t_0)\}$, otherwise do not enter the market. Given Firm 1 adopts this strategy, Firm 2's best response to this strategy is the same. For $t < \bar{t}_{\mu|\nu}$, because $L(t, \mu|\nu, t_0) < F(t, \nu|\mu, t_0)$, so it should not enter the market. For $t > \bar{t}_{\mu|\nu}$, as long as the leader's payoff is higher than that of the follower, it should enter immediately, otherwise it will be preempted by firm 1. This is exactly the same strategy specified above. ■

## 6.9 The Intersection of $L(t, B|S)$ and $L(t, S|B)$

**Lemma 3** *If* $L(0, S|B) > L(0, B|S)$ *and* $L(t^m(S), S|B) < L(t^m(B), B|S)$ *, there exists a unique* $0 < t^* < t^m(B)$ *such that* $L(t^*, S|B) = L(t^*, B|S)$.

**Proof.** Because $L(0, S|B) > L(0, B|S)$ and $L(t^m_S, S|B) < L(t^m_B, B|S)$, combine with the continuity of function $L(\cdot, \mu|\nu)$ at $t$, we know there exists a $t^*$ such that $L(t^*, S|B) = L(t^*, B|S)$. In addition, note that for $t < t^m(\mu)$, $\frac{\partial \frac{L(t,S|L)}{S}}{\partial t} - \frac{\partial \frac{L(t,B|S)}{B}}{\partial t} < 0$. Hence, we must have $\frac{\partial L(t,S|B)}{\partial t} = \frac{\partial \frac{L(t,S|B)}{S}}{\partial t} S < \frac{\partial \frac{L(t,S|B)}{S}}{\partial t} B < \frac{\partial \frac{L(t,B|S)}{B}}{\partial t} B = \frac{\partial L(t,B|S)}{\partial t}$. That is, for all $t^m_B > t > t^*$, $L(t, B|S) > L(t, S|B)$ and for all $t < t^*$, $L(t, B|S) < L(t, S|B)$, which proves the uniqueness of $t^*$. ■

## 6.10 Proof of Proposition 6

**Proof.** We have discussed the case of $L^*(B|S) > L^*(S|B)$ in the main text. It suffices to consider the remaining case with $L^*(B|S) \leq L^*(S|B)$. We claim that the equilibrium is for the leader to enter with capacity $S$ and the follower to enter later with capacity $B$. First of all, Lemma 3 shows that the leader will never enter with capacity $B$, because the expected payoff of investing in capacity $S$ is always higher than investing in $B$ for all $t < t^m(S)$.[24] Consequently, the leader has a dominant strategy to enter with $S$, which will induce the follower to choose capacity $B$. According to Proposition 5, when the leader enters with capacity $S$, the follower will not be able to force the leader out of the market, so its choice of capacity $B$ and entering at $\underline{t}_{B|S}$ is indeed optimal even if a leadership contest is allowed. Consequently, according to Lemma 2, the unique subgame perfect equilibrium is for the leader to enter

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[24] Lemma 3 shows that $L^*(B|S) < L^*(S|B)$ implies $L(t, B|S) < L(t, S|B)$ for all $t < t^m(S)$. In other words, capacity $B$ is irrelevant because its payoff is always smaller than capacity $S$.

with capacity $S$ at date $\bar{t}_{S|B}$ and the follower to enter with capacity $B$ at date $\underline{t}_{B|S}$. Both firms' payoffs are equalized: $L\left(\bar{t}_{S|B}, S|B\right) = F\left(\underline{t}_{B|S}, B|S\right)$. ■

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