Astronomy 475/575: Homework 2

Be sure to show your work so that a) I can verify that you solved the problem correctly and b) I can give partial credit even if you don't reach the correct answer.

1. Assume a young star is surrounded by a disk. The disk is geometrically thin (i.e., it looks like a knife-edge if viewed edge-on), and extends from the stellar surface to some stellocentric radius, $R_{\rm out}$. Assume the central star has $M_* = 1$ M$_\odot$, $T_* = 4000$ K, and $R_* = 2$ R$_\odot$.

   (a) Assume that the disk is heated only by irradiation from the central star. Derive an expression for the temperature of the disk as a function of stellocentric radius, $T(R)$.

   (b) Use this expression to compute the disk temperature at 1 AU.

   (c) For an accretion disk, we showed in class that the temperature profile is

   $$T(R) = \left(\frac{G M_* \dot{M}}{4 \pi \sigma R^3} \right)^{1/4} . \qquad (1)$$

   Assume now that the geometrically thin disk is heated both by accretion and irradation of the central star. Determine the accretion rate for which heating by accretion becomes dominant.

   (d) Assume $\dot{M} = 10^{-6}$ M$_\odot$ yr^{-1}. What is the temperature of the disk at 1 AU?

   (e) Discuss the effect of this accretion heating on planet formation during the early evolution of the solar system.

2. (a) Assume that the cloud out of which the Sun formed had a mass of 1 M$_\odot$, a radius of 5500 AU, and that it rotated as a solid, spherical body, with a period of 10^6 yr. Calculate the angular momentum of the cloud.

   (b) Now assume that 0.5 M$_\odot$ is distributed in a geometrically thin Keplerian disk of constant surface density, Σ. Let the disk extend from $R = 0$ to $R = R_{\rm out}$. Assume that the disk orbits a central object of mass 0.5 M$_\odot$ (you may neglect self-gravity in the disk). Derive a value for Σ in terms of $R_{\rm out}$.

   (c) Determine an expression for $dM(R)$, which is the mass of a small annulus of the disk spanning dR. Hint: ensure that your expression can be integrated to give the correct total mass.

(d) Calculate the total angular momentum of the disk, in terms of R_{out}. For simplicity, you may assume that for each small annulus of the disk, dM, the angular momentum is $dM \times v \times R$.

(e) Determine the value of R_{out} for which the disk angular momentum is equal to the angular momentum of the cloud computed in part (a).

(f) Discuss the implications of your results for the formation of the Sun and the Solar System.