Astronomy 545: Homework 3

You may collaborate on all problems this week. Since they are computational, everyone should write/run their own code.

1. Solve the Lane-Emden equation numerically for $n = 1.5$ and $n = 3$. Write your own numerical integration routine for this, using a higher-order midpoint technique (e.g., 4th-order Runge-Kutta). Your solution should include your code, and plots or tables showing the behavior of θ versus ξ.

   (a) List ξ_1, $(d\theta/d\xi)_{\xi_1}$, and $M(\xi_1) = M(R_*)$ for each of your solutions. You can compare your values to those in the book to make sure you have done this correctly.

   (b) Derive the Chandrasekhar mass, using the equation of state for relativistic degenerate matter. This will require you to make an assumption about the mean molecular weight per electron, μ_e. To compute this, you may assume the star is made of pure, fully-ionized He.

2. In this problem, we will compare our semi-analytic polytropic solutions to the numerical solutions we obtain from MESA.

   (a) Start with the $n = 3$ polytropic model you generated in Problem 1. Assume $M = 1\ \mathrm{M}_\odot$, $R = 1\ \mathrm{R}_\odot$.

   - Compute the central density.
   - Compute the central pressure (P_c) and temperature (T_c). You may assume an ideal gas equation of state.
   - Plot P/P_c and T/T_c for your polytrope model.

   (b) Repeat (a) for a $n = 3/2$ polytrope.

   (c) Generate a model of a zero-age main-sequence star with $M = 1\ \mathrm{M}_\odot$, $R = 1\ \mathrm{R}_\odot$ using MESA. Compare the central density, pressure, and temperature to your ploytrope models. Then compare P/P_c and T/T_c for your MESA and polytrope models. Which polytropic model seems like a better representation of the "actual" (i.e., numerically-determined) structure of this object?

(d) Compute $d \log P / d \log \rho$ for your MESA model. Note that this quantity is equal to $1+1/n = \gamma$ for a polytrope. Comment on what physical processes are responsible for the fact that different regions of the star appear to be governed by different values of this quantity.