Astronomy 545: Homework 6 (Extra Credit)

Due: Wed. Dec. 11, 2019

1. **Electron-Scattering Opacity**
   Here we derive the electron scattering opacity, κ_e including the dependence on composition. The significance of this problem: it described the dominant high T opacity in stars, and the X dependence was shown to be relevant for e.g. semi-convection.

   (a) Express κ_e in terms of σ_e, the cross section for a photon to interact with an electron, μ_e and any constants. Explain this simple "derivation".

   (b) For non-relativistic electrons $\sigma_e = 8\pi r_e^2/3$ where r_e is the classical electron radius. Equate electrostatic and rest mass energies to "derive" r_e. Give the resulting κ_e in terms of constants and μ_e.

   (c) Derive $\mu_e(X)$ for a mixture of (fully ionized) hydrogen and helium (and X the hydrogen mass fraction).

   (d) Evaluate κ_e numerically, for general X, to reproduce the textbook result.

2. **Convection**

   (a) Argue that if $\nabla > \nabla_{\rm ad}$ implies convective instability, that $\nabla_{\rm rad} > \nabla_{\rm ad}$ is an equivalent instability criterion.

   (b) Similarly, argue that if $\nabla < \nabla_{\rm ad}$ implies stability to convection, that $\nabla_{\rm rad} < \nabla_{\rm ad}$ is an equivalent stability criterion.

   (c) For a region of convective instability place ∇, $\nabla_{\rm ad}$, $\nabla_{\rm rad}$, $\nabla_{\rm e}$ in order, which short written justifications for each inequality.

   (d) Do the above for a region of convective stability.

3. **Helium core burning**

   (a) Describe the Horizontal branch (HB) and it's role in post-MS evolution. Specifically address: what types of stars have a HB, what nuclear burning occurs, what events occur before and after the HB (i.e. what they are called and what burning happens). Sketch and label these 3 stages (and more if desired) schematically on a HR diagram.

(b) Estimate the timescale for Helium core burning in a "typical" HB star with $L \approx 50 L_\odot$ and a He core of $0.5 M_\odot$ at the ZAHB. Assume that $\sim 10\%$ of core ^{4}He burns to ^{16}O.

4. **Fluid Equations**

(a) Show that the following relation is true

$$\rho \frac{Df}{Dt} = \frac{\partial(\rho f)}{\partial t} + \nabla \cdot (\rho f \boldsymbol{u})$$

(In class we explain that this relation is useful for relating Eulerian and Lagrangian conservation laws.)

(b) For the next two subproblems start with the basic energy equation:

$$\frac{DQ}{Dt} = -\dot{Q}_{\rm cool}$$

where $\dot{Q}_{\rm cool}$ represents cooling per unit mass (or heating if negative). First show that this heating equation can reproduce the standard stellar structure equation

$$\frac{\partial L}{\partial m} = \epsilon - \epsilon_g$$

Be sure to define all terms and briefly explain all steps.

(c) Next show that for an ideal gas the fluid energy equation can be written in the form

$$\frac{D \ln P}{Dt} = -\gamma \nabla \cdot \boldsymbol{u} - \frac{\dot{Q}_{\rm cool}}{C_V T}$$

where terms have their usual meaning, including $\gamma = d\ln P / d\ln \rho|_S$. You can use (without derivation) the following properties of an ideal gas: $\mathcal{E} = C_V T$ (internal energy), $\gamma = C_P/C_V$ and $C_P = C_V + \mathcal{R}/\mu \equiv C_V + k_{\rm B}/(\mu m_u)$. Otherwise justify and briefly explain your steps.