Astronomy 400A: Homework 4

You may collaborate on all problems this week.

1. The age of the Universe is \( \sim 13.7 \) Gyr. The main-sequence lifetime of the Sun is \( \sim 10 \) Gyr.

(a) Estimate the main-sequence lifetime of a 0.8 \( M_\odot \) star. How does this compare to the age of the Universe? Are there any star clusters in the Galaxy where you would expect to see 0.8 \( M_\odot \) stars that have left the main-sequence and headed toward the red giant branch?

(b) Estimate the main-sequence lifetime of a 10 \( M_\odot \) star. How does this compare to the age of the Universe? Are there any star clusters in the Galaxy where you would expect to see 10 \( M_\odot \) stars that have left the main-sequence and headed toward the red giant branch?

(c) Sketch an H-R diagram for a star cluster with an age of 1 Gyr. In particular, indicate the stellar mass where stars are no longer found on the main-sequence.

(d) White dwarfs are known to exist with masses \( \lesssim 0.4 \) \( M_\odot \). Explain how these came to be.

2. Using the Eddington luminosity limit, derive an upper limit for the mass and luminosity of main-sequence stars. Estimate the effective temperatures for such stars. You may assume that \( \kappa = 0.4 \) cm\(^2\) g\(^{-1}\), as appropriate for electron scattering.

3. For stars that burn hydrogen via the CNO cycle, the cores and envelopes are typically fully-ionized, and so one can assume \( \kappa \approx \) constant. However, for lower-mass stars \( (M \lesssim 1.5 \) \( M_\odot \))\), partial ionization is important in some zones, and \( \kappa = \kappa_0 \rho T^{-7/2} \) is a better approximation. Use this opacity law and assume \( n = 4 \), and derive the relation between \( L \) and \( M \). Hint: use the same sorts of homology relations as used in the constant \( \kappa \) case.

4. If hydrogen is deposited onto the surface of a white dwarf, it can undergo fusion. Assume a thin layer of material with solar composition \( (X = 0.7, Y = 0.3) \) has been deposited onto a white dwarf with \( M = M_\odot \) and \( R = 0.01 R_\odot \).

(a) Calculate the fraction of the thin layers's mass \( (f) \) that must be fused into He to supply sufficient energy to eject the entire layer. Express your answer for \( f \) in terms of physical constants and \( M, R \).

(b) Derive the dependence of \( f \) on \( M \) for \( M < M_{\text{ch}} \).