Astronomy 400A: Homework 1

Be sure to show your work so that a) I can verify that you solved the problem correctly and b) I can give partial credit even if you don't reach the correct answer.

1. Individual Problems

1. Consider a binary system where both components are visible. The period of the orbits is observed to be $\tau = 49.9$ years, the observed semi-major axis of the reduced mass is $7.61''$, and the parallax of the brighter star ("star A") is measured to be $\pi_P = 0.38''$. The ratio of the distances of the two components from their common center of mass is also measured, $a_a/a_B = 0.47$.

   (a) Calculate the total mass of the system.

   (b) Calculate the inidividual masses of the components.

   (c) The luminosities of the two components are measured (given the known distance) to be $L_A = 22.5L_\odot$ and $L_B = 0.025L_\odot$. The observed color of the B component also allows an estimate of the temperature, $T_B = 25,000$ K. Derive the radius of component "B".

   (d) Compute the bulk density of component "B" and compare it to the bulk density of the sun ($M_\odot = 1.99 \times 10^{33}$ g; $R_\odot = 6.96 \times 10^{10}$ cm). What sort of object is component B?

2. In class, we will discuss how long the Sun would shine if it was powered purely by its internal energy or if it was powered by nuclear fusion. Here, you will estimate how long it could shine if it was powered by chemical reactions. The ionization potential of Hydrogen is 13.6 eV, and thus this is a reasonable guess as to how much energy could be involved in a given chemical reaction. Assuming that every atom in the Sun could release 13.6 eV through such a reaction, and that the Sun is made entirely of Hydrogen, how long could the Sun shine?

3. At what rate is the mass of the Sun decreasing due to nuclear reactions? Has the mass of the Sun changed significantly (due to nuclear reactions) over its ~ 5 billion years of life?

2. Group Problems

1. For a star of mass M and radius R, find the central pressure and check the validity of the following inequality:

$$P_c > \frac{GM^2}{8\pi R^4}. \tag{1}$$

   Perform this calculation for the following cases:

   (a) Uniform density.

   (b) $\rho = \rho_c \left[1 - (r/R)^2\right]$.

2. Show that $P_c < (\pi/6)^{1/3} G M^{2/3} \rho_c^{4/3}$.

3. Use the Virial theorem to derive the average temperature of a star for the two cases a) and b) from problem 1. Express your answer in terms of μ, M, R, and the relevant physical constants.