PROBLEM SET 3

1. Adverse selection

In a used car market there are two types of cars: bad (B) and good (G). Assume that the utility from a bad car $u_B = \$100$, an the utility from a good car $u_G = \$1000$. Manny, the owner of Manny's Used Cars, buys cars from the general public for his lot (assume he just keeps them on his lot for now). The owners of the cars who are selling to Manny know the qualities of the cars they are selling, but Manny does not.

a. If car type is observable, what is the price paid for good and bad-type cars?

Answer:

If car type is observable, both sellers and the buyer know types of the cars. That is, the car market is a perfect information competitive market. As in lecture note, $p_B = u_B$ and $p_G = u_G$.

b. Describe the equilibrium in the market assuming that Manny can only offer the same buying price to any car in the market (i.e. cannot discriminate among sellers). What is the equilibrium used car price?

Now assume that a repair shop can perform general inspections of used cars. The costs associated with inspecting a good car $c_G = \$100$, but it costs $c_B = \$600$ for a bad car to pass inspection.

Also assume that Manny offers different purchase prices (p_I and $p_N I$) for cars which have and have not undergone inspection. In the following steps you will derive the prices such that only good cars will undergo inspection.

Answer:

Assume that a fraction, α of the used cars in the market are of type bad. Manny can only offer one price to a car in this market, while his expected utility of a used car is $EU = \alpha u_B + (1-\alpha)u_G$. With this offer price only owners of bad cars will be willing to sell the cars. Thus, the outcome of this market is that Manny will offer u_B , \$100, and only bad cars are in the market.

c. Write down the participation and self-selection constraints for the two types of cars. There should be four constraints total.

Participation constraints:

$$(1) \quad p_I >= u_G + c_G \Rightarrow p_I >= 1000 + 100$$

$$(2) \quad p_{NI} >= u_B \Rightarrow p_{NI} >= 100$$

Self-selection constraints:

$$(3) \quad \Pi(good, inspect) >= \Pi(good, not\ inspect) \Rightarrow p_I - 1000 - 100 = p_{NI} - 1000$$

$$(4) \quad \Pi(bad, inspect) <= \Pi(bad, not\ inspect) \Rightarrow p_I - 100 - 600 = p_{NI} - 100$$

d. Solve for for the p_I and p_{NI} which satisfy all these constraints. (Hint: the participation constraint is binding for good cars, and the self selection constraint is binding for bad cars.) Compare these prices to those in part (a).

Answer: We know that constraints (1) and (4) are binding.

$$p_I = 1000 + 100 \Rightarrow p_I = 1100$$

$$p_I - 100 - 600 = p_{NI} - 100 \Rightarrow p_{NI} = 500$$

e. What are the prices if c_B takes on values of \$200 or \$1000? What happens to prices as c_B increases?

Answer:
If c_B takes a value of 200:

$$1100 - 1000 - 200 = p_{NI} - 100$$

$$\Rightarrow p_{NI} = 900$$

If c_B takes a value of 1000 then:

$$1100 - 1000 - 1000 = p_{NI} - 100$$

$$\Rightarrow p_{NI} = 100$$

If c_B increases, p_{NI} will decreases but p_I is independent of c_B.

2. Moral Hazard

Consider the fire insurance model described in class.

Make the following assumptions.

- Individual's utility functions for money are $U(x) = lnx$ where x is dollars. (In class we had $U(x) = x$)

- Starting income (M) =$10,000; $K_2 = \$5,000$; $C = \$500$
- $p = 0.20; p^* = 0.75$ (so taking preventive precautions decreases the probability of fire from 0.75 to 0.25)
- Premium is fair (i.e., $K_1 = pK_2$);deductible $D = \$1000$

a. Will an individual with insurance take the preventive precautions?

Answer:
An individual with insurance will take preventive precaution if and only if.

$$EU(with\ precaution) >= EU(without\ precaution)$$

$$EU(with\ precaution) = pU[M - pK2 - C - D] + (1-p)U[M - pK2 - C]$$

$$= .2ln(10000 - .2(5000) - 500 - 1000) + .8ln(10000 - .2(5000) - 500)$$

$$= 9.023$$

$$EU(without\ precaution) = p^*U[M - pK2 - D] + (1-p^*)U[M - pK2]$$

$$= .75ln(10000 - .2(5000) - 1000) + .25ln(10000 - .2(5000))$$

$$= 9.017$$

Since $EU(with) > EU(without)$, an individual will take preventive precaution.

b. Does you answer change if D is lowered to $500? Solve for the deductible value which makes the insured individual indifferent between taking and not taking the preventive precautions.

Answer:
If D = 500:

$$EU(with\ precaution) = pU[M - pK2 - C - D] + (1-p)U[M - pK2 - C]$$

$$= .2ln(10000 - .2(5000) - 500 - 500) + .8ln(10000 - .2(5000) - 500)$$
$$= 9.036$$

$$EU(without\ precaution) = p * U[M - pK2 - D] + (1 - p*)U[M - pK2]$$
$$= .75ln(10000 - .2(5000) - 500) + .25ln(10000 - .2(5000))$$
$$= 9.061 > EU(with\ precaution)$$

Therefore, an individual will not take preventive precaution.

Find D that make $EU(with) = EU(without)$.

$$D^* = \$908$$

c. Does you answer change if p rises to 0.40? Solve for the value of p which makes the insured individual indifferent between taking and not taking the preventive precautions.

Answer:
For $p = .40$:

$$EU(withprecaution) = pU[M - pK2 - C - D] + (1 - p)U[M - pK2 - C]$$
$$= .4ln(10000 - .2(5000) - 500 - 1000) + .6ln(10000 - .2(5000) - 500)$$
$$= 8.998$$

$$EU(withoutprecaution) = p * U[M - pK2 - D] + (1 - p*)U[M - pK2]$$
$$= .75ln(10000 - .2(5000) - 1000) + .25ln(10000 - .2(5000))$$
$$= 9.017 > EU(withprecaution)$$

Therefore, an individual will not take preventive precaution.

Find the p that makes $EU(with) = EU(without)$:

$$pln(7500) + (1 - p)ln(8500) = .75ln(8000) + .25ln(9000)$$
$$p = \frac{.75ln(8000) + .25ln(9000) - ln(8500)}{ln(7500) - ln(8500)}$$
$$p = .249$$

d. Based on your answers to (b) and (c), say something about how the incentive to take preventive measures is related to D and p.

The incentive to take preventive precaution increases as D rises and decreases as p rises.

3. Franchise Bidding

A university is auctioning off its cafeteria monopoly. Assume there are 100 contending firms. Demand for cafeteria food is given by the demand curve

$p = 2 - q$. Assume firm i produces with constant marginal costs equal to i cents (so that firm 5 has marginal costs equal to \$0.05).

a. Who will win the auction? At what price?

Answer:
Firm 1, which has the lowest cost, will win with the bid equals to Firm 2's profit.
Firm 2's profit under monopoly:

$$MR = 2 - 2q$$

$$MC_2 = 0.02$$

$$FOC : MC = MR$$

$$2 - 2q = 0.02$$

$$q^* = .99$$

$$p^* = 2 - q^* = 1.0]$$

$$\Pi_2 = (p^* - MC2)q^*$$

$$\Pi_2 = (1.01 - 0.2).99 = .9801$$

The winning bid is .9801.

b. What are the ensuing profits (net of franchise costs) earned by the winning firm?

Answer:
Firm 1's maximization problem:

$$MR = 2 - 2q$$

$$MC_1 = 0.01$$

$$MR = MC_1$$

$$2 - 2q = 0.01$$

$$q_m = 0.995$$

$$p_m = 2 - q_m = 1.005$$

Firm 1's profits net of franchise costs are:

$$\Pi_1 = (1.005 - 0.01)(0.995) - 0.9801$$

$$= 0.009925$$

4. Cross-subsidization

Telantarctic is the national telephone company in Antarctica. It offers local telephone service (to the penguins??) to the distinct geographic regions of the north and south Antarctica. The marginal cost of offering a telephone call in south Antarctica is \$3, but only \$1 for calls in north Antarctica.

Assume that the demand for local telephone calls is given by the demand curve $p = 10 - q$ in both regions.

a. What are the profit maximizing prices p_N and p_S that TelAntarctic should charge in the north and south regions?

Answer: Inverse demand $q = 10 - p$
South:

$$\max_{p_s} \Pi_s = (p_s - 3)(10 - p_s)$$

$$FOC : 10 - p_s - p_s + 3 = 0$$

$$p_s = 6.5$$

North:

$$\max_{p_n} \Pi_n = (p_n - 3)(10 - p_n)$$

$$FOC : 10 - p_n - p_n + 1 = 0$$

$$p_n = 5.5$$

b. Derive the uniform price p_U, whereby TelAntarctic will just break even (i.e. such that with the price p_U , its profits in the low-cost market just outweigh its losses in the high-cost market).

Answer:
Let Π_T denote total profit. p_u is chosen such that $\Pi_T = \Pi_n + \Pi_s = 0$

$$(p_u - 1)(10 - p_u) + (p_u - 3)(10 - p_u) = 0$$

$$(p_u - 1 + p_u - 3)(10 - p_u) = 0$$

$$(2p_u - 4)(10 - p_u) = 10$$

$$p_u = 2 or p_u = 10$$

If $p_u = 10$, the quantity for the service is zero and the market of telephone disappears. Thus, $p_u = 2$.

5. Information

The introduction of restaurant hygiene grade cards in Los Angeles caused restaurants to improve their hygiene. Presumably this increased costs for restaurants, putting upward pressure on equilibrium prices. But the data also shows that restaurant meal prices fell because of the grade cards. Are these two facts consistent with each other? Explain your answer.

Answer: One effect of the grade cards may be to reduce search costs for consumers. By providing information about hygiene quality, consumers are less reliant on their own past experiences at restaurants when deciding which restaurant to go to. Consumers are more confident at trying restaurants they have not been to before. In other words, consumers are less captive to the restaurants they have tended to go to in the past. The increased mobility of consumers leads firms to compete more aggressively with each other. Intensified competition places downward pressure on prices, the opposite direction of the cost effect mentioned in the question. The competitive effect may dominate the cost effect, so it is possible the net effect is to get lower prices.