

### PROBLEM SET 3

#### 1. Adverse selection

In a used car market there are two types of cars: bad (B) and good (G). Assume that the utility from a bad car  $u_B = \$100$ , and the utility from a good car  $u_G = \$1000$ . Manny, the owner of Manny's Used Cars, buys cars from the general public for his lot (assume he just keeps them on his lot for now). The owner's of the cars who are selling to Manny know the qualities of the cars they are selling, but Manny does not.

a. If car type is observable, what is the price paid for good and bad-type cars?

*Answer:*

If car type is observable, both sellers and the buyer know types of the cars. That is, the car market is a perfect information competitive market. As in lecture note,  $p_B = u_B$  and  $p_G = u_G$ .

b. Describe the equilibrium in the market assuming that Manny can only offer the same buying price to any car in the market (i.e. cannot discriminate among sellers). What is the equilibrium used car price? *Answer:*

Assume that a fraction,  $\alpha$  of the used cars in the market are of type bad. Manny can only offer one price to a car in this market, while his expected utility of a used car is  $EU = \alpha u_B + (1 - \alpha)u_G$ . With this offer price only owners of bad cars will be willing to sell the cars. Thus, the outcome of this market is that Manny will offer  $u_B$ , \$100, and only bad cars are in the market.

c. Now assume that a repair shop can perform general inspections of used cars. the costs associated with inspecting a good car  $c_G = \$100$ , but it costs  $c_B = \$600$  for a bad car to pass inspection.

Also assume the Manny offers different purchase prices ( $p_I$  and  $p_{NI}$ ) for cars which have and have not undergone inspection. In the following steps you will derive the prices such that only good cars will undergo inspection.

Write down the participation and self-selection constraints for the two types of cars. There should be four constraints total.

*Answer:*

Participation constraints:

$$(1) \quad p_I - u_G - c_G \geq 0 \Rightarrow p_I - 1000 - 100 \geq 0$$

$$(2) \quad p_{NI} - u_B \geq 0 \Rightarrow p_{NI} - 100 \geq 0$$

Self-selection constraints:

$$(3) \quad \Pi(\text{good}, \text{inspect}) \geq \Pi(\text{good}, \text{not inspect}) \Rightarrow p_I - 1000 - 100 \geq p_{NI} - 1000$$

$$(4) \quad \Pi(\text{bad}, \text{inspect}) \leq \Pi(\text{bad}, \text{not inspect}) \Rightarrow p_I - 100 - 600 \leq p_{NI} - 100$$

d. Solve for for the  $p_I$  and  $p_{NI}$  which satisfy all these constraints. (Hint: the participation constraint is binding for good cars, and the self selection constraint is binding for bad cars.) Compare these prices to those in part (a).

*Answer:* We know that constraints (1) and (4) are binding.

$$p_I = 1000 + 100 \Rightarrow p_I = 1100$$

$$p_I - 100 - 600 = p_{NI} - 100 \Rightarrow p_{NI} = 500$$

e. What are the prices if  $c_B$  takes on values of \$200 or \$1000? What happens to prices as  $c_B$  increases?

*Answer:* Remember only two constraints are binding

$$(1): p_I - u_G - c_G \geq 0$$

$$(4): p_I - u_B - c_B \leq p_{NI} - u_B$$

Since  $c_B$  does not enter into the into (1) the price for inspected cars will not be affected by the change. However,  $c_B$  does enter constraint (4) which will affect out solution for  $p_{NI}$  If  $c_B$  takes a value of 200:

$$1100 - 100 - 200 = p_{NI} - 100$$

$$\Rightarrow p_{NI} = 900$$

If  $c_B$  takes a value of 1000 then:

$$1100 - 100 - 1000 = p_{NI} - 100$$

$$\Rightarrow p_{NI} = 100$$

If  $c_B$  increases,  $p_{NI}$  will decrease but  $p_I$  is independent of  $c_B$ .

## 2. Moral Hazard

Consider the fire insurance model described in class.

Make the following assumptions.

- Individual's utility functions for money are  $U(x) = \ln x$  where  $x$  is dollars.
- Starting income (M) = \$10,000;  $K_2 = \$5,000$ ;  $C = \$500$
- $p = 0.20$ ;  $p^* = 0.75$  ( so taking preventive precautions decreases the probability of fire from 0.75 to 0.20)
- Premium is fair:  $K_1 = pK_2 \Rightarrow K_1 = 0.2 * (5000)$ . Assume that this is the premium for all parts of the question below.
- The deductible  $D = \$1000$

a. Will an individual with insurance take the preventive precautions?

*Answer:*

An individual with insurance will take preventive precaution if and only if.

$$EU(\text{with precaution}) \geq EU(\text{without precaution})$$

$$EU(\text{with precaution}) = pU[M - pK_2 - C - D] + (1 - p)U[M - pK_2 - C]$$

$$= .2\ln(10000 - .2(5000) - 500 - 1000) + .8\ln(10000 - .2(5000) - 500)$$

$$= 9.023$$

$$EU(\text{without precaution}) = p^*U[M - pK_2 - D] + (1 - p^*)U[M - pK_2]$$

$$= .75\ln(10000 - .2(5000) - 1000) + .25\ln(10000 - .2(5000))$$

$$= 9.017$$

Since  $EU(\text{with}) > EU(\text{without})$ , an individual will take preventive precaution.

b. Does your answer change if  $D$  is lowered to \$500? Solve for the deductible value which makes the insured individual indifferent between taking and not taking the preventive precautions.

*Answer:*

If  $D = 500$ :

$$EU(\text{with precaution}) = pU[M - pK_2 - C - D] + (1 - p)U[M - pK_2 - C]$$

$$\begin{aligned}
&= .2\ln(10000 - .2(5000) - 500 - 500) + .8\ln(10000 - .2(5000) - 500) \\
&= 9.036
\end{aligned}$$

$$\begin{aligned}
EU(\text{without precaution}) &= p * U[M - pK2 - D] + (1 - p*)U[M - pK2] \\
&= .75\ln(10000 - .2(5000) - 500) + .25\ln(10000 - .2(5000)) \\
&= 9.061 > EU(\text{with precaution})
\end{aligned}$$

Therefore, an individual will not take preventive precaution.

Find D that make  $EU(\text{with}) = EU(\text{without})$ . Note there is not an analytical solution to D, but it must be solved numerically.

$$\begin{aligned}
.2\ln(8500 - D) + .8\ln(8500) &= .75\ln(9000 - D) + .25\ln(9000) \\
D^* &= \$908
\end{aligned}$$

c. Does you answer change if  $p$  rises to 0.40? Solve for the value of  $p$  which makes the insured individual indifferent between taking and not taking the preventive precautions.

*Answer:*

For  $p = .40$ :

$$\begin{aligned}
EU(\text{withprecaution}) &= pU[M - pK2 - C - D] + (1 - p)U[M - pK2 - C] \\
&= .4\ln(10000 - .2(5000) - 500 - 1000) + .6\ln(10000 - .2(5000) - 500) \\
&= 8.998
\end{aligned}$$

$$\begin{aligned}
EU(\text{withoutprecaution}) &= p * U[M - pK2 - D] + (1 - p*)U[M - pK2] \\
&= .75\ln(10000 - .2(5000) - 1000) + .25\ln(10000 - .2(5000)) \\
&= 9.017 > EU(\text{withprecaution})
\end{aligned}$$

Therefore, an individual will not take preventive precaution.

*Answer:*

Find the  $p$  that makes  $EU(\text{with}) = EU(\text{without})$ :

$$\begin{aligned}
p\ln(7500) + (1 - p)\ln(8500) &= .75\ln(8000) + .25\ln(9000) \\
p &= \frac{.75\ln(8000) + .25\ln(9000) - \ln(8500)}{\ln(7500) - \ln(8500)} \\
p &= .249
\end{aligned}$$

d. Based on your answers to (b) and (c), say something about how the incentive to take preventive measures is related to  $D$  and  $p$ .

*Answer:* The incentive to take preventive precaution increases as  $D$  rises and decreases as  $p$  rises.

### 3. Information

Evaluate the following statement: For goods of uncertain quality, so long as either the seller or buyer can determine the quality prior to the sale, there will be efficient consumption.

*Answer:*

False, if only one entity in the transaction has information then we have situation with asymmetric information. If the buyer does not know the quality, any statements made by the seller have no credibility - the seller has an incentive to lie. This is the general problem of adverse selection which leads to an inefficiently low level of quality being sold in equilibrium.

There is a possibility that the statement may be true. Such an answer should provide a clear explanation of how seller reputation or a warranty, for example, means that it is sufficient for only the seller to know the actual quality of the product.

### 4. Information

Adverse Selection

1. Describe the asymmetric information problem in the labor market
2. Who has asymmetric information?

*Answer:* 1. Effort in the labor market cannot be continuously monitored. An individual hired by a company has incentives to "free ride" on the efforts of others. Unless proper incentives are in place, an individual may not put forth maximal effort (as the employer would like) if such effort is not monitored or rewarded.

2. Asymmetric information in the labor market is a moral hazard problem. The employees have information about their effort levels that the employer does not.

### 5. Double Marginalization

The market demand curve for hot dogs is  $Q = 20 - 3p$ . Hot dogs are produced by Boca Raton, Inc. at a constant marginal cost of 1. The two fast-food stores Dogs-r-Us1 and Dogs-r-Us2 are the sole distributors of Boca Raton hot dogs in Baltimore. They face a marginal cost of 1 in addition to the cost of each hot dog which they buy from Boca Raton.

(a) Assume the Boca Raton vertically integrates with both fast-food stores. What is price and output under this scenario? What are profits? What is consumer surplus?

*Answer:*

Firms maximize joint profits:

$$\Pi = (p - 2)(20 - 3p)$$

$$FOC : 20 - 3p - 3p + 6 = 0$$

$$P^J = 13/3 \Rightarrow Q_J = 20 - 3(13/3) = 7$$

$$\Pi = (13/3 - 2)7 = 16.33$$

$$CS = \frac{1}{2}(20/3 - 13/3)7 = 8.17$$

(b) Assume that Boca Raton, Dogs-r-Us1, and Dogs-r-Us2 operate independently. Assume that they compete in a Cournot fashion. What is the output and the retail and wholesale prices in this non-integrated scenario? How much profit does each firm make? What is consumer surplus?

*Answer:* Solve backwards by starting with the distributor.

Distributors  $MC = P_w + 1$

Inverse demand function:  $P = \frac{20}{3} - \frac{Q}{3}$

Profit maximization for distributor:

$$\max_{q_1} \Pi_1 = P(Q)q_1 - C(Q) = \left(\frac{20}{3} - \frac{q_1 + q_2}{3}\right)q_1 - q_1(P_w + 1)$$

FOC:

$$\frac{20}{3} - \frac{q_1 + q_2}{3} - P_w - 1 - \frac{1}{3}q_1 = 0$$

$$\Rightarrow BR_1(q_2, P_w) = \frac{1}{2}(17 - q_2 - 3P_w)$$

$$\text{and symmetrically } BR_2(q_1, P_w) = \frac{1}{2}(17 - q_1 - 3P_w)$$

$$q_i^* = \frac{17}{3} - P_w$$

Boca Raton's demand is then  $Q = q_1 + q_2 = \frac{34}{3} - 2P_w$

Boca Raton's Problem

$$\max_{P_w} \left\{ (P_w - 1) \left( \frac{34}{3} - 2P_w \right) \right\}$$

FOC:

$$\frac{34}{3} - 2P_w - 2(P_w - 1) = 0$$

$$\Rightarrow P_w^* = \frac{10}{3}$$

$$Q^* = \frac{34}{3} - 2\frac{10}{3} = \frac{14}{3} < Q^J$$

Profits for each firm:

Boca Raton:  $\frac{98}{9}$

Downstream Firms each have:  $q_i = \frac{7}{3} \Rightarrow P_r = 5.11 < P^J \Rightarrow \Pi_i = 1.81$

$$CS = \frac{1}{2} \left( \frac{20}{3} - \frac{46}{9} \right) \frac{14}{3} = 3.63$$

(c) Consider the same scenario as above except that the two retailers not compete in a Bertrand fashion rather than Cournot. What is the output and the retail and wholesale prices in this non-integrated scenario? How much profit does each firm make? What is consumer surplus?

*Answer:*

In Bertrand competition both downstream competitors will price at  $P_r = MC = P_w + 1$ .

Then market demand is  $Q = 20 - 3(P_w + 1) = 17 - 3P_w$

Boca Raton's Problem:

$$\max_{P_w} \{(P_w - 1)(17 - 3P_w)\}$$

FOC:

$$17 - 3P_w - 3(P_w - 1) = 0$$

$$P_w = \frac{10}{3} \Rightarrow Q = 7 \Rightarrow P_r = \frac{13}{3}$$

$$\Pi^B = \frac{49}{3}$$

$$CS = \frac{49}{6}$$

Note: Since the distributors do not markup the price of the product over cost there is not double marginalization problem here. The outcome is the same when firms maximized joint profits. (d) Suppose Boca Raton can charge a franchise fee to the fast-food retailers. What is the profit maximizing fee, assuming that the franchises compete in Cournot fashion?

Boca Raton will set  $P_w = mc$  to maximize the profits of the retailers (and also joint profits) and then extract all those profits through the franchise fee.

From part (b):

$$q_i^* = \frac{17}{3} - P_w$$

$$q_i^* = \frac{17}{3} - 1 = \frac{14}{3}$$

$$P_r = \frac{32}{9}$$

$\Pi_i = 7.26 =$  franchise fee for each firm

Boca Raton's profits is 14.52.

*Answer:*

## 6. 2nd degree price discrimination

Your software company has just completed a new version of its program, a voice activated word processor. The program is valued by market segments of equal size namely professionals and students. Professionals would be willing to pay up to \$400 for the program while students would pay \$100 for the full version. A scaled down version of the program is worth \$50 to students, but is worthless for professionals. The software has already been developed and marginal production costs are zero. What are the optimal prices for each version of software?

*Answer:* The firm has several options:

1. Sell the full version to both professionals and students.
2. Sell the scaled down version to both professionals and students.
3. Sell the full version to students and the scaled down version to professionals.
4. Sell the the full version to professionals and the scaled down version to students.

For option 1, the firm will have to charge a price of \$100 for the full version to get both segments to buy the full version (a price higher than \$50 so the students don't buy the scaled back version). The firm will earn a profit of  $100 * (\text{marketsize}) + 100 * (\text{marketsize}) = 200 * (\text{marketsize})$ .

Option 2 does not make sense since the profit from the professionals will be zero.

Option 3 does not make any sense since the students have a lower WTP than professionals for the full version and professionals are not willing to pay anything for the scaled back version.

For option 4 we can set up the self-selection and participation constraints to induce the behavior we want.

### Self-Selection

Professional:  $400 - P_F \geq 0 - P_S$

Student:  $50 - P_S \geq 100 - P_F$

### Participation

Professional:  $400 - P_F \geq 0$



Student:  $50 - P_S \geq 0$

So if the student participation constraint is binding:

$$\Rightarrow P_S = 50$$

Substituting into the self-selection constraint for professionals:

$$400 - P_F \geq -50 \Rightarrow P_F = \$450$$

However, if  $P_F = 450$  then the participation constraint for professionals is violated. This is because the professional has the option of buying nothing as opposed to getting utility of  $-50$  from buying the scaled down version of the software. Setting  $P_F = 400, P_S = 50$  satisfies both the participation and the self-selection constraints. The participation constraints are binding for both groups and the self selection constraints are binding for neither group.

Option 4 is the profit maximizing option since profits are  $50 * (\text{marketsize}) + 400 * (\text{marketsize}) = 450 * (\text{marketsize})$