OPTI 509
Optical Design and Instrumentation II

Course overview and syllabus
Instructor and notes information

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There is no assigned text for the class
Reference texts:
- Optical Radiation Measurements: Radiometry - Grum and Becherer
- Radiometry and the Optical Detection of Radiation - Boyd
- Aberrations of Optical Systems - Welford

Class notes are available via web access
http://www.optics.arizona.edu/kurt/opti509/opti509.php
Course description

Course covers the use of radiometry and impact of aberrations in optical design

Discussion of optical systems via ray trace codes, ray fans, & spot diagrams.

The effects of optical aberrations and methods for balancing effects of various aberrations.

Radiometric concepts of projected area & solid angle, generation and propagation of radiation, absorption, reflection, transmission and scattering, and radiometric laws.

Application to laboratory & natural sources and measurement of radiation

Why radiometry and aberrations?

OPTI 502 gave optical design from the “optics” point of view

OPTI 509 shows tools needed to ensure enough light gets through the system (radiometry) and goes to the right place (aberrations)
Course overview

Propagation of radiation

1. Class introduction, Radiometric and photometric terminology
2. Areas and solid angles; projected solid angles, projected areas
3. Radiance, throughput
4. Invariance of throughput and radiance; Lagrange invariant, radiative transfer, radiant flux
5. E, I, M, reflectance, transmittance, absorptance, emissivity, Planck's Law
6. Lambert's law; isotropic vs. lambertian; M vs. L, inverse square and cosine laws, examples
7. Directional reflectance, simplifications, assumptions, view & form factors, basic and simple radiometer
8. The atmosphere and its effects on radiation, sources
Course overview

Detectors

9. Radiometric systems, camera equation, spectral instruments, demonstration of radiometric systems
10. Basic detection mechanisms and detector types
11. Noise
12. Figures of merit, Basic electronics; photodiodes and op-amps
13. Spectral selection terms; spectral selection methods
14. Relative and absolute calibration
15. Examples of detector calculations and vision

END OF TEST 1 MATERIAL
Course overview

Aberrations

16. Monochromatic aberrations; causes of aberrations, coordinate system; wave aberrations, tangential and sagittal rays, transverse and longitudinal ray aberrations

17. Ray fans, spot diagrams; RMS spot size

18. Spherical aberration; variation with bending; high-order spherical aberration

19. Distortion

20. Field curvature and astigmatism

21. Coma; stop-shift effects

22. Longitudinal chromatic aberrations of a thin lens, thin lens achromatic doublet, spherochromatism, secondary chromatic aberration; lateral chromatic aberration
Course overview

Image quality and imaging systems

23. Defocus and tilt; balance with defocus and tilt
24. Combined aberrations; aberration balancing; ray/wave fan decoding
25. Seidel aberration coefficients; wavefront expansion; wave fans; wavefront variance, demonstration of optical design software and optimization
26. Strehl ratio, calculations of PSFs and MTFs from raytrace data, influence of aberrations on MTFs
27. Imaging detectors - general characteristics
28. Aspheric systems, conics; two mirror system, example instrumentation: imaging systems; sensor examples

END OF TEST 2 MATERIAL
Homework accounts for 25% of the final grade
  • Homework is due by the date and time listed on the assignment
  • Late homework has a 20% deduction if turned in prior to posting of solutions
  • 40% deduction if turned in after solutions are given out (one week after the due date)

One midterm, in-class exam is given worth 35% covers topics 1-15

Exam 2 covers topics 16-28 plus one comprehensive question and is worth 40% of the grade and given during final exam period

Exams are closed book, closed notes

Equation sheet will be given for use on the exam
Propagation of Radiation

OPTI 509

Lecture 1
Radiometric and photometric terminology
Radiometry - Definition

Radiometry is the science and technology of measuring electromagnetic radiant energy

- Basis for or part of many other fields
  - Astronomy
  - Optical design
  - Remote sensing
  - Telecommunications

- Field of study in and of itself
  - Metrology
  - Calibration
  - Detector development

- Photometry is similar except that it is limited to the visible portion of the electromagnetic spectrum
Radiometric systems

All radiometric systems have the same basic components:

- Source
- Object
- Transmission medium
- Optical system
- Detector
- Signal processing
- Output
Radiometric sources

Source illuminates the object

- Anything warmer than 0 K can be a source
- Source partially determines the optical system and detector
  - Source type defines the spectral shape/output
  - Source output is combined with the object properties to get the final spectral shape at the entrance of the optical system
  - Source also defines the power available to the optical system
Transmission medium is what the radiation from the source and object travels through to the optical system.

- Transmission medium scatters, absorbs, and emits radiation and confuses the object signal.
- Can be used to enhance the spectral shape and power of the energy at the optical system.
- Typically is viewed as a degradation of the energy at the optical system.
Object

Object is the ultimate source of energy collected by the optical system

- Can attempt to infer the properties of the object from the measurements
  - Characterization of reflectance standards
  - Characterization of a calibration standard

- Known properties of the object can be used to infer the properties of the optical system and detector (radiometric calibration)
Optical system collects the radiation and sends it to the detector

- Optical system is used to condition the energy into a more useful form for the detector and problem
  - Spectral filtering
  - Larger entrance pupil to collect more energy
  - Neutral density filter to reduce energy
  - Limit the field of view of the detector

- In radiometry, simpler is better
Detectors convert the energy of the photon into a form that is easier to measure such as electric current.

- At this point in the system, the rest of the work is selecting and characterizing the detector.
- Selection of the detector is based on the spectral range of the problem and other issues:
  - Operational requirements
  - Cost, size, availability
- Characterize detector to understand relationship between input energy and output.

Signal Processing → Output

Detector

Optical System

Source

Transmission Medium

Object
Signal processing is necessary to convert the detector output to something useful to the user.

- Signal processing can improve the output of the detector:
  - Integration/averaging of multiple measurements
  - Amplification of the detector output (and noise)

- Methods for signal processing can feed back into choices for the other portions of the system.
Variations in radiometric systems

Each radiometric system can have slightly different configurations but the basic elements are always present.
Radiometric systems - example

Images below are of an Exotech radiometer being absolutely calibrated in the RSG’s blacklab.
Radiometric systems - examples

Images below show an example of radiometric calibration of using a spherical integrating source

- Righthand image includes an ASD FieldSpec FR monitoring the spectral characteristics of the SIS output for an airborne imager
- Lefthand system is effectively identical to the radiometric system shown on the previous viewgraph
Radiometric systems

One goal of this course is to illustrate the importance of viewing radiometry as the entire system.

- Easy to concentrate on the equipment
  - Optical system
  - Detectors
  - Signal processing
- Ignores the source, the transmission medium, and object
  - What is the spectral output of the source
  - How does the energy from the source interact with the object
- Design of the optical system should optimize the output
- Consider the radiometer viewing the sphere source and the panel
  - Both sources are spectrally similar
  - Sphere source has lower output
  - Optimizing the output of the radiometer for the sphere source could make the output while viewing the panel unusable
Electromagnetic radiation

Radiometry is the study of the measurement of electromagnetic radiation

- Thus, radiometry is impacted by the ray, wave, and quantum nature of electromagnetic radiation

- Material in this course is dominated by the ray nature of light
  - Geometric optics
  - Ignore diffraction in most cases
  - Not worry about the particle nature of light

- Also ignore coherent light effects in most cases

- Radiometric concepts will still apply in the coherent case and in the presence of diffraction
  - Some modifications are required in the details
  - Basic philosophy is still the same
Electromagnetic spectrum

- **Gamma Rays**
- **Cosmic Rays**
- **X-rays**
- **Ultraviolet**
- **Visible Light**
- **Infrared**
- **Short Radio Waves**
- **Microwave and Radar**
- **Long Radio Waves**
- **FM, TV**
- **AM**

**Frequency (Hertz)**

\[10^{22} \quad 10^{19} \quad 10^{16} \quad 10^{13} \quad 10^{10} \quad 10^{7} \quad 10^{4}

**Wavelength (meters)**

\[10^{-14} \quad 10^{-11} \quad 10^{-8} \quad 10^{-5} \quad 10^{-2} \quad 10^{1} \quad 10^{4}\]
Electromagnetic spectrum

Commonly used labels

- This class focuses on the 0.2 to 100 μm range
  - Shorter wavelengths are absorbed completely in the earth’s atmosphere
  - Shorter wavelengths dominated by the particle nature of light
  - Longer wavelengths dominated by diffraction effects
- This spectral range covers >99% of the total energy emitted by objects warmer than 273 K
- Spectral regions are delineated by either detector, source, or transmission medium effects
- All of the wavelength ranges given are approximate
- Visible part of the spectrum
  - 0.35 to 0.7 μm (350 to 700 nm)
  - Wavelength range over which the human eye is sensitive
Electromagnetic spectrum

More labels

- Ultraviolet (UV) - less than 0.35 μm
- Near Infrared (NIR) - 0.7 to 1.1 μm
  - Lower end is the upper end of the visible range
  - Upper end is the limit of which silicon-based detectors are sensitive
- VNIR - Visible and near infrared - 0.35 to 1.1 μm (region over which silicon detectors work well)
- Shortwave Infrared (SWIR) - 1.1 to 2.5 μm
  - Lower limit is related to the upper limit of silicon detectors
  - Upper limit is the wavelength at which the sun’s output becomes less dominant relative to emitted energy from the earth
- Mid-wave Infrared (MWIR) - 2.5 to 7.0 μm
- Thermal Infrared (TIR) - 8.0 to 16.0 μm
  - Also called LWIR
  - Emissive portion of the spectrum for the earth-sun case
Equipment terminology

Will use several instrument types as examples in this course

- Instruments are labeled according to their usage
- Monochromator
  - Produces quasi-monochromatic light
  - “One” wavelength as a source or in measurement
- Spectrometer
  - Measures a spectrum suitable for locating spectral features
  - Not an absolute device (cannot give accurate values of radiance)
- Spectroradiometer
  - Spectrometer for which the spectral response can be determined
  - Can give the radiance of a spectral feature
- Radiometer is the general term for an instrument used in radiometry
- Spectrophotometer
  - Relative instrument that can determine the reflectance or transmittance spectrum of a sample
  - Strictly speaking only applies to visible light
Radiometric quantities

List several key terms and units for radiometric quantities for reference

- Much of what follows will discuss these parameters and how they relate to each other in greater detail

- Radiant Energy - $Q$
  - Energy traveling as electromagnetic waves
  - Units of J [Joules] in radiometric units

- Radiant Flux (or power) - $\Phi$
  - Rate at which radiant energy is transferred from a point on a surface to another surface
  - Typical units of W [Watts or J/s]

- Radiant flux density is the power per unit area -$[W/m^2]$
  - Radiant exitance - $M$
  - Irradiance - $E$

- Radiant intensity - $I$ [W/sr]
- Radiance - $L$ [W/(m$^2$ sr)]
Radiometric quantities

The terms just given are based on radiant energy

- Key to the usage of units is consistency
  - Using a given unit does not change the final conclusion
  - Often a matter of preference
  - There are some situations where choosing a given set of units will simplify the problem

- This course will use radiometric units
  - More general since it covers a wider portion of the EM spectrum
  - Instructor is most familiar with these units

- In addition to units, there are the labels as well
  - Note that radiant flux is oftened used interchangeably with power
  - Radiant flux density has two terms for the same units - irradiance and radiant exitance
  - Intensity is sometimes used to refer to \( \text{W/}(\text{m}^2 \text{ sr}) \) \{radiance\}
  - Flux is sometimes used to refer to \( \text{W/m}^2 \) \{radiant flux density\}

- Terms on the previous viewgraph are those used in this course
Photometric quantities

Will not use photometric quantities in this course but they are given here for reference

- Photometric units are based on the measurement of light
  - Determined relative to a standard photometric observer
  - Visible part of the EM spectrum
  - Used in fields related to visual applications such as illumination

- Basic photometric units are the candela and lumen
  - Candela is luminous intensity of 1/60 of 1 cm² of projected area of a blackbody operating at the melting point of platinum
  - 1 lumen is equivalent to 1/685 Watts at 555 nm

- Luminous Energy - Q
  - Similar to radiant energy
  - Quantity of light
  - Units of lumen-seconds (lm s) or talbots
Photometric quantities

Other units have analogies to radiometric quantities as well

- Luminous Flux - $\Phi$
  - Similar to radiant flux
  - Rate light is transferred from point on a surface to another surface
  - Typical units of lumen (lm) where a lumen is equal to the flux through a unit solid angle from a point source of unit candela

- Luminous exitance (M), Illuminance (E)
  - Similar to radiant flux density, radiant exitance, and irradiance
  - Units of lumens/m$^2$ or lux for both

- Luminous Intensity - I
  - Similar to radiant intensity
  - Luminous flux per unit solid angle
  - Units of candela

- Luminance - L
  - Similar to radiance
  - Units of candela/m$^2$ or the nit
Photon units

Can use the number of photons as the base unit as well and this leads to photon quantities

- Not used in this course, but often easier to use for some detector applications
- Photon number is similar to radiant energy

\[ N_p = \int dN_p = \frac{1}{h} \int Q_v \frac{dv}{v} = \frac{1}{hc} \int Q_\lambda \lambda d\lambda \]

- \( v, \lambda, \) and \( c \) are the frequency, wavelength, and speed of light
- \( N_p \) is number of photons
- \( Q_v \) and \( Q_\lambda \) are the radiant energies in frequency and wavelength
- \( h \) is Planck’s constant

- Photon flux is similar to power/radiant flux [photons/second]
- Photon exitance and photon irradiance [photons/m²]
- Photon intensity [photons/sr]
- Photon radiance [photons/(s sr m²)]
Radiant flux

Radiant flux is a key quantity when determining the behavior of radiometric systems

- Detector of a radiometer collects photons over the field of view of the system
- Radiant flux is a quantitative unit related to the number of photons
- Radiant energy is the quantity more related to the energy collected by the detector
  - Some situations will warrant the use of radiant energy
  - Statistical analysis of detector output (and optical collection) is one case where radiant energy is preferred
  - Remove the integration time effects by using radiant flux
- Goal in the design of almost all radiometric systems is to maximize the radiant flux
- Examine how to maximize radiant flux by
  - Use of collection optics and detector area
  - Increasing field of view
  - Spectral integration
Radiant exitance and irradiance

Radiant flux density has two labels to clarify the geometry of the energy transfer:

- Radiant exitance refers to energy leaving a surface:
  - Usually restricted to the case of emitted energy
  - Symbol is $M$

- Irradiance refers to energy incident on a surface:
  - Symbol is $E$
  - Irradiance is useful when considering the amount of energy incident on a detector or similar surface

Radiant Exitance

Irradiance
Radiance

Radiance is a critical unit for understanding the source and transferring source energy to the radiometer

- Will show that radiance is conserved with distance
  - Assumes that there is no absorption by the medium or optics
  - Makes it easier to compute the incident energy onto a radiometric system

- Use of radiance removes sensor-related effects
  - Area of the collection optics and/or detector
  - Field of view of the radiometer

- Radiant intensity will likewise allow us to readily transfer energy from a source to radiometer to compute radiant flux

- Note that both radiance and radiant intensity have an sr unit
  - Steradian
  - Takes into account solid angle which is covered in lecture 2
Propagation of Radiation

OPTI 509

Lecture 2
Areas and solid angles, projected areas, projected solid angles
Solid angle

Image below is from a 2-D CCD array airborne framing camera system (whole image taken at one time)

- Objects that look larger in this image subtend a larger solid angle at the sensor
- McKale Center subtends a larger solid angle than Meinel Building
Solid angle

Steradian unit of solid angle shows up in radiance and radiant intensity

- Solid angle is a three-dimensional angle
  - Unit is steradian [sr]
  - Mathematically, solid angle is unitless
  - Carry the unit for “clerical” purposes

- Concept of solid angle and steradian not that different from two-dimensional angle (What is a radian, anyway?)

- Steradian unit is defined in terms of a unit sphere
  - Steradian is defined as the solid angle subtended by an area that is equal to the radius of the sphere squared
  - Recall that the surface area of a sphere is $4\pi R^2$
  - Thus, by extension, there are $4\pi$ sr in a sphere
Solid angle “can be shown” in spherical coordinates to be

\[ d\Omega = \sin \theta d\theta d\phi \]

- Where the terms are as follows
  - \( \Omega \) is the solid angle
  - \( \theta \) is the polar or zenith angle
  - \( \phi \) is the rotational or azimuth angle

- Integrating the above gives

\[
\Omega = \int_{\phi_{\text{min}}}^{\phi_{\text{max}}} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \sin \theta d\theta d\phi
\]

and

\[
\Omega = (\phi_{\text{max}} - \phi_{\text{min}})(\cos \theta_{\text{min}} - \cos \theta_{\text{max}})
\]

where the cosine difference is “flipped” due to a negative sign in the integration.

- Above assumes that we are determining solid angle subtended on a circular surface of a sphere
Solid angle of a hemisphere

From the previous formula, it should be clear that the solid angle subtended by a hemisphere is $2\pi$ sr

- The limits on azimuth, or rotation, are 0 to $2\pi$
  - Limits on zenith are 0 to $\pi/2$
  - Then $\Omega=(2\pi-0)(\cos[0]-\cos[\pi/2])=2\pi$ sr

- The solid angle of a sphere is $4\pi$ sr which is what was seen earlier

- These calculations are rather simplistic
  - Unfortunately, typical solid angle calculations not always this basic
  - Thus, need a method to allow computation of solid angle for more complicated situations
Solid angle of a sphere

What about a spherical object as seen from the outside?

- This is a straightforward case analogous to determining the solid angle of the entire earth as seen by a satellite in orbit.
- Diagram at right illustrates the problem.
- The “trick” to this problem is that the solid angle is determined by the tangent view:
  - Limits on zenith are 0 to θ.
  - Then \( \Omega = (2\pi - 0)(\cos[0] - \cos[\theta]) \)
- \( \cos\theta = \frac{(S^2 + 2SR)^{1/2}}{(S + R)} \)
- Then \( \Omega = 2\pi(1 - \frac{(S^2 + 2SR)^{1/2}}{(S + R)}) \)
Solid angle of a square

A bit more difficult is something that is not circular in shape such as the solid angle subtended by a square

- Make the problem a bit easier by considering that the square has length 2S on each side and the distance to the square is S
- There is no rotational symmetry so cannot do the azimuthal integration easily
- Switch to Cartesian coordinates and the solid angle is written in terms of distance and radial direction from the observation point

\[
\Omega = \int \int_S \frac{\hat{n} \cdot d\vec{r}}{r^2}
\]

- The following relationships allow the integral to be put in Cartesian coordinates
  \[ r^2 = x^2 + y^2 + z^2 \]
  \[ \hat{n} \cdot d\vec{r} = \cos \theta dx dy \]
  \[ \cos \theta = \frac{z}{r} = \frac{z}{\sqrt{x^2 + y^2 + z^2}} \]
Solid angle of a square

Use the previous relationships to rewrite the integral in terms of x, y, and z

- This gives

\[
\Omega = \int \int \frac{z \, dx \, dy}{(x^2 + y^2 + z^2)^{3/2}}
\]

- Next assume the square is oriented perpendicular to the z axis at a distance S from the observation point (z=S)

\[
\Omega = \int_{-a}^{a} \int_{-a}^{a} \frac{a \, dx \, dy}{(x^2 + y^2 + a^2)^{3/2}}
\]

- Solving this integral gives \(2\pi/3\) sr
  - Makes sense from a qualitative sense
  - Assume a cube of side 2S
  - Each face of the cube is identical in size and equal distance from the center of a reference sphere
  - Six faces into \(4\pi\) sr of a sphere gives \(4\pi/6=2\pi/3\) sr
Solid angle of a disk

What about the solid angle subtended by a flat disk (thus rotationally symmetric)

Consider the geometry to the right
- S is the distance to the disk
- R is the radius of the disk
- dr is an incremental change in the disk radius
- S’ is the distance to edge of dr
- r is the radius to edge of dr

Then the differential solid angle subtended by a thin ring of the disk is

\[ d\Omega = \frac{(2\pi \cos \theta)rdr}{(S')^2} \]

Integrating this realizing that \( \cos \theta = S/r \) and \( S' = (r^2 + S^2)^{-1/2} \) gives

\[ \Omega = \int_0^R \frac{2\pi S r dr}{(r^2 + S^2)^{3/2}} \]
Solid angle of a disk

The disk integral, while not trivial can be solved from a standard table of integrals

Thus,
\[ \Omega = \int_{0}^{R} \frac{2\pi S r dr}{(r^2 + S^2)^{3/2}} = -2\pi S \frac{1}{\sqrt{r^2 + S^2}} \bigg|_{0}^{R} \]

Doing a bit of algebra gives
\[ \Omega = -2\pi S \frac{1}{\sqrt{r^2 + S^2}} \bigg|_{0}^{R} = -2\pi S \left[ \frac{1}{\sqrt{R^2 + S^2}} - \frac{1}{S} \right] = 2\pi \left[ 1 - \frac{S}{\sqrt{R^2 + S^2}} \right] \]

This is the analytic solution for the solid angle subtended by a disk.

This is readily done, but still, there must be an easier way.
Solid angle of a disk

Assuming large distances gives us a simplification

- Binomial expansion of the disk solution gives

\[
\Omega = 2\pi \left[ 1 - \left( 1 - \frac{1}{2} \frac{R^2}{S^2} + \frac{3}{8} \frac{R^4}{S^4} + \ldots \right) \right] = \pi \left[ \frac{R^2}{S^2} + \frac{3}{4} \frac{R^4}{S^4} + \ldots \right]
\]

- Next, assume that the distance \( D \) is large relative to \( R \)
  - Higher order terms go to zero
  - Left with

\[
\Omega = \pi \left[ \frac{R^2}{S^2} + \frac{3}{4} \frac{R^4}{S^4} + \ldots \right] = \pi \frac{R^2}{S^2} = \frac{\text{Area}_{\text{disk}}}{S^2}
\]

- Thus, the solid angle for cases when the distances are large relative to the object can be determined by the ratio of the area of the object to the distance squared
Solid angle - simplified formula

Solid angle can be adequately computed by the ratio of the area of the object of interest to distance squared

Then

\[ \Omega = \frac{Area}{S^2} \]

This actually goes back to the original definition for the steradian
- Area equal to the radius of the sphere squared
- Could do all of our calculations this way
- Trick is determining the area on the sphere that is enclosed by our physical object

That is, need to project the physical area onto the sphere

This projection gets easier at small solid angles (small objects at large distances)
Solid angle approximation

The next step is to determine when this area over distance squared approximation “fails”

- Failure depends on the application and the level of error that can be tolerated

- Assume that the area of the sphere that is intercepted by the solid angle can be assumed to be a disk
  - Solid angle subtended by the circular portion of the sphere is $2\pi(1-\cos\theta)$
  - Solid angle of the disk for our simplified formula is $\pi R^2/D^2 = \pi \tan^2 \theta$
Solid angle approximation

The error due to making the area approximation can be determined as a function of $\theta$

- Exact formulation is based on the spherical coordinates integration
- Approximate formula is the ratio of area of the disk to the distance squared
- Difference is small for small $\theta$ angles
  - Alternatively, the approximation is accurate for small solid angles
  - $< 2\%$ error out to half angles of 10 degrees
Solid angle approximation

Could also use $\sin^2$ instead of $\tan^2$ if the distance $S$ is to the edge of disk

- Difference is still smaller for small $\theta$ angles
  - Alternatively, the approximation is accurate for small solid angles
  - < 2% error out to half angles of 16 degrees

- Slightly better approximation than the tangent formulation
Solid angle computations

Sine approximation will work most of the time in this class but application will determine accuracy requirement

- Must have an angle less than 11.5 degrees for errors <1%
- Angles less than 3.7 degrees are needed for 0.1% errors
- The errors in terms of size are
  - 2% error when the distance squared is about four times the area
  - 1% error when the distance squared is about eight times the area
  - 0.1% inaccuracy when the distance squared is 80 times the area
Solid angle and optics

F-number (F/#) and numerical aperture (NA) are both related to solid angle subtended by the collection optics

- Recall F/#=f/D_{pupil} where D_{pupil} is the entrance pupil diameter and f is the effective focal length
  - Then if \( \alpha \) is the half angle it follows that F/#=1/(2\tan \alpha)
  - Likewise, if the system is well-designed (aplanatic and following the Abbé sine condition) you can show that F/#=1/(2\sin \alpha)

- Numerical aperture for the case of a system operating in air is NA=\sin \alpha

- Then \( \Omega=\pi/(4 \text{ F/}#^2) = \pi(\text{NA})^2 \)
  - Will see something similar to this later in the Camera Equation
  - Hopefully make sense physically
    - Faster systems (small F/#) have larger solid angles
    - Systems with a larger NA have larger solid angles
Units and quantities

Key to radiometry is the interplay between the quantities given previously

- Remind ourselves of the radiometric units and quantities
  - Radiant Energy - Q in J [Joules]
  - Radiant Flux (or power) - $\Phi$ in W [Watts or J/s]
  - Radiant flux density - [W/m$^2$]
    - Radiant exitance - M
    - Irradiance - E
  - Radiant intensity - I [W/sr]
  - Radiance - L [W/(m$^2$ sr)]

- Radiant flux strongly determines the quality of the signal output

- Radiance is critical for understanding the physical nature of the source

- Radiant intensity and radiant flux density are useful in transferring energy from source to sensor

- Determine the relationships between each of these quantities
Simplified relationships

At one level, the relationships between the energy quantities follow logically from the units of each

- True when the areas are much smaller than the distance between them and radiance does not vary much spatially
  - Object of interest is located far from a sensor
  - Change in radiance from object is small over the view of a sensor

- Radiant flux
  \[ \Phi = L \times A \times \Omega \]
  \[ \Phi = E \times A \]

- Irradiance
  \[ E = L \times \Omega \]

- Not quite this simple
  - Solid angle calculation is not simple
  - Area also has subtle effects
Projected area

Cannot claim that objects with the same solid angle are the same size

- The moon and the sun subtend about the same solid angle at the earth’s surface (think about a solar eclipse) but they are not the same size

- The football field and parking garage at right subtend about same solid angle
  - Garage is a 3-D object with three parking levels below the roof
  - Football field is a 2-D object
  - Stadium seats are a 3-D object tilted relative to the camera
Projected area

The stadium seats and parking garage present an interesting example

- The camera sees the projected area of the stadium seats
- Projected area is the planar area seen from the observation point
- Clear that the angle of the object relative to the observer plays a role in the solid angle
- Note that it is also important to get the right distance as shown by the parking garage
Projected area

In the simplest case, the projected area and actual area are related by a cosine factor:

- Projected area is the physical area projected onto the plane that is orthogonal to the observation point.

Then \( \text{Area}_{\text{projected}} = \text{Area} \times \cos \theta \)

- \( A \) is the physical area of the object.
- \( \theta \) is the angle between the normal to the physical area and the observation point.
Physical area

Physical area encompassed by a given solid angle depends upon the distance and the angle to the observer.

- For the case at right, the solid angle between the on-axis and off-axis cases is constant.
- This would be the case for a well-designed radiometer pointing off-axis.
- The distance between the object on axis to the same object off-axis is related by cosine.

\[ S_{\text{off-axis}} = \frac{S}{\cos \theta} \]

- The area seen by the instrument off-axis is

\[ \text{Area}_{\text{off-axis}} = \frac{\text{Area}_{\text{on-axis}}}{\cos^3 \theta} \]

- Assumes that the area is much less than the distance squared.
Projected solid angle

Projected solid angle (or weighted solid angle) is related in philosophy to the projected area

- The angle $\psi$ is the angle between the plane of the observer to that of the plane of the object

$$\Omega_{projected} = \int_{\varphi_{\text{min}}}^{\varphi_{\text{max}}} \int_{\theta_{\text{min}}}^{\theta_{\text{max}}} \cos \psi \sin \theta d\theta d\phi$$

- In most cases, $\psi=\theta$, but this is not always the case

- Using projected solid angle allows the geometry of the source or detector to be treated separately when doing flux computations

- The cosine weighting factor can also be added later when doing the energy computations
  - Reduces confusion as to whether using projected solid angle or solid angle
  - Fits in nicely with the cosine law that we will see later
Projected solid angle - hemisphere

Classic illustration of projected solid angle is the hemispheric case

- Recall that solid angle subtended by a hemisphere is $2\pi \text{ sr}$
- Project this solid angle to a flat plane gives

$$\Omega_{\text{projected}} = \int_0^{\pi/2} \int_0^{2\pi} \cos \theta \sin \theta d\theta d\phi = 2\pi \left( \frac{\sin^2 \theta}{2} \right) \bigg|_0^{\pi/2} = \pi \text{ sr}$$

- $\psi = \theta$ in this case
- Most cases the difficulty is determining what $\psi$ is
Simplified formulas

Rewrite the simplified formulas including projected area and solid angle effects

- We’ll see more details on this later related to specifically what areas and what solid angles to use
- Radiant flux
  \[ \Phi = L \times A \times \cos\theta \times \Omega \]

  \[ \Phi = E \times A \times \cos\theta \]

- Irradiance
  \[ E = L \times \Omega \]

- \[ \Omega = \frac{A_{\text{projected}}}{D^2} \]

Projected Area

\( \Omega \) of the Object as seen by detector
Propagation of Radiation

OPTI 509

Lecture 3
Radiance, throughput

“Say... Now I'm starting to feel kinda warm!”
Why use radiance?

Radiance can either be incident onto a surface or come from a surface area in a specified direction

- One of the most important radiometric quantities
  - It is conserved with distance in a non-scattering, non-absorbing medium
  - Removes all geometric dependencies
- Units of radiance are $W/(m^2 \text{ sr})$
- Diagrams below attempt to illustrate the concept

![Diagram illustrating radiance](image-url)
Radiance

Radiance is the radiant flux in a specified direction as the differential area and solid angle go to zero

- Physically, we are counting the number of photons that are travelling through an infinitely small solid angle from an infinitely small area in a direction from the area given by $\theta$

\[
L = \lim_{\Delta A \to 0} \lim_{\Delta \Omega \to 0} \left[ \frac{\Delta \Phi}{\Delta A \Delta \Omega \cos \theta} \right] = \frac{d^2 \Phi}{dA d\Omega \cos \theta}
\]

- Radiant energy is the basic energy unit
  - $d\Phi = dQ/dt$
  - Will almost always exclusively go to radiant flux rather than radiant energy

- Radiance is the basic radiometric unit from which the other quantities will be determined

- Ignore spectral aspect now for simplicity of notation
Radiance

Examine radiance from a geometric viewpoint first by examining the energy traveling between two areas

- Consider a bundle of rays that passes through both areas
  - Projected area of $A_1$ that is seen by $A_2$ is simply $A_1 \cos \theta_1$
  - Likewise, the area of $A_2$ that is seen by $A_1$ is simply $A_2 \cos \theta_2$

- Solid angles can be shown to be
  - $\Omega_2 = A_2 \cos \theta_2 / S^2$ is the solid angle subtended by area $A_2$ at area $A_1$
  - $\Omega_1 = A_1 \cos \theta_1 / S^2$ is the solid angle subtended by area $A_1$ at area $A_2$
Radiance

Product of projected area of one surface and solid angle subtended by other is the entire bundle

- This product gives identical values for both directions
- Case of $A_1$ looking at $A_2$
  \[ A_1 \cos \theta_1 \Omega_2 = A_1 \cos \theta_1 \frac{A_2 \cos \theta_2}{S^2} \]
- Case of $A_2$ looking at $A_1$
  \[ A_2 \cos \theta_2 \Omega_1 = A_2 \cos \theta_2 \frac{A_1 \cos \theta_1}{S^2} \]
- This $A\Omega$ product will show up again later
Radiance conservation

Can now show that the radiance leaving area $A_1$ is identical to that incident on area $A_2$

- Implies radiance is conserved
- Now consider the energy passing through the areas
  - Concept is that our $A\Omega$ product encompasses all rays that pass through both areas
  - Consider the rays to be related to energy

- Radiant flux from all points on area $A_1$ that passes through all points on area $A_2$ is $L_1A_2 \cos\theta_2\Omega_1$
- Radiant flux from all points on area $A_2$ that passes through all points on area $A_1$ is $L_2A_1\cos\theta_1\Omega_2$
- Total number of photons collected is the same in either case
  - $A\Omega$ product is independent of direction
  - One concludes that $\Phi_1=\Phi_2$
  - Then $L_1=L_2$
- Radiance is conserved in a lossless medium
Basic radiance and conservation

Basic radiance takes into account changes in index of refraction

- Assume there are no other losses at the boundary of two indexes of refraction.
- The amount of radiant energy through the boundary does not change
- Change in index of refraction causes the incident radiance to alter direction
  - Change in direction leads to a change in solid angle
  - Change in solid angle alters the radiance (remember the per unit solid angle in radiance)
- From a qualitative standpoint, expect the radiance to be larger in medium 2
  - The same radiant flux is confined to a smaller solid angle in medium 2
  - The radiance is larger for medium 2
Basic radiance

Show this more quantitatively using Snell’s Law and the relationship between radiant flux and radiance

- Recall
  - \( \text{d}\Phi = L \text{d}A \text{d}\Omega \cos\theta \)
  - Snell’s Law is \( n_1\sin\theta_1 = n_2\sin\theta_2 \)
  - Derivative of Snell’s law gives \( n_1\cos\theta_1 \text{d}\theta_1 = n_2\cos\theta_2 \text{d}\theta_2 \)

- Determine the radiance onto the area of the interface \((L_1)\) and the radiance from the interface \((L_2)\)

- The ratio of the radiance on either side of the boundary is

\[
\frac{L_1}{L_2} = \frac{\frac{d\Phi_1}{d\Omega}}{\frac{d\Phi_2}{d\Omega}} \frac{dA \cos\theta_2}{dA \cos\theta_1}
\]

- Substituting for the solid angle gives

\[
\frac{L_1}{L_2} = \frac{\frac{d\Phi_1}{d\Omega}}{\frac{d\Phi_2}{d\Omega}} \frac{dA \cos\theta_2 \sin\theta_2 \text{d}\theta_2 \text{d}\phi_2}{dA \cos\theta_1 \sin\theta_1 \text{d}\theta_1 \text{d}\phi_1}
\]
Basic radiance

The radiant fluxes in both media must be identical since there is no absorption (this will come up repeatedly)

- Derivation of Snell’s law shows that the refracted beam and incident beams are in the same plane and this means that \(d\phi_1 = d\phi_2\)
- Likewise, the areas are identical
- Then
  \[
  \frac{L_1}{L_2} = \frac{\cos \theta_2 \sin \theta_2 d\theta_2}{\cos \theta_1 \sin \theta_1 d\theta_1}
  \]

- Substituting for Snell’s law such that \(\sin \theta_2 = (n_1/n_2)\sin \theta_1\) and for \(\cos \theta_2 \, d\theta_2 = (n_1/n_2)\cos \theta_1 \, d\theta_1\)
  \[
  \frac{L_1}{L_2} = \frac{n_1 \sin \theta_1}{n_2} \frac{n_1 \cos \theta_1 \, d\theta_1}{n_2 \cos \theta_1 \sin \theta_1 \, d\theta_1} = \frac{n_1^2}{n_2^2}
  \]
- Or \(L_2 = \frac{L_1 n_2^2}{n_1^2}\) and the radiance in medium 2 is larger
Throughput, etendue

The $A\Omega$ product contains all of the geometric effects related to the radiometric system

- Product is so pervasive and important that it is studied in and of itself.
- $A\Omega$ is the throughput of the system.
- Also called the geometric extent.
- The situation we are working towards is the case of a detector collecting photons:
  - The entire area of the detector collects photons.
  - Each elemental area on the detector has a finite field of view (solid angle) over which to collect photons.
- Throughput is not concerned with direction of energy flow.

\[ T = A\Omega = A_2 \frac{A_1}{S^2} = A_1 \frac{A_2}{S^2} \]
Throughput

Previous formulation assumes large distances and small areas

- The full formulation requires use of the integral form

\[ T = \int \int dA d\Omega = \int \int \int dA \sin \theta d\phi d\theta \]

- This still assumes that the two areas are on axis
  - The more general case is below
  - This adds an additional cosine factor in the approximate formula

\[ T = \int \int \int \cos \theta dA \sin \theta d\phi d\theta \approx A \Omega \cos \theta \]
Throughput

All you are really trying to do is to convert the geometry of any arbitrary case to that of the simple situation

\[ T = A\Omega = A_2 \frac{A_1}{S^2} = A_1 \frac{A_2}{S^2} \]
Why do we care?

Throughput calculations allow for “quick” comparisons of radiometric systems

- Determine which system should collect more energy
- As an example consider the following
  - Radiometer with a circular detector 0.1 mm in radius
  - Optical system is a simple tube that is 1 cm in radius and 10 cm long
- Thus, S=10 cm, and both radii are more than a factor of 10 smaller than the distance (thus we can justifiably use our approximation)
- Then \[ T = \frac{\pi \times 0.1^2 \times \pi \times 10^2}{100^2} = 9.9 \times 10^{-4} \text{ mm}^2 \text{ sr} \approx 9.9 \times 10^{-10} \text{ m}^2 \text{ sr} \]

\( R_{\text{detector}} = 0.1 \text{ mm} \quad \quad R_{\text{aperture}} = 1 \text{ cm} = 10 \text{ mm} \)

Tube length = 10 cm = 100 mm
Increasing etendue

Hopefully clear that increasing solid angle increases the throughput

- Increase the solid angle seen by the detector
  - Decrease tube length (though if it is too short we can no longer use the simplified formulation)
  - Increase the aperture size

- Could increase the solid angle of the detector but will do this as an increase in area

\[ \text{R}_{\text{aperture}} = 1 \text{ cm} \]
\[ \text{R}_{\text{detector}} = 0.1 \text{ mm} \]

\[ \text{Tube length} = 5 \text{ cm} \]

\[ \text{R}_{\text{aperture}} = 2 \text{ cm} \]
\[ \text{R}_{\text{detector}} = 0.1 \text{ mm} \]

\[ \text{Tube length} = 10 \text{ cm} = 100 \text{ mm} \]
Increasing etendue

Increasing the area of the detector is the most straightforward way to increase the throughput

- Ignoring other effects such as vignetting that can limit the field view
- That is, the example here assumes that entire detector can see the entire entrance aperture
- The case shown here and those on previous viewgraph give a throughput that is larger by a factor of four from the original case

\[ R_{\text{detector}} = 0.2 \text{ mm} \quad R_{\text{aperture}} = 1 \text{ cm} = 10 \text{ mm} \]

Tube length = 10 cm = 100 mm
Magnitude of throughput

Is the example shown a large value for throughput?

- It is difficult to say what is considered a large throughput.
- Need to consider the source energy.
  - A small throughput will work fine when looking at the sun (do not need a 36-inch telescope to count sunspots).
  - Same throughput is not adequate to look for magnitude 10 stars.
- Also need to consider the detector and signal processing package.
- However, as an example that may be familiar consider a typical digital camera.
  - Large detector size would be around 20 micrometers.
  - Fast camera would have an F/# of 1.8.
  - Then the throughput would be:

\[
T = A\Omega = \left[\pi \times (2.0 \times 10^{-5})^2 \right] \left[\frac{\pi}{4(1.8)^2}\right] = 3.0 \times 10^{-10} \text{ } m^2 \text{sr}
\]
Throughput cautions

Can use throughput for more complicated optical systems but must not overextend the simplistic thinking

- Assume that we have a camera system with a manual lens
  - Adjustable f-stop
  - We can vary the F/# of the lens
  - Recall that $\Omega = \pi/(4 \ F/\#^2)$
  - Then changing the F/# by a factor of square root of 2 will change the throughput by a factor of 2

- Ignores other effects like
  - Stray light
  - Reflections
  - Poorly designed optics
  - Variability in the grain size of the film

- Still allows us to design a camera lens so each click of the f-stop means about a factor of two more light (for smaller F/#)
Throughput calculations

The subtlety in throughput calculations is to ensure that the correct areas/solid angles are being used

- Need to ensure that the tilts of the surfaces are taken into account
  - Projected areas
  - Projected solid angles

\[
T = \int_{\phi} \int_{\theta} \int_{A} \cos \theta dA \sin \theta d\theta d\phi \approx A \Omega_{proj}
\]

- Less obvious aspect is to ensure that the limiting areas are used
  - This is where the source can play a role
  - Consider our earlier case
  - The original radiometer’s throughput is \(9.9 \times 10^{-10} \text{ m}^2\text{sr}\)
  - In this case the entrance aperture and detector are the limiting areas

\(R_{\text{detector}} = 0.1 \text{ mm}\)

\(R_{\text{aperture}} = 1 \text{ cm} = 10 \text{ mm}\)

Tube length = 10 cm = 100 mm
Throughput calculations

Now consider the two cases shown below of an extended source and a small source

- Assume in case 1 that the radiometer is viewing a large diffuser screen
  - The diffuser screen fills the entire field of view of the sensor
  - The throughput is the same as computed before

- Can see how distance can play a role in this in that it is the solid angle subtended by the source not the physical size that is important
Throughput calculations

Consider the second case where the same radiometer now views a “smaller” source

- Smaller source can be something physically smaller or the same source at a larger distance
- Diffuser underfills the field of view of the sensor
  - The throughput of the system is smaller than previously computed
  - The throughput of the radiometer is still the same
  - It is the throughput of the radiometric system that has changed
- Should always compute the throughput of the radiometric system
- Still have not included the source energy in the calculation
Basic throughput accounts for refractive effects when the area and solid angle are not in the same medium.

- In a similar fashion as basic radiance can show that basic throughput (or optical extent) is
  \[ T_{\text{basic}} = n^2 T \]
- Consider identical radiometers viewing an identical source
- Upper radiometer is operating in air
- Lower radiometer is operating in water
  - Throughput of the lower case is then \((n_{\text{air}}/n_{\text{water}})^2 T_{\text{air}}\)
  - Can see that the throughput in water is smaller
Propagation of Radiation

OPTI 509

Lecture 4

Invariance of throughput and radiance, Lagrange invariant, Radiative transfer, radiant flux
Interesting aspect to throughput is that it does not matter which area/solid angle pair is used

- Assumes that the pairing of solid angle and area is done correctly
  - One phrase used is “no snow cones”
  - Referring to the radiometer example, use the area of the detector and the solid angle of the aperture as seen by the detector

- Straightforward when using simplified formulas
  - Simply make sure both areas are used
  - Still have to be concerned about which angles to use for the projected area and projected solid angle effects

- Often an issue when doing integral formulation
Throughput invariance

Once it is ensured that both areas are used then it should be clear that either solid angle/area pair can be used

- Recall earlier simplified example repeated below
  - Throughput can be computed using the solid angle subtended by the area $A_1$ as seen by area area $A_2$
  - Or, use the solid angle subtended by the area $A_2$ seen by $A_1$

\[ T = A \Omega = A_2 \frac{A_1}{S^2} = A_1 \frac{A_2}{S^2} \]

- Why do we care?
  - Goal is to compute the amount of energy collected by a detector through an optical system or from a source of known size
  - Seems that the area of the detector and the solid angle of the source (or optics) as seen by the detector would always be used

- Some cases are much easier when you “flip” the calculations
Throughput Invariance

Throughput is summing all of the solid angles subtended by an area as seen by all points on a second area.

\[ T = \int_{\Omega_1} \int_{A_2'} \cos \theta dA d\Omega \]

Or

\[ T = \int_{\Omega_2} \int_{A_1'} \cos \theta dA d\Omega \]

BUT NEVER

\[ T = \int_{\Omega_1} \int_{A_1} \cos \theta dA d\Omega \]
Throughput invariance

Physically, we are saying that we don’t care which direction the photons are traveling

- Assume the system is designed to operate left to right and collects 100 photons
  - Have to see the photons to collect them (field of view or solid angle aspect)
  - Also need something for the photons to travel through (area)
- Reverse the direction
  - Collecting area now becomes the source
  - Original source area becomes the collector
  - Still collect 100 photons
- Remember that this assumes the correct pairing of areas
  - Things become tricky when the source does not fill the field of view
  - Optical system drives the solid angle/area pair for the overfill case
  - Size of source limits the solid angle/area pair for the underfill case
Throughput invariance plays a big role when considering an optical system

- Use the simple optical system below as an example
- Energy from a source of area $A_{\text{source}}$ enters a single lens system
  - Area $A_{\text{lens}}$
  - Exiting photons just fill the detector with area $A_{\text{detector}}$
- Optical system (entrance pupil) subtends a solid angle, $\Omega_{\text{entrance}}$, as seen from the source
- Optical system (exit pupil) subtends a solid angle $\Omega_{\text{exit}}$ as seen from the detector
Throughput invariance

Then the throughput of the radiometric system can be written in several ways:

- In terms of the lens collecting the source energy it is \( T = A_{\text{source}} \Omega_{\text{entrance}} \).
- The detector collecting energy from lens gives \( T = A_{\text{detector}} \Omega_{\text{exit}} \).
- Then using throughput invariance can also write:
  - Then \( T = A_{\text{lens}} \Omega_{\text{source}} \),
  - Then \( T = A_{\text{lens}} \Omega_{\text{detector}} \).
- All four are equivalent.
- Can use the entrance and exit pupils in the more complicated optical case:
  - This allows the designer to work in either image or object space.
  - \( T = A_{\text{source}} \Omega_{\text{entrance}} = A_{\text{entrance}} \Omega_{\text{source}} = A_{\text{detector}} \Omega_{\text{exit}} = A_{\text{exit}} \Omega_{\text{detector}} \).
When invariance helps

Consider the case of the throughput for a system that has two areas separated by distance $H$

- Assume one area is much larger than the other area and $H$
- For example
  - Circular area with radius 1000 m
  - Second circular with 1 m radius
  - Separation distance $H=1$ m
- Simplistic approach would be to use the areas and in this case the throughput would be $(\pi \times 1^2)((\pi \times 1000^2)/(1^2))=9.87 \times 10^6$ m$^2$ sr
- Can’t do this because the areas are not small relative to the distance

\[
A_1 = 3.14159 \text{ m}^2 \\
A_2 = 3141590 \text{ m}^2
\]
When invariance helps

Need to use the integral form of throughput in this case of a large area relative to the distance

- One approach would be to use the area \( A_2 \) and the solid angle subtended by \( A_1 \) at that distance
  - Then
  \[
  T = \int_{A_1} \int_{A_2} \cos \theta dA d\Omega
  \]
  - Not trivial because the cosine term depends on what part of the area \( A_2 \) you are using

- In other words, the smaller disk changes its appearance/solid angle as you move along the large area
When invariance helps

Flipping the problem around and using the solid angle of the large disk and area of the smaller simplifies things

- Thus, use Area, $A_1$ and the solid angle subtended by the larger area
- Integral formulation becomes
  \[ T = \int_{0}^{2\pi} \int_{0}^{89.94^\circ} \cos \theta \sin \theta dA d\theta d\phi \]
- Readily soluble,
- Further, in the limit of an infinitely large area, $A_2$ (only an extra 0.06 degrees) we get a throughput of $\pi A_1$
- When would we see this situation?
  - Bare detector viewing the ground or sky
  - Attempts to illuminate a large area using a lamp
Another invariant is the Lagrange invariant and this too is related to throughput and radiance invariance.

- Recall that the Lagrange invariant relates the chief and marginal rays.
  - The size of the image (chief ray) is related to the size of the optical system (F/#).
  - Small angle approximation.
  - \[ N = n(H_{\text{marginal}} \alpha_{\text{chief}} - H_{\text{chief}} \alpha_{\text{marginal}}) \] where \( n \) is the index of refraction.
- Solid angle of the image is related to the solid angle of the object.
Lagrange invariant

Assume that the system is paraxial and objects are small relative to distances

- Then $\sin \alpha \approx \alpha$ giving $\Omega = \pi \alpha^2$
- Large distance versus object size allows us to use the area over distance formula where $A = \pi H^2$
- Then $H \alpha = (A\Omega/\pi^2)^{1/2}$ and

$$N = \frac{n}{\pi} \left( \sqrt{\Omega_{\text{chief}} A_{\text{marginal}}} - \sqrt{\Omega_{\text{marginal}} A_{\text{chief}}} \right)$$

- At the image plane, $H_{\text{marginal}} = 0$ and

$$N^2 = \left( \frac{n}{\pi} \right)^2 \Omega_{\text{marginal}} A_{\text{chief}}$$

- At the pupil plane, $H_{\text{chief}} = 0$ and

$$N = \frac{n}{\pi} \left( \sqrt{\Omega_{\text{chief}} A_{\text{marginal}}} \right)$$
Lagrange invariant

Look at the problem more generally in terms of the relationship between throughput and Lagrange invariant

- Take the pupil plane case
- Then

\[ N = \frac{n}{\pi} \sqrt{\Omega_{\text{chief}} A_{\text{marginal}}} = \frac{n}{\pi} \sqrt{T_{\text{image plane}}} \]

- Generalizing for throughput
  - Invariance of throughput
  - Recall example through a lens
  - Show that the Lagrange invariant and throughput are related via

\[ N = \frac{n}{\pi} \sqrt{T} \]

- Another way to think about this is that the Lagrange invariant is related to the light gathering ability of the radiometric system
Throughput and radiance

Can relate three quantities to one another - Radiance, Throughput, and Radiant Flux

- Usually know two of these for a given radiometric system and can then infer the third for future use

- For example, consider a calibrated system
  - Know the relationship between radiant flux through the detector and the output from the signal processing system
  - Have a known radiance source incident on the system
  - Measure the reported radiant flux
  - Infer the throughput

- Once throughput is known we can then look at an arbitrary source
  - Record the radiometer output and convert to radiant flux
  - Infer the radiance from the throughput and the radiant flux

- Clearly, need to determine the radiant flux versus output relationship
  - Done theoretically at some level using known detector properties
  - This then relies on knowledge about the detector
Consider the example of a radiometric system viewing a source of known radiance

- Single lens system coupled to a fiber coupled to another lens system which then illuminates the detector
  - Illustrated schematically below
  - All of the light collected by the first lens system is captured by the fiber optic
  - All of the light exiting the fiber optic is captured by the second lens system
  - All of the light from the last optical element is collected by the detector

- Goal is to compute the radiant flux on the detector
Radiant flux and invariance

The fact that radiance is conserved and that throughput is invariant simplifies the calculation

- All that is needed is to know
  - Area of the entrance pupil
  - Area of the source as seen by the first lens system
  - Distance from the entrance pupil to the source
  - Second two are equivalent to the field of view or NA

- Then the radiant flux through the pupil is

\[ \Phi = L_{\text{source}} A_{\text{source}} \Omega_{\text{entrance}} \]

- This has to be the same radiant flux as that incident on the detector
  - Photons are not lost anywhere along the way
  - If a photon is collected by the entrance pupil it must hit the detector

- Admittedly, this is a simplified problem
  - Ignore losses at interfaces and in the fiber
  - Assumes perfect coupling
  - These factors can be readily calculated permitting the radiant flux on the detector to be determined
Equation of radiative transfer

Take these examples a bit further and look more closely at the relationship between radiant flux and radiance

- Equation of radiative transfer
  \[ d^2 \Phi = \frac{L(\theta, \phi, d) \cos \theta_1 dA_1' \cos \theta_2 dA_2'}{S^2} \]

  - Seen a portion of this is in the throughput calculations
  - Also shown in converting from one energy “type” to another

- Note that radiance is written as a function of distance to account for transmission losses

- The areas, angles, and distance are just as before in previous discussions

- The first cosine factor allows for tilt of the source radiance area to account for the projected area normal to the second area

- The second cosine factor takes into account projected solid angle
Equation of radiative transfer

Radiant flux depends on the apparent size of the source, size of the collector, and the radiance from the source.

- This becomes clearer when using the integral form of the equation:

\[
\Phi = \int_{A_1} \int_{A_2} \frac{L(\theta, \phi, d) \cos \theta_1 \cos \theta_2}{S^2} \, dA_1 \, dA_2
\]

- This problem is not straightforward in general because:
  - Distance can vary across the areas we are using
  - Radiance can vary as a function of angle
  - Angles can vary across the areas we are using
  - Radiance can be attenuated as we travel through are transmission media
An alternative form for the equation of radiative transfer can be written in terms of solid angle.

- Simpler in format, giving
  \[ \Phi = \int_{\Omega} \int_{A} L(\theta, \phi, d) \cos \theta d\Omega \ dA \]

- Can be more difficult to use
  - Distance effect is included in the solid angle
  - One of the cosine factors is taken into account in the solid angle calculation
  - Second cosine factor leads to the projected solid angle

- This was the form used earlier

- Will see later that this form can make sense once the cosine law is introduced
Radiance invariance

In reality, this is conservation of radiance but the concept of radiance invariance matches throughput invariance

- The primary reason that radiance plays such a key role in radiometry is that it does not vary with location
- That is, radiance is conserved in a lossless medium
- This is a result of the fact that radiance was designed to be this way
- Physically, as one moves away from a source of radiation nothing changes geometrically from a radiance point of view
  - The differential area over which the radiation was emitted does not change size as you move away from it
  - Differential solid angle in limit to zero solid angledoes not change
  - Think of it as following a single “ray” of photons leaving a surface
Radiance conservation

Optical system used cannot change the radiance (once index of refraction effects are taken into account)

- Assumes a lossless set of optics (kind of like the frictionless surface and weightless string)
  - The radiance on the focal plane of a radiometer is IDENTICAL to the radiance from the source if the focal plane is in the same medium as the source
  - Applies to both imaging and non-imaging systems

- One advantage to this thinking is that now there is no need to worry about the sensor
  - Can deal with the radiance on the entrance aperture
  - Helpful because models predicting the radiance incident on an optical system have an easier time than optical design packages
  - For example, consider a 33-element fisheye lens with less than perfect anti-reflection coating

- One goal in radiometry is to determine the response of the system to these input radiances
Radiant flux is not conserved

Can see from the equation of radiative transfer that radiant flux is not conserved with distance

- Recall throughput depends on the two areas being considered and the distance between them
- Throughput decreases with the distance between the areas
- Radiant flux can be shown to be $\Phi=TL$
  - Thus, radiant flux decreases with distance as well
  - Using $T=AO$, then $\Phi=LAO$
  - Once again, the actual formulation is an integral

$$\Phi = \int_{\Omega_{\text{receiver}}} \int_{\Delta_{\text{source}}} L_{\text{source}} \cos \theta dA d\Omega = \int_{\Omega_{\text{source}}} \int_{\Delta_{\text{receiver}}} L_{\text{source}} \cos \theta dA d\Omega \approx LAO \cos \theta$$

- If $\theta \approx 0$, then $\cos \theta \approx 1$ and if radiance is relative constant over the area and solid angle we get back to the simplified formulation of $\Phi=LAO$
  - For solid angle and throughput only worry about geometry factors
  - Now, also need to ensure that the radiance behaves properly within the integral in order to simplify
Examples of radiance and radiant flux

Consider case below where both radiometers are at the same distance but different solid angles

- Radiant flux measured by the two radiometers will differ significantly
- Radiance is still identical for both sensors

Homogeneous surface - radiance same in all directions and positions
Radiance and radiant flux

Solid angle same for both sensors
Area of the source that is seen by Sensor 1 is four times that of Sensor 2

Height = H

Sensor 1

Height = 1/2 H

Sensor 2

Homogeneous surface
Radiance and radiant flux

Solid angle seen by Sensor 2 is twice that seen by sensor 1.
Area seen by both radiometers is identical.

Height = H
Height = 1/2 H
Propagation of Radiation
OPTI 509

Lecture 5
E, I, M, reflectance, transmittance, absorptance, emissivity, Planck’s Law
Summarize quantities so far

At this point we have the following radiometric quantities excluding radiant energy

- **Radiant Flux - \( \Phi \)**
  - Rate at which radiant energy is transferred from a point on a surface to another surface
  - Typical units of W [Watts or J/s]

- **Radiant Flux Density - \( E \) and \( M \)**
  - Rate at which radiant energy is transferred per unit area
  - Radiant Exitance denoted by \( M \) and units of W/m\(^2\)
  - Irradiance denoted by \( E \) and units of W/m\(^2\)

- **Radiant Intensity - \( I \)**
  - Radiant flux per unit solid angle
  - Units of W/sr

- **Radiance - \( L \)**
  - Radiant flux per unit solid angle per unit area
  - Units of W/(m\(^2\) sr)
Radiant flux density

Have examined radiant flux (W), radiance (W/[m^2sr]), and throughput (m^2sr)

- Now look at other quantities that can be formed by combining radiant flux and other geometric factors
- First look at the ratio of radiant flux to the area \( -d\Phi/dA \)
- Radiant flux density is the general term applied to this quantity
- Units are W/m^2
- There is no directionality to radiant exitance (hence no solid angle unit)
  - Referenced relative to the normal to the area
  - More of an issue for incident energy
- Make a terminology distinction between the incident and exiting energy
  - Irradiance refers to the radiant flux density incident on a surface
  - Radiant exitance refers to the radiant flux density leaving a surface
Radiant intensity

What is left to examine is radiant flux per solid angle

- Intensity, I, is the radiant flux per unit solid
  - Units are W/sr
  - Intensity is not invariant

- Intensity is especially useful when dealing with the energy from a point source
  - One way to view a point source is that it emits energy without an area
  - Radiance in this case would not be defined since there is no area that is emitting

- Simplistic way to determine the intensity from a point source is to divide the radiant flux by $4\pi$

\[ I = \frac{\phi}{4\pi} \]
Spectral quantities

Add a further complication in that all of the quantities shown vary with wavelength

- The spectral variation is both a hindrance and an aid
- Need to account for the spectral nature of the quantities
  - Spectral radiant flux, $\Phi_\lambda$ [W/(\mu m)]
  - Spectral radiant exitance, $M_\lambda$ [W/(m$^2$ \mu m)]
  - Spectral irradiance, $E_\lambda$ [W/(m$^2$ \mu m)]
  - Spectral radiant intensity, $I_\lambda$ [W/(sr \mu m)]
  - Spectral radiance, $L_\lambda$ [W/(m$^2$ sr \mu m)]

- The idea is that these quantities are at a specific wavelength only
  - Can do this theoretically as will be seen shortly
  - In real life, the spectral quantities are averages over small wavelength intervals

$$\langle X_\lambda \rangle = \frac{\int X_\lambda d\lambda}{\int_{\Delta \lambda} d\lambda}$$
Spectral radiance

Spectral radiance is the quantity that removes all of the geometrically- and spectrally-related parameters

- Examine the case of a radiometer collecting energy
- Radiant flux is the more important parameter to ensure that we have appropriate output quality
  \[ \Phi = \int \int \int L(\theta, \phi, d) \cos \theta d\Omega \ dA \ d\lambda \]
- Spectral radiance is more important when concerned about the target or object
  \[ L_\lambda = \frac{d^3\Phi}{\cos \theta dA d\Omega \ d\lambda} \]
  - Removes all sensor related effects such as
    - Spectral width
    - Detector size
    - Solid angle of collection
  - Can compare outputs from widely different sensors
Spectral radiance - example

Consider the case of studying the spectral output from a laboratory sample illuminated by a lamp source

- Two “identical” radiometers
  - Radiometer 1 has a detector area that is two times that of radiometer 2
  - Radiometer 1 collects over twice the spectral interval of radiometer 2
  - Radiometer 1 has twice the solid angle of collection than radiometer 2
- Radiant flux of radiometer 1 is eight times that of radiometer 2
  - This is typically a good thing
  - Science/application must still allow this to be done
- The incident spectral radiance is identical for both radiometers
- Difficulty is inferring the spectral radiance from the radiant flux
  - More than one spectral radiance distribution can give the same radiant flux
  - Usually assume the band-averaged radiance is equivalent
More radiometric terms

Since we’re defining terms, it’s worth bringing up four more quantities

- Emissivity
- Transmittance
- Absorptance
- Reflectance

All of these are unitless ratios

All of them can be determined from any of our energy quantities
  - Differences between directional and non-directional
  - Ignore the directional aspects for now
  - Define all of the parameters in terms of radiant flux at this point
  - Examine later the reflectance from a directional standpoint as an example
  - Numerator and denominator must have the same units
Reflection, reflectance, reflectivity

Reflection is the process in which radiant energy is "thrown" back from a surface

- Reflectance is the ratio of the reflected radiant flux to the incident radiant flux
  \[ \rho = \frac{\Phi_{refl}}{\Phi_{inc}} \]
  - Diffuse and specular (Fresnel)
  - More on this later

- Reflectivity strictly refers to the reflectance of a layer of material
  - The layer is thick enough that there is no change in reflected energy as the layer increases in thickness
  - In essence, it is the reflectance of an infinitely thick layer
  - Not used much anymore

- Reflectance is also spectral in nature but still no units
  \[ \rho_\lambda = \frac{\Phi_{\lambda,refl}}{\Phi_{\lambda,inc}} \]
Transmission, transmittance, transmissivity

Transmission is the process in which radiant energy passes through a surface or material

- Spectral transmittance is the ratio of the transmitted radiant flux to the incident radiant flux
  \[ \tau_\lambda = \frac{\Phi_{\lambda,\text{trans}}}{\Phi_{\lambda,\text{inc}}} \]

- Diffuse and regular transmission
  - Diffuse transmission is transmission that occurs independent of the laws of refraction
  - Regular transmission is transmission without diffusion
  - Total transmittance is the sum of the diffuse and regular transmittance

- Examine transmittance more quantitatively later in terms of the properties of the material
Transmission, transmittance, transmissivity

Other transmittance terms that are used in radiometry are included here for completeness

- Spectral internal transmittance is the transmittance within a layer and is the ratio of the radiant flux through the layer entry to that exiting the layer.

- Spectral transmissivity is the spectral internal transmittance through a layer of unit length under conditions when the boundary has no influence.

- Transparent medium has a transmission that has a high regular transmittance.

- Translucent medium transmits light by diffuse transmission.

- Opaque medium transmits no radiant energy.

- $4\pi$ transmittance:
  - Ratio of the forward and backward radiant fluxes leaving a surface to the incident radiant flux.
  - Typically measured in an integrating sphere.
Absorption, absorptance, absorptivity

Absorption is the process in which incident energy is retained without reflection or transmission

- Spectral absorptance is the ratio of absorbed spectral radiant flux to the incident spectral radiant flux

\[ \alpha_{\lambda} = \frac{\Phi_{\lambda,\text{abs}}}{\Phi_{\lambda,\text{inc}}} \]

- The energy is converted to a different form
- Spectral internal absorptance is absorptance within a layer and is the ratio of the radiant flux absorbed in a layer to that exiting a layer
- Spectral absorptivity is the spectral internal absorptance through a layer of unit length under conditions when the boundary has no influence

- Absorption coefficients
  - Characteristics of the material
  - Will use these later to define better the transmittance
- \( 4\pi \) absorptance is the one’s complement to the \( 4\pi \) transmittance
Emission, emissivity, emittance

Emissivity is the ratio of the amount of energy emitted by an object to the maximum possible

- Spectral emissivity is then

\[ \varepsilon_\lambda = \frac{\Phi_{\lambda,\text{emitted}}}{\Phi_{\lambda,\text{maximum}}} = \frac{\Phi_{\lambda,\text{emitted}}}{\Phi_{\lambda,\text{blackbody}}} \]

- Will see later that an object emitting the maximum amount of energy is a blackbody
- As before, emission refers to the process of emitting energy
- Emittance is not used in practice, so emissivity is used with reflectance, absorptance, and transmittance
Conserving energy

Photons are either absorbed, reflected, or transmitted in this course

- That is
  \[ \Phi_{\text{inc}} = \Phi_{\text{refl}} + \Phi_{\text{abs}} + \Phi_{\text{trans}} \]

- Then
  \[ 1 = \rho + \alpha + \tau \]

- Or in spectral terms
  \[ 1 = \rho_\lambda + \alpha_\lambda + \tau_\lambda \]

- Knowledge or measurement of any two parameters allows the derivation of the third
  - Directionality effects (radiance instead of radiant flux) complicates this
  - Knowledge of the two is not always trivial
Spectral $\alpha, \rho, \tau, \varepsilon$

Ideally, these quantities would be monochromatic values, but there are also band-integrated and band-averaged quantities.

- As described previously, the monochromatic values we use will be band-averaged values:

$$\left\langle X_\lambda \right\rangle = \frac{\int X_\lambda d\lambda}{\Delta \lambda}$$

- Band integrated values, or band-limited values are not divided by the wavelength interval:

$$X_\lambda = \int_{\Delta \lambda} X_\lambda d\lambda$$

- As has happened numerous times already, this is still simpler than real life:
  - Weighting by the source energy
  - More on this later
A blackbody is one for which the absorptance is unity at all wavelengths

- Also a body for which the amount of energy emitted is a maximum
- Can see this using the figure below
  - Blackbody at some temperature $T$
  - Absorbs all energy incident on it
  - Inside an isothermal enclosure at temperature $T$
- Local thermodynamic equilibrium (not lasers or gas discharges)
  - Object must have the same temperature as the enclosure
  - Object must emit as much as it absorbs
    - If it emits more it will have to cool
    - If it absorbs more it has to heat
Planck’s Law

Planck’s Law allows the spectral radiant exitance of an object to be determined based on temperature.

- In the case of a blackbody, the spectral radiant exitance is a function only of the temperature (and the wavelength).
- Derivation based on quantization of energy modes of oscillators.

\[
M_{BB\lambda} = \frac{2\pi\hbar c^2}{\lambda^5 \left[ e^{(\hbar c/\lambda kT)} - 1 \right]} \quad [W / (m^2 m)]
\]

\[
c = 2.99792 \times 10^8 \quad [m / s]
\]

\[
h = 6.62607 \times 10^{-34} \quad [J \cdot s] \quad \text{(Planck's constant)}
\]

\[
k = 1.38065 \times 10^{-23} \quad [J / K] \quad \text{(Boltzmann's constant)}
\]
Planck’s Law

Can substitute for the constants in Planck’s law and rewrite it in a “simpler” fashion

- Constants given here require that the input temperature and wavelength have appropriate units
  - Temperature must be in Kelvin
  - Wavelength must be in μm

\[
M_{BB\lambda} = \frac{C_1}{\lambda^5 \left[ e^{(C_2/\lambda T)} - 1 \right]} \quad [W/(m^2 \mu m)]
\]

\[
c_1 = 3.74151 \times 10^8 \quad [(W\mu m^4) / m^2]
\]

\[
c_2 = 1.43879 \times 10^4 \quad [\mu m K]
\]

- Constants here and in previous formulation have a certain level of uncertainty to them
  - Not an issue in this class
  - Can be an issue when attempting extremely high accuracy measurements
Graybody is an object for which the emissivity is not a function of wavelength but it is not unity

- That is, \( \varepsilon \neq f(\lambda) < 1.0 \)
- Then

\[
M_{BB\lambda} = \frac{\varepsilon C_1}{\lambda^5 [e^{(C_2/\lambda T)} - 1]} \quad [W/(m^2 \mu m)]
\]

\[
c_1 = 3.74151 \times 10^8 \quad [(W\mu m^4) / m^2]
\]

\[
c_2 = 1.43879 \times 10^4 \quad [\mu m K]
\]

- The shape of the Planck curve is identical, it is simply translated downward
  - That is, the output at all wavelengths is reduced
  - The spectral radiant exitance is reduced by the same factor at all wavelengths
Planck’s Law

Once you are given the temperature you can develop a Planck curve

- Planck curves never cross
- Curves of warmer bodies are above those of cooler bodies
- Given a spectral radiant exitance one can compute an equivalent temperature
  - Do this at one wavelength
  - Can do it over a spectral range
Planck’s Law

On linear scale it is difficult to display the broad range of temperatures that can be seen.

- Using a log-log scale shows dramatically the large range of exitance that is seen.
- Also note the large differences in exitance between the two objects.
- Curves do not cross each other as well.
- Finally, the shapes of the curves do not change at all in this presentation.
Planck’s Law

Planck’s Law was the first use of quantum physics

- Quantization of the energy was necessary to obtain satisfactory agreement between measurements across the entire spectral range
  - Wien approximation was used for shorter wavelengths (short of the peak)
    \[ M_{BB\lambda} = \frac{2\pi\hbar c^2}{\lambda^5} e^{-(\hbar c / \lambda kT)} \]
  - Rayleigh-Jean approximation works for longer wavelengths (beyond the peak) but fails at short wavelengths (ultraviolet catastrophe)
    \[ M_\lambda = \frac{2\pi c kT}{\lambda^4} \]
- Note that the goal was to match a set of measurements
Planck’s Law - frequency space

Can also write Planck’s law in terms of frequency as well as in terms of wavelength

Then the spectral radiant flux is

\[ M_{BB\nu} = \frac{2\pi h \nu^3}{c^2 \left[ e^{(h\nu/kT)} - 1 \right]} \]

Note that the equation above does not simply replace \( \nu = c/\lambda \)
- Rather have to ensure that \( M_{\lambda} d\lambda = M_{\nu} d\nu \) (in absolute terms)
- The problem is that \( d\nu = -c (d\lambda/\lambda^2) \)
- These are not linearly related

Leads to an interesting feature - the peak in frequency space does not match the peak in wavelength space converted to frequency
- See this in Wien’s Law
- Leads to the interesting question as to whether a system should be optimized in wavelength or frequency space
Planck’s Law - with index

Strictly speaking, Planck’s Law should include an effect caused by the index of refraction of the medium

- Originally assumed vacuum or material with index close to unity
- Planck’s Law was derived for frequency space
  - Energy is linearly related to frequency in frequency space
  - Conversion to wavelength space from frequency depends upon the index of refraction

\[ M_{BB\lambda} = \frac{2\pi \hbar c^2}{n^2 \lambda^5 \left[ e^{(\frac{\pi}{\hbar \lambda k T})} - 1 \right]} \]

- We will ignore the index of refraction term because the index of air <1.0003
  - Thus, unless we are doing work requiring extremely high accuracy we can ignore this factor
  - If you work in frequency space you can ignore the factor because frequency does not change with index
Band-averaged spectral emissivity

Revisiting spectral emissivity, one should actually determine it via a weighting by the blackbody curve

- Recall emissivity was given as the ratio of the radiant fluxes emitted by an object to that emitted by a blackbody.
- Now write band-averaged spectral emissivity in terms of radiant exitance

\[
\left\langle \varepsilon_\lambda \right\rangle = \frac{M(\Delta \lambda, T)}{M_{BB}(\Delta \lambda, T)} = \frac{\int \varepsilon(\lambda) M\text{\_BB}(\lambda, T) d\lambda}{\int M\text{\_BB}(\lambda, T) d\lambda}
\]

- Where \(M(\Delta \lambda, T)\) is the band-integrated radiant exitance of an object at temperature \(T\) over the wavelength interval \(\Delta \lambda\) (at a given \(\lambda\)).
- \(M_{BB}(\Delta \lambda, T)\) is the band-integrated radiant exitance of the blackbody.

- Thus, there is the spectral, total or band-integrated, and band-averaged emissivity.
- Still have not even considered the directionality aspect.
Wien’s Law describes the relationship between the peaks of Planck curves and the temperature of the object.
Wien’s Law

Obtain Wien’s Law by differentiating Planck’s Law and setting equal to zero

- Recall that there are two versions of Planck’s Law - frequency and wavelength
- Then the wavelength of maximum emission derived from the wavelength version of Planck’s Law is

\[ \lambda_{\max}(M_\lambda) = \frac{2.898 \times 10^3}{T} [\mu m] \]

- Wavelength of maximum emission as determined from the frequency version of Planck’s Law is

\[ \lambda_{\max}(M_\nu) = \frac{5.100 \times 10^3}{T} [\mu m] \]

- Brings up the question as to which is more relevant for energy calculations
Stefan-Boltzmann Law

Stefan-Boltzmann Law gives the radiant exitance (not spectral radiant exitance) from the object

- Done empirically at first then with classical thermodynamics
- Conceptually, it is the integral of Planck’s Law across all wavelengths
- Planck’s Law gives spectral radiant exitance - [W/(m² μm)]
- Stefan Boltzmann Law gives radiant exitance - [ W/m²]

- \( M = \varepsilon \sigma T^4 \ [W/m^2] \) Where \( \sigma = 5.67 \times 10^{-8} \ [W/(m^2K^4)] \)
- Emissivity factor assumes a gray body
- Warmer objects emit more energy - by a power of four
- Corollary is that \( \varepsilon = M/\sigma T^4 \)
Kirchhoff’s Law

Kirchoff’s law relates emissivity and absorptivity

\[ \epsilon_\lambda = \alpha_\lambda \]

- Good absorbers are good emitters law
- Kirchoff’s Law says **nothing** about the amount of energy emitted since this will also depend on the temperature and wavelength
- Planck’s law gives the maximum amount of energy that can be emitted
  - A cold object that is an excellent absorber may not emit that much energy
  - A piece of black paper absorbs very well but is too cold to emit a significant amount of energy in the visible
- Thermal equilibrium
- Further, this allows us to write

\[ 1 = \rho_\lambda + \epsilon_\lambda + \tau_\lambda \]
Propagation of Radiation

OPTI 509

Lecture 6
Lambert’s law; isotropic vs. lambertian, M vs. L, inverse square and cosine laws, examples
Blackbody emission - directionality

It can be shown that the radiance emitted by a blackbody must be invariant with angle

- Consider the small area $dA_{BB}$ which is a blackbody inside a blackbody enclosure, both at the same temperature

- Radiant flux collected by a small area $dA$ located a distance $R$ from the blackbody, $dA_{BB}$ at an angle $(\theta, \phi)$
  \[
  d\Phi(\theta,\phi) = L_\lambda(\theta,\phi)dA_{BB}\cos\theta dA/R^2
  \]

- Radiant flux emitted by the area $dA$ towards the blackbody area $dA_{BB}$ is
  \[
  d\Phi(0,0) = L_\lambda(0,0)dA(dA_{BB}\cos\theta/R^2)
  \]

- Emitted must equal absorbed thus the conclusion is that $L_\lambda(0,0)=L_\lambda(\theta,\phi)$

- Must also be true in the case when $dA_{BB}$ is not surrounded by a blackbody enclosure since the enclosure would be a perfect absorber
Lambert’s cosine law

A surface for which the radiance does not change with angle follows Lambert’s cosine law

- Lambert’s cosine law states the radiant intensity from a lambertian surface varies with the cosine of the angle from the surface

\[ I = I_0 \cos \theta \]

- Recall that radiance has a per unit area dependence
- Area varies w/cosine of the angle thus radiance independent of angle
- The variation of radiant intensity with \( \cos \theta \) offsets the change in projected area so that radiance is constant

\[ I = I_0 \cos \theta \]

\[ L = L_{\text{lambertian}} \]
Lambertian surface

A surface that follows Lambert’s cosine law is a lambertian surface

- Lambertian surface plays a central role in radiometry and other fields
  - Simplifies radiant flux calculations and other calculations as will be seen later
  - Model applies to both emitted and reflected energy
- But, must keep in mind that it is a model
- Radiance from a lambertian surface does not vary with angle
- Blackbodies are lambertian sources
  - Know the spectral radiant flux from a blackbody - Planck’s Law
  - Know the wavelength of maximum emission - Wien’s Law
  - Know the radiant exitance - Stefan Boltzman Law
  - Radiance as a function of angle is a constant
Lambertian surface

Radiance from a lambertian surface is independent of view angle

- All of this knowledge can be applied to an arbitrary surface if we assume that it is a blackbody
- Applies to gray bodies because we simply need to add a spectral emissivity
- For surfaces that are not blackbodies, we can still make assumptions that they are nearly blackbodies
  - Over small wavelength ranges
  - Over small angles of emission
  - Do this because we may not have any other information
  - Blackbody assumptions might be valid over a small range of wavelengths and viewing angles
- The one thing we don’t know yet is the amount of spectral radiance in all directions (that is, Planck’s Law for spectral radiance)
M vs. L for lambertian

Want the relationship between radiant exitance and radiance from a lambertian surface

- Obtain the radiant exitance from a surface by integrating the radiance through the entire hemisphere from the surface
- Because of the large range of angles, we cannot simply use $M = L\Omega$

$$M_{\text{lambertian}} = \int_{\text{hemisphere}} L_{\text{lambertian}} \cos \theta d\Omega$$

- Simplifies because radiance does not vary with angle

$$M_{\text{lambertian}} = L_{\text{lambertian}} \int_{\text{hemisphere}} \cos \theta d\Omega$$
Planck’s law for radiance

Have seen that the integral over the hemisphere weighted by cosine gives $\pi$

- In spherical coordinates the integration is

$$M_{\text{lambertian}} = L_{\text{lambertian}} \int_0^{\pi/2} \int_0^{2\pi} \cos \theta \sin \theta d\theta d\phi$$

$$M_{\text{lambertian}} = L_{\text{lambertian}} \pi$$

- Then it can be shown that Planck’s Law can also be written in terms of radiance

$$L_{BB\lambda} = \frac{M_{BB}}{\pi} = \frac{C_1}{\pi \lambda^5 \left[ e^{(C_2/\lambda T)} - 1 \right]} \quad [W / (sr \ m^2 \ \mu m)]$$

- Note that the derivation of Planck’s law can be done either for radiance or radiant exitance and then the other determined from lambertian aspect of the blackbody
  - Factor of $\pi$ can be included in the constants thus may not appear in the radiance version
  - The $\pi$ factor then shows up in the numerator of the radiant exitance
Now have a relationship between radiance and irradiance for the specific case of a lambertian surface

- $M_{\text{lambertian}} = \pi L_{\text{lambertian}}$
- Spectral radiant exitance given by Planck’s Law
- Wavelength of maximum emission described by Wien’s Law
- Radiant exitance is described by Stefan Boltzman Law
- Spectral radiance as a function of angle from a blackbody is a constant and it’s value is gotten from Planck’s Law/$\pi$
- Radiance from a black body is Stefan Boltzman Law/$\pi$
- This approach and methodology shows up repeatedly
  - Look for a theoretical relationship or model that is convenient to work with (In this case we will assume a lambertian surface)
  - Check that the model applies to our real life case (It won’t be perfect, but if it makes life much easier it still helps)
Irradiance on a surface

Now want to flip the radiance around and see how the irradiance on a surface behaves.

Images here are of the same hill in Nevada about 1 week after a snow event.

Can probably guess which side of the hill is to the south.

Why is there snow in areas exposed to the sun?
Cosine law

Irradiance on a surface is proportional to the cosine of the angle between the normal to the surface and the incident irradiance.

- \[ E = E_0 \cos \theta \]
  where \( E_0 \) is the incident irradiance normal to the surface
- \( E_0 \) is the maximum incident irradiance
- Irradiance is zero for the tangent case
Radiometric Laws - Cosine Law

Can also view the Cosine Law as a projected area effect

- Irradiance is a per unit area quantity
  - The same amount of radiant flux [Watts] spread out a larger area gives less irradiance
  - Area increases as the angle of incidence increases

- The beam of energy below is confined to the tube shown
  - The same amount of radiant flux hits both surfaces
  - In case B, the tilt of the second surface allows more surface area to be illuminated thus giving lower irradiance

- The increase in area is inversely proportional to the cosine of the tilt (area gets larger as angle increases)
Irradiance and directionality

Irradiance must always be defined relative to a given normal to a surface

- The useful amount of irradiance that a surface can “use” is only in the normal direction
- Attempt to illustrate this with the cases below
  - The radiant exitance from the lamp is normal to a unit area of the lamp
  - The irradiance from the lamp at the surface is relative to a normal defined by the direction from the lamp to the reference surface
  - The irradiance on the actual surface then includes the cosine factor
1/R² law

Irradiance from a point source is inversely proportional to the square of the distance from the source (1/R²)

- Also called the distance squared law
- Only true for a point source
- Can still be used for cases where distance from the source is large relative to the size of the source
  - Factor of five gives accuracy of 1%
  - Sun can be considered a point source at the earth
- Can understand how this law works by remembering that irradiance has a 1/area unit and looking at the cases below

In both cases, the radiant flux through the entire circle is same

Area of larger circle is 4 times that of the smaller sphere and irradiance for a point on the 2D circle is 1/4 that of the smaller circle
1/R^2 law

Given the irradiance (or radiant exitance) at one location, the irradiance can be computed at another location:

- Assume the output of a lamp source is known to be 100 W/m^2 at a distance of 50 cm
- Then the output of the lamp will be 25 W/m^2 at a distance of 100 cm

\[ E(100\,cm) = E(50\,cm) \frac{50^2}{100^2} = (100) \left[ \frac{1}{4} \right] \]

- Assumes the lamp is a point source (typical lamp size <2 cm)
- Still need the irradiance/radiant exitance at that initial distance

Difficulties encountered when trying to use the radiant exitance from a point source and translating this over distance:
- Strictly cannot do this since a point source has no area, thus no radiant exitance
- In this case, need radiant intensity
Lens falloff

Can combine distance-square law and cosine law to determine incident irradiance on a planar surface

- Important when looking at the irradiance reaching a detector in a sensor
- Diagram below shows the irradiance from a radiance source onto the plane surface
- Want the irradiance at the off-axis point in terms of the on-axis irradiance

$E_0 \sin^2 \theta$
Lens falloff

It is “plainly” evident that the irradiance at the off-axis location will be $E_0 \cos^4 \theta$

- The distance between the source and plane surface increases by a cosine factor
  - Solid angle subtended by source depends on $1/R^2$ (if source small enough)
  - Thus, the irradiance at the off-axis point decreases by $\cos^2$
- Also have an angle of incidence onto the plane that gives an additional cosine factor
- The fourth cosine factor comes from the change in solid angle of the source as seen from the off-axis point

$$E_0 \frac{D}{\cos \theta}$$
Radiometric Laws - Lens falloff

The impact is that images will appear darker at the edges

- Image at right is a fisheye lens image of the ground
- The surface is reasonably close to lambertian
  - Bright spot at the top is the hot spot
  - Bright spot at the bottom is the specular reflection
  - These will be covered in more detail later
- Edge of the scene is darker primarily due to the lens falloff
The cosine$^4$ factor is even more noticeable when the effect is corrected as in the righthand image.
Cosine⁴ falloff

Lens falloff is also known as cosine⁴ falloff and is true for any extended source

- Extended source can be a diffuse source of some finite size
  - The ground outside for an imaging system
  - Could be a fully illuminated lens focusing light on the image plane
- Examples of “detectors” include what we normally would think of such as CCD arrays and film
- Similar falloff effects for sources illuminating a projector screen or a diffuser panel
- Technically, the formulation is only good for small extended sources
  - If the source is too large, then our simplifying assumptions start to breakdown and need the full integral
  - Still cos⁴ law is a useful tool
- Just keep in mind it is a theoretical idea - I have never seen a cos⁴ falloff in a real system
- Finally, it is straightforward to avoid this problem (Imax does this)
Point source falloff

For a point source we no longer have a solid subtended by the source

- The solid angle was the source of three of our cosines
  - The solid angle of the source changed with off-axis position
  - Area changed by a cosine and distance gave us two cosines

- In the case of the point source, there is no radiance from the source
  - Have irradiance, intensity, or radiant flux
  - Then it is possible to compute the irradiance for the on-axis point
  - Simply $E_0 = \frac{E_{\text{source}}}{D^2}$

- Off-axis you have the same formulation except the distance is larger
  - $D_{\text{off-axis}} = D / \cos \theta$
  - This gives a $\cos^2$ factor for $E_{\text{off-axis}}$

- Including the cosine factor due to the cosine incidence gives
  \[ E_{\text{off-axis}} = E_0 \cos^3 \theta \]
Example of irradiance calculation

Compute the irradiance from the sun at the top of the earth’s atmosphere at the equator

Since we are not computing the spectral irradiance, we start with Stefan-Boltzman’s Law to determine the radiant exitance from the sun at the surface of the sun.

\[ M_{\text{sun}} = \sigma T^4 \]

\[ = (5.67 \times 10^{-8})(6000^4) = 7.348 \times 10^7 \text{ W/m}^2 \]
Irradiance calculation example, cont’d.

Now have the radiant exitance from a small area on the surface of the sun. To get irradiance, the next step is to determine the total radiant flux from the sun as a whole.

Thinking from a logical view, we want units of [W] based on a known [W/m²]

Assuming that the radiant exitance is constant over the entire surface, then simply multiplying by the area of the sun should get us what we want.

Radius of the sun is \(6.957 \times 10^8\) m; thus the radiant flux of the sun is

\[
\Phi_{\text{sun}} = M_{\text{sun}} \times A_{\text{sun}} = M_{\text{sun}} \times 4\pi(R_{\text{sun}})^2
\]

\[
\Phi_{\text{sun}} = (7.348 \times 10^7)4\pi(6.957 \times 10^8)^2 = (4.469 \times 10^{26}) \text{ [W]}
\]
Irradiance calculation, cont’d.

Knowing the radiant flux from the sun we can compute the irradiance at the top of the earth’s atmosphere.

Assume that the solar radiant flux is equally distributed across the entire orbit of the earth.

Then the irradiance on the earth is

\[ E = \frac{\Phi}{A} = \frac{\Phi}{4\pi(D_{\text{earth-sun}})^2} \]

\[ E = \frac{(4.4693 \times 10^{26})}{4\pi(1.496 \times 10^{11})^2} \]

\[ E = 1589 \text{ W/m}^2 \]

Typically accepted value is measured to be 1367 W/m².
Rather than use the radiant flux approach, we can also use the radiance from the surface of the sun propagated to the earth.

Assume the sun is a blackbody and thus lambertian, then

\[ L_{\text{sun}} = \frac{M_{\text{sun}}}{\pi} = \frac{\sigma T^4}{\pi} = \frac{(5.67 \times 10^{-8})(6000^4)}{\pi} = 2.339 \times 10^7 \text{ W/(m}^2\text{sr)} \]

The advantage to this is that the radiance is constant with distance, thus we know the radiance at the top of the earth’s atmosphere.

Then integrating over solid angle of the sun will give us the irradiance at earth orbit.

\[ E_{\text{sun}} = \int L_{\text{sun}} \cos \theta \, d\Omega_{\text{sun}} = \int \left( \frac{M_{\text{sun}}}{\pi} \right) \cos \theta \, d\Omega_{\text{sun}} \]
Irradiance calculation, alternate approach

Because the sun is small relative to the distance and we are viewing at near normal, the integration simplifies to

\[ E = L \Omega \]

\[ E_{\text{sun}} = L_{\text{sun}} \Omega_{\text{sun}} \]

\[ = (L_{\text{sun}})(A_{\text{sun}}/D^2_{\text{earth-sun}}) \]

\[ = (L_{\text{sun}})(\pi R^2_{\text{sun}}/D^2_{\text{earth-sun}}) \]

\[ = (2.339 \times 10^7 \text{ [W/(m}^2\text{sr)]}) \pi (6.957 \times 10^8)^2/(1.496 \times 10^{11})^2 \]

\[ = 1589 \text{ W/m}^2 \]

And this matches the answer we obtained from the other approach
Irradiance example, alternate approach 2

Can also use Intensity

- The sun is far enough away that it can be viewed as a point source.
- Then the radiant intensity from the sun is $\Phi/4\pi$ W/sr
  
  $$I_{\text{sun}} = \Phi_{\text{sun}} / 4\pi = (4.469 \times 10^{26}) / 4\pi$$

  $$= 3.5563 \times 10^{25} \text{ W/sr}$$

- Then integrating over solid angle of the earth as seen by the sun and then dividing by the area of the earth will give us the irradiance at earth orbit
  
  $$E_{\text{sun}} = \int_{\Omega_{\text{earth}}} I_{\text{sun}} d\Omega / A_{\text{earth}} \approx I_{\text{sun}} \Omega_{\text{earth}} / A_{\text{earth}} = I_{\text{sun}} / D_{\text{sun}}^2 = 1589 \frac{W}{m^2}$$

  - Note - we are assuming that the sun is a point source
  - Thus, there is no area of the sun
  - This may seem to conflict our throughput discussion of using the two separate areas (collector and source)
  - However, we took into account the area of the sun in determining the radiant flux.
Irradiance example, alternate approach 2

Worthwhile to examine the geometry of the radiant intensity approach a bit

- Note, we don’t really need the area of the earth
- We could have just as easily used a generic unit area at the earth-sun distance
  \[ E_{sun} = I_{sun} \Omega_{unit\_area} / 1m^2 = I_{sun} / D_{sun}^2 = 1589 \, \frac{W}{m^2} \]

- What is being done in this case is determining how much radiant intensity is being collected by the unit area, hence the use of the solid angle of the receiver
- The real trick in this approach is that we need to know the size of the sun to determine the radiant flux to get the radiant intensity

\[ I_{sun} = 3.5563 \times 10^{25} \, W/sr \]
Irradiance example, yet another way

Could always rely on the $1/R^2$ law to determine the irradiance on the earth from the sun

- We know the radiant exitance at a given distance
  
  \[ M_{\text{sun}} = 7.348 \times 10^7 \text{ W/m}^2 \text{ at the surface of the sun} \]

- Then the irradiance is simply
  
  \[
  E_{\text{sun\_at\_earth}} = M_{\text{sun}} \left( \frac{R_{\text{sun}}}{D_{\text{earth\_sun}}} \right)^2 = 7.348 \times 10^7 \left( \frac{6.957 \times 10^8}{1.496 \times 10^{11}} \right)^2 = 1589 \left[ \frac{W}{m^2} \right]
  \]

- This approach is the least aesthetically pleasing
  - The radiant exitance we started with is clearly not from a point source
  - We can view the radius of the sun factor as taking us back to a point source, but this is a bit disturbing as well

- However, the method works because we have accounted for all of the relevant geometry factors that are important
  - Earth-sun distance
  - Size of the sun
Temperature of the earth

Assuming that the earth is a blackbody, compute the temperature it will have when in radiative balance with the sun.

If the earth is in balance with the incoming solar energy then the emission from the earth equals the incident solar irradiance.

Earth absorbs energy as a disk

Earth emits energy as a sphere

\[ \Phi_{\text{incident}} = E_{\text{incident}} \pi (R_{\text{earth}})^2 \]

\[ \Phi_{\text{emitted}} = M_{\text{emitted}} 4 \pi (R_{\text{earth}})^2 \]

\[ E_{\text{incident}} \pi (R_{\text{earth}})^2 = M_{\text{emitted}} 4 \pi (R_{\text{earth}})^2 \]

\[ T^4 = \frac{E_{\text{incident}}}{4 \sigma} = \frac{7 \times 10^9}{4 \times 7.007 \times 10^8} \]

\[ T = 289.3 \text{ K} \]
Propagation of Radiation

OPTI 509

Lecture 7
Directional reflectance, simplifications, assumptions, view & form factors, basic and simple radiometer
Recall the earlier discussion of spectral reflectance as the ratio of reflected energy to the incident energy:

\[ \rho_{\lambda} = \frac{\Phi_{\lambda,\text{refl}}}{\Phi_{\lambda,\text{inc}}} \]

- Ignored the directionality of the reflectance in this earlier discussion:
  - Diffuse
  - Specular or Fresnel

- The problem with reflectance is that it can be highly dependent upon the geometry of the situation:
  - Whether the incident energy is hemispheric or directional
  - Whether the reflected energy is hemispheric or directional
  - Thus, have to know both the incident angles and view angles

- Will search for the holy grail of reflectance -lambertian surface
Specular and diffuse reflectors

There are no perfect diffusers or perfect specular reflectors

- Diffuse surface something that is nearly lambertian
- Mirrors are examples of a surface that is nearly specular
Examples of specular reflection
Examples of directional reflectance

Bare soil

Backscatter Forward Scatter
Examples of directional reflectance

Spruce forest

Backscatter

Forward Scatter
Specular or Fresnel reflection

Reflection from a smooth surface between two differing indexes of refraction

- Air-glass interface for example
- Reflectance can be computed based on Maxwell’s equations and ensuring consistent boundary conditions
  - Reflectance depends only on the angle of incidence and refractive index
  - No spectral dependence except related to index of refraction
  - Fresnel reflectance is polarized
- Fresnel equations give the reflection and transmission coefficients
- Reflectance and transmittance used for energy calculations are the squares of these coefficients
- Unpolarized transmittance and reflectance determined as the average of the two polarized components
Fresnel reflection

Reflection effect occurs even at normal incidence at the index of refraction interface

- Reflectance and transmittance for normal incidence can be shown to be

\[ \tau = \left( \frac{2n_1}{n_1 + n_2} \right)^2 \quad \rho = \left( \frac{n_2 - n_1}{n_2 + n_1} \right)^2 \]

- Where \( n_1 \) is the index of refraction of the incident medium
- \( n_2 \) is the index of the transmitted medium

- Graph here shows the reflectance as a function of the ratio of \( n_2/n_1 \)
  - Values around 1.5 correspond to an air/glass interface
  - Large ratios relate to several detector materials in air
Fresnel reflection versus angle

Can compute the reflected and transmitted components from the interface based on angle of incidence and index

- Variety of forms of Fresnel equations based on what parameters have been substituted
- Simple mathematical form is in terms of the incident ($\theta_{inc}$) and refracted ($\theta_{ref}$) angles
  - Index of refraction contained within the refracted angle
  - Equations here are for reflectance not reflection coefficient

\[
\begin{align*}
\rho_{\parallel} &= \left[ \frac{\tan(\theta_{inc} - \theta_{ref})}{\tan(\theta_{inc} + \theta_{ref})} \right]^2\\
\rho_{\perp} &= \left[ \frac{\sin(\theta_{inc} - \theta_{ref})}{\sin(\theta_{inc} + \theta_{ref})} \right]^2\\
\tau_{\parallel} &= \left[ \frac{2 \sin \theta_{ref} \cos \theta_{inc}}{\sin(\theta_{inc} + \theta_{ref}) \cos(\theta_{inc} - \theta_{ref})} \right]^2\\
\tau_{\perp} &= \left[ \frac{2 \sin \theta_{ref} \cos \theta_{inc}}{\sin(\theta_{inc} + \theta_{ref})} \right]^2
\end{align*}
\]

- Above assume that the magnetic permeabilities are nearly unity (or at least similar)
Fresnel reflection versus angle

Example here shows the reflectance and transmittance for an air/glass interface

- Note that the parallel component of the reflected beam becomes completely polarized around 55 degrees
- Transmitted beam goes toward zero at 90-degree incident angle
  - See this with a piece of paper
  - Illustrated with the photograph shown on viewgraph 11-4
Fresnel reflection versus angle

Consider the case where the index of refraction of the transmitted media is much larger

- Graphs here were generated with an $n_2=3$
- Greater loss at normal incidence than for the smaller index case
- Large indexes such as this are not uncommon at air-detector interfaces
Specular reflection

There are three aspects of Fresnel/specular reflection that are critical in radiometry

- The first is that there is reflection of energy at all optical interfaces
  - Energy is reflected from the desired direction leading to loss of signal
  - The reflected energy may go in unanticipated directions leading to stray light
  - Anti-reflection coatings help alleviate this

- The second aspect in radiometry is when the object is a specular surface
  - Design of the radiometric system must take this into account
  - Geometry of the source/optical system must ensure that sufficient energy gets to the detector
  - Geometry of the source/optical system must compensate for the case where too much energy gets to the detector

- Polarization sensitivity and effects
Diffuse reflectance is the situation that occurs in bulk scatterers or rough surfaces

- Bulk scatterer is one in which the light penetrates the “surface” and exits the material after multiple scattering events
  - Even if the light went in only one direction (except backscatter) after an individual scatter the exiting light would be diffuse
  - Bulk scatterer must be thick enough to limit the effect of the substrate

- Surface scatter is an issue in mirror fabrication and leads to stray light issues
Geometry of reflectance

The geometry of the situation significantly affects the outcome of the reflectance results

- Consider the case where a radiometer does not view the specular direction of a Fresnel reflection
- Figures below illustrate the basic geometries that are encountered in determining reflectance
- Not shown are the directional limits that are actually desired

![Bi-hemispheric](image1)
![Conical-hemispherical](image2)
![Hemispherical-conical](image3)
![Bi-conical](image4)
Hemispheric reflectance

Also referred to as bi-hemispheric reflectance & is when incident & reflected radiance are over a hemisphere

- Term albedo is often used and this strictly refers to the hemispheric reflectance averaged over all wavelengths
- The problem is determining the reflected and incident radiances

\[
\rho = \frac{\Phi_{\text{reflected}}}{\Phi_{\text{incident}}} = \frac{E_{\text{reflected}}}{E_{\text{incident}}} = \frac{\int_{0}^{2\pi} \int_{0}^{\pi/2} L_{\text{reflected}} \cos \theta \sin \theta \, d\theta \, d\phi}{\int_{0}^{2\pi} \int_{0}^{\pi/2} L_{\text{incident}} \cos \theta \sin \theta \, d\theta \, d\phi}
\]

- If the surface is lambertian then

\[
\rho = \frac{\pi L_{\text{reflected}}}{E_{\text{incident}}}
\]
Lambertian reflectance

Reflected radiance from a lambertian surface does not vary with view angle

- The radiance does vary with the angle of incidence
  - Cosine law effect on the incident irradiance
    \[ E_{inc} = E(\theta) \cos \theta \]
  - Incident irradiance can be determined by integrating the incident radiance over the hemisphere in this case
    \[ E_{inc} = \int \int L(\theta, \phi) \cos \theta \sin \theta d\theta d\phi \]
Reflectance factor

Want reflectance, but characterizing the incident energy is troublesome. Reflectance factor avoids this

- Reflectance factor is the ratio of the reflected energy from the sample of interest to what would be reflected from an ideal lambertian surface

\[ RF = \frac{\Phi_{\text{sample}}}{\Phi_{\text{lambertian}}} = \frac{E_{\text{sample}}}{E_{\text{lambertian}}} \]

- Rewrite in terms of the incident radiance

\[ RF = \frac{\Phi_{\text{reflected}}}{\Phi_{\text{lambertian reflected}}} = \frac{E_{\text{reflected}}}{E_{\text{lambertian reflected}}} = \frac{\int_{0}^{2\pi} \int_{0}^{\pi/2} L_{\text{reflected}} \cos \theta \sin \phi \, d\phi \, d\theta}{\pi L_{\text{lambertian reflected}}} \]

- The above is valid for the case where the reflected energy is collected over a hemisphere
Reflectance factor

Reference reflected measurements to known lambertian surface with the same incident and reflected geometry

- No measurement of incident radiation is needed
- Incident radiation is essentially inferred from the measurement of the reflectance factor
- If the surface under study is lambertian then the reflectance factor is simply the reflectance
- One difficulty is ensuring that the geometries match between the sample and the lambertian
- Still need someone at some point to either determine the reflectance factor of a repeatable standard or a lambertian surface
Reflectance factor

Have one or two groups make the time and effort to characterize the reflectance of a standard reference

- National Institute of Standards and Technology (NIST) in the US
  - NIST characterizes the reflectance of a powder (polytetrafluoroethylene, PTFE, Halon) pressed into a sample of specified thickness and density
  - NIST measures the incident energy and can compute what would be reflected from a lambertian surface

- Using the computed $\Phi_{lambertian}$ and the measured $\Phi_{sample}$ gives the reflectance factor of the standard

- Then make a version of the standard in another laboratory
  - Take an arbitrary sample and measure the reflected energy from the sample and the standard
  - The reflectance factor of the sample is

$$RF_{sample} = RF_{standard} \frac{\Phi_{sample}}{\Phi_{standard}}$$
Hemispheric-conical RF

Hemispheric incident irradiance and a conical reflected radiance

- The integration is not over the hemisphere

\[
RF = \frac{\int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} L_{\text{reflected}} \cos \theta \sin \theta d\theta d\phi}{\int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} L_{\text{lambertian}} \cos \theta \sin \theta d\theta d\phi} = \frac{\int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} L_{\text{reflected}} \cos \theta \sin \theta d\theta d\phi}{L_{\text{lambertian}} \left( \phi_2 - \phi_1 \right) \frac{\sin^2 \theta_2 - \sin^2 \theta_1}{2}}
\]

- The incident geometry has no bearing on the form of the integrals above
- What the incident geometry does is impact the value of the reflected radiance

- Example of a hemispheric-conical situation is an instrument with a small field of view looking at a surface under cloudy skies
- If the surface under study is also lambertian, we obtain the reflectance
Conical-hemispheric RF

Conical incident irradiance and hemispheric reflected radiance gives a form identical to the bi-hemispheric case

- Identical in form as the bi-hemispheric case
  \[
  RF = \frac{\int_0^{\pi/2} \int_0^{2\pi} L_{\text{reflected}} \cos \theta \sin \theta \, d\theta \, d\phi}{\pi L_{\text{lambertian}}}
  \]

- In this case, however, the incident light is considered over only a small cone of incident angles
  - As a thought problem consider an isotropic incident source and a lambertian reflector
  - Conica-hemispheric geometry leads to a reflected radiance that is much smaller than for the hemispheric case

- Reciprocity means that the conical-hemispheric and the hemispheric-conical are the same

- An example is measuring the upwelling radiance with a full 180-degree field of view where the incident beam is from a flashlight
Bi-directional reflectance is the holy grail of reflectance but cannot be derived via measurements

- Bi-directional case has infinitely small solid angle of reflected radiation and incident radiation over an infinitely small solid angle

- Can develop a reasonably good directional source (e.g., lasers)
  - Tougher to obtain an output from a sensor with an infinitely small solid angle but we can get close (integrate for a long time)
  - Still not the bi-directional reflectance

- Actually measure the bi-conical reflectance
  - Incident source has a small solid angle and is reasonably constant over the solid angle
  - Reflected radiance is reasonably constant over a small solid angle of a sensor
  - Then, bi-conical reflectance and bi-directional are similar
The bidirectional reflectance factor is what is reported in the literature

- As mentioned, the best that can be done is bi-conical
- In general, reference to reflectance, directional reflectance, and reflectance factor typically means BRF
- Reciprocity is also assumed valid here as well

\[
RF = \frac{\int_{\phi_1}^{\phi_2} \int_{\theta_1}^{\theta_2} L_{\text{reflected}} \cos \theta \sin \theta d\theta d\phi}{L_{\text{lambertian}}^{\text{reflected}} (\phi_2 - \phi_1) \frac{\sin^2 \theta_2 - \sin^2 \theta_1}{2}}
\]

- Note that the form of the bi-conical case is identical to the hemispheric-conical
Bi-directional reflectance distribution function allows for the conversion from irradiance to radiance

- Can be determined by the ratio of the reflected radiance to the incident irradiance
  \[
  BRDF = \frac{L_{\text{reflected}}}{E_{\text{incident}}} \quad [sr^{-1}]
  \]
  - Thus, has units of sr\(^{-1}\)
    - BRDF has infinite value for a specular surface
    - BRDF=\(1/\pi \) sr\(^{-1}\) for a perfect lambertian reflector (hemispheric reflectance of unity)

- BRF=(\(\pi\))BRDF for all cases

- Major advantage to BRDF is that if it is truly known, then it is possible to derive the reflectance of a surface regardless of view/sensor effects
Lambertian surface

It should be clear by now that the use of a lambertian surface makes life very simple

- Recall

\[ \rho = \frac{\pi L_{\text{reflected}}}{E_{\text{incident}}} \]

- This now becomes \( L_{\text{refl}} = (\text{BRDF})(E_{\text{inc}}) \) where the BRDF = \( \rho / \pi \) sr\(^{-1} \)

- Simple model that allows us to know the radiance in all directions given an incident irradiance
  - Reflected radiance is independent of view angle
  - Reflected radiance varies as the cosine of the incident angle since the incident irradiance is normal to the surface

- BRDF of this surface is \( \rho \)
- BRDF is \( \rho / \pi \) sr\(^{-1} \)
Example of BRF usage

Can illustrate the use of BRF through the measurement of reflectance of a field sample

- This approach is used throughout remote sensing
- Show below the measurement of a desert test site for calibration
  - Compare measurements of the surface to those of a reference of known BRF
  - Ratio of the two allows the BRF of the surface to be determined
- Step 1 is then to determine the BRF of the reference
Laboratory BRF

Goniometric measurements in a “blacklab” are used to characterize the BRF of the field reference

- Start with a NIST-traceable standard of reflectance (pressed PTFE powder)
  - NIST gives the 8-degree hemispheric BRF
  - Powder must be pressed to specific standards

- BRF is found only for the zero-degree-viewing case since this simulates its use in the field

- Measurements of field reference compared to the NIST standard
Radiometric radiative transfer equation

Remind ourselves again of the basic formulation for computing the radiant flux

- Use the dual area form of the equation to allow form factors later
  \[ \Phi = \iint_{A_1}^{A_2} \frac{L(\theta, \phi, d) \cos \theta_1 \cos \theta_2}{S^2} \, dA_1 \, dA_2 \]

- First simplification is to ignore transmission losses
  - Not saying there will not be such losses
  - Just want to make the equation easier to write for now
  - Bring it back later but will decouple transmission effects from the integration and simply include a transmittance term at the end
  - Then
    \[ \Phi = \iint_{A_1}^{A_2} \frac{L(\theta, \phi) \cos \theta_1 \cos \theta_2}{S^2} \, dA_1 \, dA_2 \]

- If the radiance is lambertian (or equivalently does not change much over \( \theta \)) then
  \[ \Phi = L \iint_{A_1}^{A_2} \frac{\cos \theta_1 \cos \theta_2}{S^2} \, dA_1 \, dA_2 \]
Further simplifications

Case where the areas are far apart simplifies the integrals

- All we are left with is
  \[ \Phi = \frac{L A_1 A_2 \cos \theta_1 \cos \theta_2}{S^2} \]

- If we recognize that the combination of areas, cosines, and distance give us solid angle we can write
  \[ \Phi = L A \Omega \cos \theta \]

- And, for normal incidence on the “collector” we get
  \[ \Phi = L A \Omega \quad and \quad E = L \Omega \]

- Remind ourselves that this assumes
  - Large distance relative to the areas (distance/linear dimension=20 gives an uncertainty < 0.1%) 
  - Little change in radiance over the areas (lambertian would be best)
Form factors

Go back to the integral formulation for the case of the lambertian source

- In this case there is the radiation portion \( L \) and geometric portion (everything else)

\[
\Phi = L \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{S^2} dA_1 dA_2
\]

- Geometric term also known as a configuration factor, view factor, and others
  - Use this idea to develop the concept of a form factor
  - Cluster all of the geometric terms together
  - Precompute this form factor for a set of commonly-used setups
Form factors

Form factor is not simply the geometry but is the ratio of the radiant flux on surface 2 to that emitted from 1

- $F = \Phi_{on\_2}/\Phi_{from\_1}$

- Remember that we can use area of detector and solid angle of source or solid angle of the detector and area of source for radiant flux

- Form factor in terms of the radiant fluxes allows for a parameter that includes both geometries

- Radiant flux from the first surface is simply $\Phi_{from\_1} = M_1A_1$

- The radiant flux through $A_2$ is

$$\Phi_{on\_2} = \frac{M_1}{\pi} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{D^2} dA_1 dA_2$$
Form factors

Then the ratio of the two radiant fluxes from the previous viewgraph gives

\[
F_{1-2} = \frac{M_{\text{from } 1}}{\pi} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{D^2} dA_1 dA_2 = \frac{1}{A_1 \pi} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2}{D^2} dA_1 dA_2 = \frac{\Omega_2}{\pi}
\]

- Now bring back invariance of throughput \((A\Omega)\)
  - Throughput for the case of area 1 and solid angle of surface 2 is
    \[
    A_1 \Omega_2 = \pi F_{1-2} A_1
    \]
  - Throughput for the case of area 2 and solid angle of surface 1 is
    \[
    A_2 \Omega_1 = \pi F_{2-1} A_2
    \]
- Invariance of throughput then means that \(F_{1-2} A_1 = F_{2-1} A_2\)
Form factors

Can thus compute the form factor in terms of which ever geometry is easier

- Form factor effectively allows for the determination of the solid angle
- Then $\Phi = LA\Omega = L_{\text{from}_1A_1}\pi F$
- Realizing that $\pi L = M$ gives $\Phi = M_{\text{from}_1A_1F}$
- Again, in the case of a lambertian source
  - Can precompute form factors for typical geometries
  - Use the form factor to give the radiant flux
- This approach gives more accurate results than obtained using more simplified geometric assumptions
- Can still use this for cases where radiance does not change much over the solid angles and areas of interest
Form factor - example

Geometry below would represent the simple radiometer and spherical integrating source problem

- $A_1$ can represent the exit port area
- $A_2$ is the detector area
- Both areas are those that would be seen by the other based on the design of the system (takes into account projected area effects)
- Reverts back to the simplified formulation as the radius becomes small relative to the distance
  - $\Phi = L(A_1)(A_2/H^2) = MA_1F_{1-2}$
  - Then $F_{1-2} = (1/\pi)(A_2/H^2) = R_2^2/H^2 = S_2^2$

$$F_{1-2} = \frac{1}{2} \left[ X - \left( X^2 - 4 \frac{S_2^2}{S_1^2} \right)^{1/2} \right]$$

where $S = \frac{R}{H}$ and $X = 1 + \frac{(1 + S_2^2)}{S_1^2}$
Form factor - example

Use the previous viewgraph’s geometry for a “real” problem

- Use the approximate geometry of the RSG’s black laboratory
  - 10-cm radius lambertian source
  - 1-cm radius collector
  - Distance between the two is 0.5 m
  - Assume that the radiance from the source 100 W/(m²sr)

- Then the parameters are
  - \( S_1 = \frac{R_{source}}{Distance} = 0.2 \)
  - \( S_2 = \frac{R_{detector}}{Distance} = 0.02 \)
  - \( X = 26.01 \)
  - Form factor = 0.0003845
  - \( \Phi = MAF = \left( \pi L \right) \left( \pi R_{source}^2 \right) \left( F \right) = 0.003795 \text{ W} \)

- Approximation approach gives
  - \( \Phi = \frac{L A_{source} A_{detector}}{(Distance)^2} = 0.003948 \text{ W} \)

- About 4% difference
Form factor - example, continued

Compare the simplified approach versus form factor as a function of distance

- Larger difference at smaller distances
- Makes sense since the approximate formulations are no longer valid
- 1% difference at approximately a distance of 1 m (about a factor of five in distance to source size)
- Of course this still assumes that the source is uniform spatially and angularly
Form factor - example, continued

Normalize the distance to the size of source

- 1% error at a factor of 5
- 0.1% at a factor of 16

Implication for RSG work is the more exact formulation is needed
  • In reality, the source size is about 2 cm due to FOV limits
  • Example illustrates a typical laboratory problem that requires the use of more accurate formulations than will be relied on this course
Form factor - another example

This case would relate to the situation of measuring the output from a spherical blackbody

- Assume the area in this case is a small differential area centered on the sphere
- In our simplification approach, $H$ becomes large relative to $R$
  - Treat the sphere as a disk
  - Then $\Phi = L(A_1)(A_2/H^2) = L(A_1) \left( \pi R^2/H^2 \right)$
- The form factor should simplify to what we had for the previous case
  - Then $F_{1-2} = (1/\pi)(A_2/H^2) = R^2/H^2$
  - Note, that this matches the form factor below

$$F_{1-2} = \left( \frac{R}{H} \right)^2$$
Form factors, examples

This case could relate to the situation of looking at the effects of off-axis light

- Stray light test or MTF analysis
- The theoretical value allows us to determine how close the sensor output matches the theory
  - Expect differences because no instrument is perfect
  - Large differences would indicate a possible problem in the design or our understanding of the sensor

\[ F_{1-2} = \frac{1}{2} \left[ 1 - \frac{Z - 2S^2}{(Z^2 - 4S^2)^{1/2}} \right] \]

where \( S = \frac{R}{D} \) and \( Z = 1 + \left( \frac{R}{D} \right)^2 + \left( \frac{H}{D} \right)^2 \)
Simple radiometer - small source

Simple radiometer consists of an aperture and detector separated by a given distance

- Make our simplifying assumption that distances and sizes allow us to use $\Phi = LA\Omega$

- Ask the following questions?
  - What happens to the radiant flux when source area increases but stays smaller than the FOV?
  - Detector area increases?
  - Aperture area increases?
  - Either distance decreases?

![Diagram of a simple radiometer with source, aperture, detector, and FOV labeled.](image-url)
Simple radiometer - large source

Now examine the case of an extended source

- Ask the following questions?
  - What happens to the radiant flux when source area increases?
  - Detector area increases?
  - Aperture area increases?
  - Distance between source and aperture decreases?
  - Distance between aperture and detector decreases?

![Diagram of a simple radiometer with an extended source, showing various distances and areas.](image-url)
Basic radiometer

Basic radiometer is similar to the simple radiometer except now include a lens at the front aperture

- Place the detector at the focal distance of the lens for simplicity
  - Focal plane
  - Can place the detector at other locations with similar results
- Focal length and detector size define the resolution angles
- Lens size and focal length (equivalently, f/#) define the speed angles

Resolution angles

Speed angles
Basic radiometer

Produces an image of the source while giving a better-defined field of view

- Field of view of the simple aperture radiometer is not well defined for similar reasons as a pinhole can “image” the sun
- Chief ray defines the FOV of the basic radiometer
- Still use our simple formulation for radiant flux except now we should consider the transmittance of the lens and reflections from it
- Addition of the lens makes the lens the limiting aperture and the detector size the field stop (assume no vignetting)
Examine the case again of the small source as we did before with the simple radiometer

- Ask the following questions?
  - What happens to the radiant flux when output area increases but stay smaller than the FOV?
  - Detector area increases?
  - Aperture area increases?
  - Distance between source and aperture decreases?
  - Distance between aperture and detector decreases?
Now consider the extended source which is larger than the FOV of the radiometer

- Ask the following questions?
  - What happens to the radiant flux when output area increases?
  - Detector area increases?
  - Aperture area increases?
  - Distance between source and aperture decreases?
  - Distance between aperture and detector decreases?

![Diagram of a radiometer system with extended source, lens, and detector.]
Basic and simple radiometers - summary

Consider the two radiometer types and the radiant flux through the detector for the extended source case

- **Simple radiometer**
  - Collection area is defined by the area of the detector
  - Solid angle is determined by the size of the aperture and the distance between the detector and aperture

  \[ \Phi = L_S A_d \frac{A_a}{S_d^2} \]

- **Basic radiometer**
  - Collection area is defined by the area of the detector
  - Solid angle is determined by the size of the aperture and the distance between the detector and aperture

  \[ \Phi = L_S A_d \frac{A_a}{S_d^2} \]
Basic and simple radiometers - summary

Consider the two radiometer types and the radiant flux through the detector for the small source case

- Simple radiometer
  - Collection area is defined by the area of the detector
  - Solid angle is determined by the size of the source and the distance between the detector and source

\[
\Phi = L_s A_d \frac{A_s}{(S_s + S_d)^2}
\]

- Basic radiometer
  - Collection area is defined by the area of the aperture
  - Solid angle is determined by the size of the source and the distance between the source and aperture

\[
\Phi = L_s A_a \frac{A_s}{S_s^2}
\]
Basic and simple radiometer - summary

Can take the small source to the extreme of the point source

- There is no radiance in this case
- Use the radiant intensity of the source and

\[ \Phi = I_s \frac{A}{S^2} \]

- In the case of the simple radiometer

\[ \Phi = I_s \frac{A_d}{(S_s + S_d)^2} \]

- In the case of the basic radiometer (with lens)

\[ \Phi = I_s \frac{A_a}{(S_s)^2} \]
Basic and simple radiometer - summary

The lens improves the output for the point source but has no effect in the extended source case

- Include the transmittance of the lens we actually lose power in the extended source case
- The lens is the limiting aperture in both the extended and point source case
- Size of source has an impact on the simple radiometer
  - Detector defines the throughput when the FOV is underfilled
  - Size of the tube opening defines the throughput for the case when the radiometer FOV is overfilled
- Rays that would strike the wall of the radiometer and not reach the detector are redirected by the lens towards the detector
Propagation of Radiation

OPTI 509

Lecture 8 - The atmosphere and its effects on radiation, sources
Effect of the Atmosphere on Radiation

Atmosphere can refract, scatter, and absorb energy (transmit means nothing happened)

- Refraction refers to the bending of radiation from its original path
  - Only a major factor in long path lengths through the atmosphere
  - Weather radar and horizontal view remote sensing must consider it
  - Mirages are a refractive phenomenon
  - We will not be concerned with refraction in this course

- Scattering and absorption together also referred to as attenuation or extinction

- Scattering refers to the redirection of radiation after interaction with a particle in the atmosphere
  - Molecular scattering
  - Aerosol scattering

- Absorption refers to the incorporation of the radiation within the particle to alter the energy state of the particle
  - Gaseous absorption
  - Aerosol absorption
Atmospheric composition

Understanding what is in the atmosphere and where it is will be important for determining the impact

- Effect of different gases in the atmosphere depends upon their location and concentration - both horizontally and vertically
  - Will be a difference between lab and open air
  - Concepts still the same in both lab and open air

- Composition of the atmosphere has two components
  - Molecules
  - Aerosols

- Molecules make up what we typically think of as atmospheric gases
  - 78% Nitrogen
  - 21% Oxygen
  - Argon
  - Water vapor, carbon dioxide

- Aerosols are everything else
Scattering by aerosols and molecules is radically different

Curve below shows the scattering phase function of a particle.

Phase function is effectively a probability function of where the photon will go after hitting an aerosol or molecule.

The larger the value at a given $\Theta$ angle, the larger the probability.

Aerosols have strongly peaked scattering probability in the forward direction.

Molecular case has been multiplied by 10 (red line) to make it visible relative to the aerosol curve.

Molecules have a peanut-shaped probability.

$\Theta=90$ or 270 degrees.
Polarization example

Molecules also strongly polarize when scattering as seen in the two images below taken by a digital camera with a polarizing filter in front of the lens.
Example of forward scattering

Images below show the solar aureole due to forward scattering by aerosols
Third image illustrates the brightness one obtains by including the solar beam in the image as opposed to only the aureole
Absorption

Gaseous absorption is highly dependent upon wavelength, atmospheric composition, and concentration

- Gases causing the largest impact are carbon dioxide and water vapor
- Wavelengths < 0.3 μm are almost completely absorbed
- Gaseous absorption in VNIR is small relative to longer wavelengths
  - Ozone from approximately 0.5 to 0.7 μm
  - Oxygen absorption at 0.76 μm
  - Water vapor bands at several band locations (largest at 0.94 μm)
- SWIR, MWIR, and TIR affected by strong absorption features preventing surface radiation from reaching the top of the atmosphere
- “Window” region is from 8-12 μm
- Other gases also absorb but impact can be readily decreased
Extinction coefficient and optical depth

Extinction coefficient is an inherent property of a given material related to how efficiently it scatters or absorbs.

- Optical depth includes the distance that a photon must travel to determine the probability that a photon is scattered or absorbed.
  - Extinction coefficient has units of per distance \([m^{-1}]\).
  - Optical depth is unitless.

\[
\delta_\lambda = \int_0^s k_\lambda \, ds
\]

- Alternate form uses the volume extinction coefficient and density.

\[
\delta_\lambda = \int_0^s \mu_\lambda \rho ds
\]

- Large optical depth can occur because.
  - Material is an efficient scatterer or absorber at a given wavelength.
  - The physical path through the material is large.
Optical depth

Optical depth can be divided into separate components due to absorption and scattering

- Extinction coefficients can be treated in like fashion
- Optical depths can be summed to give a total optical depth
  \[ \delta_{\text{total}} = \delta_{\text{scatter}} + \delta_{\text{absorption}} \]
- Scattering optical depth can be split into molecular and aerosol
  \[ \delta_{\text{scatter}} = \delta_{\text{molec}} + \delta_{\text{aerosol}} \]
- Absorption is the sum of individual gaseous components
  \[ \delta_{\text{absorption}} = \delta_{H_2O} + \delta_{O_3} + \delta_{CO_2} + \ldots \]
- This can then be used to understand how radiance changes as it passes through a scattering or absorbing material

- The radiance coming out of the box depends on
  - The amount going in
  - The amount created within the box
  - The material in the box and how it interacts with light
Radiative transfer

The other radiative transfer equation describes how radiance varies in a scattering and absorbing media

- Law of conservation of energy
- Start with the differential form - Schwarzschild’s equation
  \[- \frac{dL_\lambda}{d\delta} = L_\lambda - J_\lambda\]
- The solution gives the integral form
  \[L_\lambda(\delta, \theta, \phi) = \int_0^\delta J_\lambda(\delta', \theta, \phi)e^{-\delta'} d\delta' + L_\lambda(0, \theta, \phi)e^{-\delta}\]
- Ignore the source terms for the rest of this course for simplicity (or we will use very simplified arguments to determine it)
- Then we are left with Beer’s Law
  \[L_\lambda(\delta, \theta, \phi) = L_\lambda(0, \theta, \phi)e^{-\delta}\]
Optical Depth and Beer’s Law

Beer’s Law relates incident energy to the transmitted

\[ L_{\text{transmitted}} = L_{\text{incident}} e^{-\delta} \]
\[ E_{\text{transmitted}} = E_{\text{incident}} e^{-\delta} \]
\[ \Phi_{\text{transmitted}} = \Phi_{\text{incident}} e^{-\delta} \]

- Increase in optical depth means decrease in transmittance
- The exponential term of Beer’s law is the transmittance
  - Then, in terms of optical depth we have
    \[ \tau = e^{-\left(\delta_{\text{molec}} + \delta_{\text{aerosol}} + \delta_{\text{absorption}}\right)} \]
  - Total optical depth has been substituted with the sum of the component optical depths
- Beer’s Law is also used in a logarithmic form to help linearize the problem
  \[ \ln(E_{\text{transmitted}}) = \ln(E_{\text{incident}}) - \delta \]
Absorption

MODTRAN3 output for US Standard Atmosphere, 2.54 cm column water vapor, default ozone, 60-degree zenith angle and no scattering
Absorption

Same curve as previous page but includes molecular scatter

Transmittance

Wavelength (micrometers)
Absorption

At longer wavelengths, absorption plays a stronger role with some spectral regions having complete absorption.
Absorption

![Diagram showing absorption spectra with peaks at different wavelengths for H₂O and CO₂.](image-url)
Absorption

The MWIR is dominated by water vapor and carbon dioxide absorption
Absorption

In the TIR there is the “atmospheric window” from 8-12 μm with a strong ozone band to consider.
Atmospheric composition

The biggest problem in radiometry due to atmospheric composition is the spatial and temporal variability:

- Amount of water vapor can change dramatically from measurement to the next.
- Multiple ways to avoid this problem:
  - Remove the atmosphere
    - Vacuum chambers
    - Nitrogen purge
  - Operate at a wavelength that is not sensitive to water vapor influences
  - Shorten path lengths
- Not trivial to avoid this effect if one must operate in open air:
  - Can still shorten path lengths and change wavelengths
  - Science application must permit this
Atmospheric effects, example

MODIS Airborne Simulator data
Atmospheric effects, example

#12 = 1.75 microns
#13 = 1.80 microns
#14 = 1.85 microns
#15 = 1.90 microns
#16 = 1.95 microns
#17 = 2.00 microns
#18 = 2.05 microns
#19 = 2.10 microns
#20 = 2.15 microns
#21 = 2.20 microns
#22 = 2.25 microns
#23 = 2.30 microns
#24 = 2.35 microns
#25 = 2.40 microns
Atmospheric effects, example
Atmospheric effects, another example

Results here show percent differences between laboratory calibrations and field verifications

- Note the odd behavior of three instruments at 905 nm
- One idea is that this is a residual water vapor absorption effect in the laboratory calibration
Radiometric sources

Recall that the source illuminates the object in the radiometric system

- Anything warmer than 0 K can be a source
- Source partially determines the optical system and detector
  - Source type defines the spectral shape/output
  - Source output is combined with the object properties to get the final spectral shape at the entrance of the optical system
  - Source also defines the power available to the optical system
Sources play a key role in the characterization of the optical system and object

- Characterization of an instrument in the laboratory for use in sunlight is one example
  - Laboratory source could be a lamp (2900 K blackbody)
  - Source used in operation of instrument could be sun (5800 K blackbody)
  - Geometric effects reduce the solar output such that the laboratory source has larger output at $\lambda > 1 \, \mu m$
  - Solar output dominates at shorter wavelengths

- Spectral leak at short wavelengths will not be seen in the laboratory

- Same spectral leak at a short wavelength could dominate the signal

- Thus, the spectral shape of the source needs to be well matched to the application, or well understood
Spectral effects of sources

As an example of the impact of the spectral nature of the source consider the measurement of emissivity:

- Assume that the emissivity of a material is desired across the 7 to 15 μm spectral range.
- Two sources are used to determine the emissivity:
  - 290 K blackbody
  - 330 K blackbody
- Emissivity varies linearly across the spectral range.
- Derived emissivity from the two sources differs by more than 1%.
Natural sources

Natural sources are a driving influence in the design of laboratory and artificial sources

- Examples of natural sources include the sun, skylight, thermal emission from the ground and sky
- Rare to see a natural source in the laboratory due to the difficulty to control the output level
  - Several groups are now using light pipes and heliostats to bring natural light into the laboratory
  - Must figure out ways to offset absorption and scattering effects
- Advantage to natural sources is that they simulate the source which might be used for the operation of the system
- A great deal of effort is made to attempt to simulate natural sources
  - Human eye and brain has adapted to natural conditions and these are viewed as normal
  - Background skylight
  - Sunlight
Sources - Solar radiation

Studying the sun as a radiometric source is helpful both from a modeling and measurement standpoint.

- The sun is the primary source of energy that drives the earth’s weather and climate.
- Peak of solar radiant exitance is at approximately 0.45 μm.
- Distance from the earth to sun varies from 0.983 to 1.0167 AU.
  - AU is an astronomical unit and is the ratio of the actual distance to the mean distance.
  - Using $1/R^2$ gives a difference of 7% in the irradiance between minimum and maximum distance.
  - Mean distance is $1.496 \times 10^{11}$ m.
  - Mean radius $6.957 \times 10^8$ m.
Solar radiation

Solar constant has been studied quantitatively since the early 1900s

- Samuel Langley’s work for the Smithsonian Institute used ground-based measurements from mountain tops
- Next improvement was balloon-borne instruments
- 1960s and later used rocketsondes and space-based instruments
- Currently agreed upon value for solar constant is 1367 W/m² (based on the work of Frohlich)
Solar radiation - spectral

Solar constant is not as well understood at the spectral level

- Solar variability at very short wavelengths
- Calibration uncertainty at longer wavelengths
- Knowledge in the VNIR is better than the SWIR
- Two models shown here are both used in the remote sensing community
  - MODTRAN-based model is from Chance-Kurucz based on measurements at Kitt Peak, Arizona
  - WRC is based on the Frohlich work

![Graph showing spectral irradiances for MODTRAN-based and WRC-Based models with wavelength from 0.70 to 0.90 micrometers.](image)
Knowledge has improved since the 1990s due to several shuttle-based experiments

- These shuttle-based results are still being evaluated
- Will require several historical works to be revised
- The newer results should point to which of the current set of spectral curves is the most accurate
- Curve here shows the percent difference between the WRC and MODTRAN-based models
- Note that in many cases the uncertainty requirements of some sensors that are measuring terrestrial energy are exceeded in some bands solely by the differences in solar models
Solar radiation - blackbody model

Could also use the Planck curve to simulate the output of the sun

- Note that one single curve would not be sufficient
- Equivalent temperature to obtain the 1367 W/m² would be 5778 K
Terrestrial radiation

Blackbody approach works a bit better with terrestrial surfaces

- Planck curves below show typical terrestrial temperatures
- Peak is in the TIR
- Spectral emissivity must be taken into account
Terrestrial radiation

Would still need to modify the Planck curve output to take into account the spectral emissivity of the surface

- Most terrestrial surfaces can be adequately approximated as gray bodies
- Plots here are measured emissivities of several materials

![Graphs of emissivity vs. wavelength for different materials](image)
Solar-terrestrial comparison

It can be shown that solar energy dominates in VNIR/SWIR and emitted terrestrial dominates in the TIR

- Takes into account the earth-sun distance
- Sun emits more energy than the earth at ALL wavelengths
- It is a geometry effect that allows the wavelength regions to be treated separately

![Graph showing spectral radiant exitance vs. wavelength](image)
Solar-terrestrial comparison

The same plot in linear space shows how the peak spectral output of the sun exceeds that of the earth.
Solar/Terrestrial comparison

Putting the two curves on a relative scale highlights the difference between the solar reflective and the emissive parts of the spectrum.

The graph shows two curves: one representing solar irradiance and the other representing terrestrial exitance. The x-axis represents wavelength (micrometers), and the y-axis represents spectral radiant exitance in W/(m²·micron). The graph demonstrates the spectral distribution of solar and terrestrial radiation, with peaks in different wavelengths indicating the absorption and emission characteristics of sunlight and the Earth's outgoing radiation.
Skylight

Plot here illustrates a modeled output from the sky for a given view-sun geometry

- Plot will also vary depending upon atmospheric conditions
- Curve is clearly dominated by the solar output
- Now have two parts of the three part puzzle why the sky is blue
**Artificial sources**

Common sources used in the laboratory include lasers, incandescent bulbs, LEDs, and discharge lamps

- Just as there are large differences in the output of the sun and earth with wavelength there are differences with artificial sources
- The source used depends heavily on the application
- Lasers are often used because of their spectral purity
  - Downside is often the variability of the output
  - Also need to find a source at the appropriate wavelength
- Discharge lamps or fluorescent sources are used at shorter wavelengths
  - Require approximately a 50,000 K blackbody to simulate blue sky
  - Discharge lamps can give higher output at short wavelengths relative to a blackbody
- LEDs have broader spectral output than lasers
  - Can be more stable but still better to have controlled output
  - Mixtures of colors can be used to get the appropriate spectrum
Broadband versus spectral

Curves below show typical spectral outputs from both a fluorescent tube (left) and incandescent bulb (right)

- Fluorescent tube output is characterized by several spiked features
- Incandescent bulb has a peak at a wavelength longer than the peak output of the sun
- These two factors can lead to interesting color effects in clothing and paints
Artificial sources

Artificial sources have received widespread attention outside the laboratory in reference to illumination studies

- Computer displays
- Nighttime illumination
  - Inside the home
  - Streetlights, traffic signals, vehicular lighting
- Large amount of research in developing energy efficient lighting that can simulate the spectral output of normal daylight conditions
  - Incandescent bulbs with appropriate coatings
  - Halogen lamps that are hotter than standard tungsten filament bulbs
  - Fluorescent lamps with different mixtures of gases and internal coatings
  - Mixtures of LEDs
- Development of artificial lighting is a good example of where radiometry (and photometry) is critical to understanding the problem
“Standard” Lamps

Full name is a standard source of spectral irradiance or spectral irradiance standard

- Filament lamp sources work on the simple concept of heating the filament to the temperature needed to obtain the desired spectral output
- Image at right shows an FEL secondary standard
  - FEL just refers to an ANSI designation
  - Obtained from commercial vendor
- Typical operation parameters are 120 V at 8 A current
- Tungsten filament
- Filament operates at around 3000 K
Radiometric sources

A major issue in radiometry is developing sources that satisfy needed outputs both spectrally and absolutely.

- Example here compares the output of the sun at the top of the earth’s atmosphere to that of an FEL lamp standard.
  - FEL lamp is at the standard distance of 50 cm.
  - Solar values are based on the WRC model.
- Note the low lamp output in the blue part of the spectrum.
- Large output of lamp relative to the sun at long wavelengths also causes problems.

![Graph of spectral irradiance vs. wavelength (micrometer)]
Lamp sources

In addition to FEL lamps are other tungsten lamps for laboratory usage

- Photo here shows a DXW lamp that is not absolutely calibrated and is much less expensive than an FEL standard

- Difficulties with lamp sources are
  - Degradation
  - Changes in tungsten spectral emissivity
  - Change in tungsten resistance during usage

- Use of halogen improves the lamp stability over time
Spherical integrating source

SIS relies on the fact that light from a lamp source goes through multiple reflections prior to exiting the sphere

- Images below show the RSG’s 40-inch SIS
- Attempt to coat sphere with near-lambertian, spectrally-flat material
- SIS provides a near-lambertian source with some extended size
- The lamp alone provides a near-point source
Blackbody sources

Laboratory blackbodies are designed to have emissivities that are near unity

- SIS provides a uniform illumination for radiance calibration at short wavelengths
- Laboratory blackbody provides a good source for longer wavelengths with much the same principles
- $M_{\text{emitted}} = \epsilon \sigma T^4$
  - Trick is to have knowledge of the temperature
  - Original traceability to national standards was based on the temperature sensors for a specific conical design
Blackbody sources

Typical design of a blackbody incorporates some type of cavity that enhances the absorption of light

- Conical cavity is one of the easiest to produce
  - Spherical design also works well (just like the SIS)
  - Other designs are also used

- The idea is to have a large surface area to emit the radiant exitance but requiring multiple bounces for the photons to exit the port