

**Economics 696, Homework 3**  
**Due March 14 (Friday)**

The data for this exercise are in the file `hw3.dat`, available on the course web site. It contains data on 10 years of labor earnings for 516 males from the Panel Study of Income Dynamics. The values in the data set are residuals from least-squares regressions of log earnings on a variety of characteristics.

Consider the following model:

$$y_{it} = \alpha_i + \rho y_{i,t-1} + u_{it}, \quad i = 1, \dots, n, t = 1, \dots, T.$$

Here  $T = 9$  and  $n = 516$ . Assume that the  $u_{it}$  are i.i.d.  $N(0, \sigma^2)$ .

(1). Assume a diffuse prior for  $\sigma^{-2}$  and  $(\rho, \alpha_1, \dots, \alpha_n)$ . What is the posterior distribution for  $\sigma^{-2}$  and  $\rho$ ? Report the (marginal) posterior mean and .025 and .975 quantiles for  $\rho$ .

(2). Now assume that

$$\alpha_i | y_{i0} \sim N(\tau y_{i0}, \sigma_v^2).$$

Explain why you might expect  $\alpha_i$  to be correlated with  $y_{i0}$ .

Consider the following prior specification:  $(\rho, \sigma, \tau, \sigma_v)$  are mutually independent with

$$\sigma^{-2} \sim \frac{\chi^2(1)}{.01},$$

$$\sigma_v^{-2} \sim \frac{\chi^2(1)}{.01},$$

$$\rho \sim N(0, 10^4),$$

$$\tau \sim N(0, 10^4).$$

Describe a Gibbs sampling routine for this model.

(3). Write a program to implement the Gibbs sampling routine you developed in part (2). Run the algorithm for 500 iterations, and plot the sequence of draws for the parameters  $\rho$ ,  $\tau$ ,  $\sigma$ , and  $\sigma_v$ . Discard the first 100 draws and use the remaining draws to report posterior .025, .5, and .975 quantiles for  $\rho$ ,  $\tau$ ,  $\sigma$ , and  $\sigma_v$ .

(4). Repeat (3) using a different set of starting values for the Gibbs sampler.