Economics 696, Homework 2
Due March 5

For some of the calculations below, you will find it useful to perform kronecker products and reshapings of matrices. In Matlab, the functions `kron`, `reshape` do this. Also, you will need to draw from a multivariate $\mathcal{N}(\mu, \Sigma)$ distribution. Some packages offer a routine to do this, or you can use the method described in LN4 based on scalar normal random draws.

PART I.

The data for this exercise are available in plain ascii form in the file `hw2i.dat`. `ccrs` is the continuously compounded real stock return, based on the value-weighted index of stocks traded on the NYSE with the Consumer Price Index used for deflation. (So a real dollar invested in the stock fund at the beginning of the month returns exp(ccrs) real dollars at the end of the month.) `ccrtb` is the continuously compounded U.S. Treasury Bill return, and `dpr` is the dividend-price ratio. There are 499 monthly observations for the period June 1952 through December 1993. (In `hw2i.dat`, the first column of observations is `ccrs`, the second column is `ccrtb`, and the third column is `dpr`.)

1. Consider the VAR model

$$Y_t | Y_0 = y_0, \ldots, Y_{t-1} = y_{t-1} \sim \mathcal{N}(\Pi x_t, \Sigma), \qquad t = 1, 2, \ldots,$$

where $x_t = (1, y_{t-1})'$.

(a) Write a program to calculate the mean of $\sum_{t=1}^{H} Y_{T+t}$, conditional on Π, Σ, and on y_T. The following identity is useful:

$$(I + B + \ldots + B^{L-1}) = (I - B)^{-1}(I - B^L)$$

(assuming that $I - B$ is nonsingular).

(b) Write a program to calculate the variance of $\sum_{t=1}^{H} Y_{T+t}$, conditional on Π, Σ, and on y_T.

2. Pretend that Σ is known, with the following value:

$$\Sigma = \sum_{t=1}^{T} (y_t - \hat{\Pi} x_t)(y_t - \hat{\Pi} x_t)'/T,$$

where $\hat{\Pi}$ is obtained from the least-squares estimates.

(a) Let Y_t = (ccrs ccrtb)'. Suppose that the prior density for Π is constant, and calculate the posterior means and standard deviations for the coefficients in Π.

(b) Take $J = 1000$ draws from the posterior distribution of Π. Compare the means and standard deviations of these draws with your results in (a).

(c) A (real) dollar invested in the stock fund at T will return $W_{T+H} = \exp(\sum_{t=T+1}^{H} Y_{1,T+t})$ at $T + H$. For each of the first five draws from (b), use your programs from (a) to calculate the density of W_{T+H}, conditional on Π (and Σ) and y_T. Use a ten year horizon ($H = 120$), and for y_T, use the sample mean of Y_t ($t = 1, \ldots, T$). Plot the five densities on a single graph.

(d) Explain how to calculate the predictive density for W_{T+H}, conditioning only on the data $y_0, \ldots, y_T$. Illustrate the calculation using the five densities you have calculated in (c), and (on a separate graph) plot this predictive density (based on five draws).

3. Now suppose that the investor exploits the information on the dividend-price ratio, using a VAR with Y_t = (ccrs ccrtb dpr)'.

(a) Repeat the calculations in 2(a) and 2(b). (Recalculate Σ to reflect the change in x_t.)

(b) Repeat the calculation in 2(c). Plot the five densities on the same graph with the five densities from 2(c).

(c) Repeat the calculation in 2(d). Plot the predictive density on the same graph with the predictive density from the model that ignores the dividend-price ratio.

PART II.

Consider the probit model:

$$\Pr(y_i = 1|x_i) = \Phi(\alpha + \beta x_i),$$

where y_i and x_i are scalars, and conditional on α, β, x, the y_i are independent.

The data for this exercise are available in ascii format in the file `hw2ii.dat`. (The first column of `hw2ii.dat` contains the vector of observations on y_i, and the second column contains the vector of observations on x_i.)

Consider two priors for α, β:

$$p(\alpha, \beta) \propto 1,$$

$$p(\alpha, \beta) \propto \exp(-\frac{100}{2}\beta^2),$$

(a) Calculate the marginal posterior $p(\beta|y, x)$ under the two prior specifications. To perform the calculations, use the *discretization* method discussed in Lecture Note 5. Discretize the parameter space for α as

$$\alpha \in \{-10.0, -9.9, \ldots, 9.9, 10.0\}.$$

Likewise, discretize the parameter space for β as

$$\beta \in \{0, .01, .02, \ldots, .49, .50\}.$$

Plot the two marginal posterior distributions on one plot, and discuss their similarities and/or differences.

(b) Write a data augmentation algorithm, assuming a uniform prior on both α and β, to simulate the posterior distribution of (α, β). Initialize the parameters at $(0, 0)$ and run the algorithm for 500 iterations. Save the draws for α and β; you do not need to save the draws for any latent variables for the purposes of this exercise.

Plot α and β over time and discuss whether it appears that they have "converged." Provide an approximate posterior mean and posterior variance for β. Discard the first 100 iterations and plot a histogram of the last 400 draws for β.