**Economics 696, Homework 1**

**Due Feb 15, 2008** (Note: this is a Friday. You may turn the homework in to my mail box in the Economics department by the end of the day.)

**1.** Consider the following loss function:

$$L(\theta, a) = \begin{cases} p \cdot |\theta - a| & \text{if } \theta > a \\ (1-p) \cdot |\theta - a| & \text{if } \theta \leq a \end{cases}$$

for some given $p \in [0, 1]$.

(a) Discuss what this loss function looks like for different values of $p$.

(b) Show that the Bayes estimate under this loss function is the $p$th quantile of the posterior distribution for $\theta$.

**2.** Consider the **classical regression model:**

$$Y_i | X_1 = x_1, \ldots, X_n = x_n, \beta, \sigma \overset{\text{ind.}}{\sim} \mathcal{N}(x_i'\beta, \sigma^2), \qquad i = 1, \ldots, n,$$

where $x_i$ is a $k \times 1$ vector and $y_i$ is a scalar. Let $z = (y_1, x_1', \ldots, y_n, x_n')$. Let $\tau \equiv 1/\sigma^2$. Suppose we use conventional diffuse priors, as in Lecture Note 4:

$$p(\beta, \tau) \propto \frac{1}{\tau}.$$

**(a)** Suppose $a \in \mathbb{R}^k$. What is the posterior distribution of $a'\beta$ given $\tau$ and $z$?

**(b)** Derive the (marginal) posterior distribution of $\tau$,

$$p(\tau | z) = \int p(\beta, \tau | z) d\beta.$$

The following may be useful:

**Scaled Chi-square Distribution:** Suppose that $W$ has a $\chi^2$ distribution with $\nu$ degrees of freedom. This has density

$$f_W(w) = c \cdot w^{\frac{\nu-2}{2}} \exp\left(-\frac{1}{2}w\right),$$

where $c$ is a constant such that the density integrates to one. Then, if $\gamma$ is a positive number, $U \equiv \gamma \cdot W$ has density

$$f_U(u) = c \cdot u^{\frac{\nu-2}{2}} \exp\left(-\frac{1}{2\gamma}u\right).$$

**(c)** Recall that if $U \sim \mathcal{N}(0,1)$, and $W \sim \chi^2_\nu$, and $U$ and $W$ are independent, then $U/\sqrt{W/\nu}$ has a $t$ distribution with $\nu$ degrees of freedom, denoted

$$\frac{U}{\sqrt{W/\nu}} \sim t(\nu).$$

Show that

$$\sqrt{H}(a'\beta - a'b)|z \sim t(n-k),$$

where $b$ is the vector of least-squares coefficients. What is $H$?

**3. Autoregressive Model:** Use the data in the matlab file `hw1.dat`, which contains observations on log real GDP per capita for the U.S. Suppose that the following model applies:

$$Y_t|Y_0 = y_0, \ldots, Y_{t-1} = y_{t-1}, \beta, \sigma \sim \mathcal{N}(\beta_1 + \beta_2 y_{t-1}, \sigma^2), \qquad t = 1, 2 \ldots$$

**(a)** Using diffuse priors, provide point estimates based on squared error loss for the $\beta$ coefficients. Does the process appear to be stationary?

**(b)** Calculate the optimal forecast for $Y_{T+1}$ conditional on $y_0, \ldots, y_T$ based on squared error loss:

$$L(y, a) = (y-a)^2.$$

**(c)** Repeat **(b)** calculating the optimal forecast for $Y_{T+5}$.