Economics 696F, Homework 2

Due Friday, February 3, 2006

For this homework, you may use any standard statistical package, such as Matlab or Stata. Provide the code used to generate the simulations, and make sure the code is clearly commented and explained in your own words.

1. Consider the following model:

$$\begin{aligned} y_i &= \beta x_i + u_i \\ x_i &= \gamma z_i + \{\gamma v_i + \lambda u_i\}. \end{aligned}$$

Suppose that u_i, v_i, and z_i are independent mean-zero normal random variables with variance σ_u^2, σ_v^2, and σ_z^2, respectively. Assume i.i.d. sampling for $i = 1, \ldots, n$.

Consider the IV estimator b_{IV}

$$b_{IV} = \frac{\sum_{i=1}^n z_i y_i}{\sum_{i=1}^n z_i x_i}.$$

(a) Show that this estimator is a GMM estimator. Derive the asymptotic distribution of b_{IV}, and provide a consistent estimator of its variance.

(b) Set $\sigma_u = \sigma_v = \sigma_z = 1$, $\beta = 1$, and $\lambda = 1$. Assume a sample size of $n = 100$. Set $\gamma = 2$ and approximate the sampling distribution of b_{IV} by Monte Carlo, using 1000 replications (so you should end up with 1000 independent draws for b_{IV}). Plot a histogram of the draws for b_{IV}.

Calculate the asymptotic approximation to the distribution of b_{IV} for these parameter values, and compare it to the finite-sample distribution. Explicitly compare the mean, variance, and the overall shape of the two distributions.

(c) Repeat the experiment in part (b) but with $\gamma = .5$. For this part and the next, when forming the histograms, it may be useful to only use the draws for b_{IV} that are within the interval $[-8, 8]$ to make the histograms easier to read.

(d) Repeat the experiment but with $\gamma = .1$. Comment on how the quality of the asymptotic approximation relates to γ.

(e) Explain what would happen if $\gamma = 0$. How does this relate to the conditions for consistency and/or asymptotic normality of the GMM estimator?