Economics 696F, Final Review Questions

For the final exam, you must work completely on your own. You may use any books, lecture notes, and other printed reference materials you wish, but you may not discuss your exam with any other person while you are working on it.

The following questions indicate some of the types of questions you may be asked on the final, but the final may cover any topic discussed in the course.

1. Suppose you have data on log wages (y_i), and a vector of regressors x_i containing a constant and variables like education, experience, and demographic characteristics. Suppose that

$$y_i = x_i'\beta + \epsilon_i.$$

   (a) Consider the moment equation $E[x_i\epsilon_i] = 0$. Show that the least squares coefficient can be interpreted as a GMM estimator, and derive estimators for the standard error based on homoskedasticity and heteroskedasticity.

   (b) Suppose we also know that $E[y_i] = C$, where C is a known constant. Show how to incorporate this restriction in a GMM framework to obtain an estimate of β. Explain how to obtain the optimal 2-step GMM estimator, in enough detail so that a skilled programmer could implement the estimator. Is this estimator "better" than the LS estimator? Explain your reasoning.

   (c) Suppose that $E[y_i|x_i] = x_i'\beta$. Does this lead to other possible GMM estimators which might improve upon your previous estimator?

2. Suppose we have a logit model with a binary regressor:

$$Pr(y_i = 1|x) = \frac{\exp(\gamma_1 + \gamma_2 x_i)}{1 + \exp(\gamma_1 + \gamma_2 x_i)},$$

   where $x_i = 0$ or 1 for all $i = 1, \ldots, n$.

   (a) Write down the logit log-likelihood, and obtain as simple a characterization of the logit MLE as possible.

   (b) Show that the logit model can be written as a linear probability model

$$Pr(y_i = 1|x_i) = \beta_1 + \beta_2 x_i.$$

   Determine β_1 and β_2 in terms of the logit parameters γ_1, γ_2.

(c) Let $\hat{\beta}_1$ and $\hat{\beta}_2$ be the LS coefficients in a (linear) regression of y_i on a constant and x_i. Show how $\hat{\beta}_1$ and $\hat{\beta}_2$ can be converted into estimates of the logit coefficients γ_1, γ_2, and prove that the resulting estimator of (γ_1, γ_2) is consistent.

3. Suppose that y_i are IID Uniform on $[-\theta, \theta]$, for $\theta \in [a, b]$, $0 < a < b < \infty$.

   (a) Derive the maximum likelihood estimator. Obtain as simple a characterization as possible.

   (b) Is the MLE consistent? Justify your reasoning as completely as possible.

   (c) Suppose that the prior distribution for θ is uniform on $[a, b]$. Explain (providing pseudo-code) how to sample from the posterior distribution for θ given a random sample of size n using a Metropolis algorithm.

4. Suppose that x_i is uniform $[0, 1]$, and that $y_i = x_i + N(0, 1)$. Suppose we estimate the regression function of y_i given x_i using k-nearest neighbor nonparametric regression.

   (a) Provide pseudo-code for implementing the k-nearest neighbor regression.

   (b) Suppose that we use $k = 1$, and consider the estimate of $E[y_i | x_i = 0]$. What is the bias of this estimator? Try to provide as explicit a characterization of this bias as possible.