Economics 522A, Midterm Review Questions

1. Consider the following probability mass function:

$$f(x|\theta) = \begin{cases} .4-\theta & x=1 \\ .3 & x=2 \\ .1+\theta & x=3 \\ .2 & x=4 \\ 0 & \text{otherwise} \end{cases}$$

   where $-.1 \leq \theta \leq .4$.

   (a) Suppose we have a random sample of size 1 from $f(x|\theta)$. Give the critical region for a uniformly most powerful test (with level .1) of $H_0 : \theta = 0$ vs. $H_1 : \theta = .2$.

   (b) Would the critical region be the same for a uniformly most powerful test (with level .1) of $H_0 : \theta = 0$ vs. $H_1 : \theta > 0$? Why?

2. Consider the following probability mass function:

$$f(x|p) = \begin{cases} p & \text{if } x=0 \\ p^2 & \text{if } x=1 \\ 1-p-p^2 & \text{if } x=3 \\ 0 & \text{otherwise} \end{cases}$$

   Suppose $\{3,0,0,3,0,3\}$ is a random sample of size $n=6$ from the PMF $f(x|p)$.

   (a) Calculate the MLE of p given this sample of data.

   (b) What is the asymptotic distribution of $\sqrt{n}(\hat{p}_{ML} - p)$? (Use $\hat{p}_{ML}$ to provide an estimate of the asymptotic variance.)

   (c) Perform a Wald test (with size .05) of $H_0 : p = .2$ vs $H_1 : p \neq .2$, using your estimate of the variance matrix from the previous part.

   (d) Suppose $n = 1$. What is the critical region for the uniformly most powerful test with size .24 of $H_0 : p = .2$ vs $H_1 : p = .4$?

3. Suppose that $X_1, \ldots, X_n$ are IID with PDF

$$f(x;\theta) = \theta x^{\theta-1}, \qquad \text{for } 0 \leq x \leq 1,$$

   and 0 otherwise.

   (a) Suppose that we have $n = 100$ and $\sum_{i=1}^{n} \log X_i = -25$. Test the hypothesis that $\theta = 3$ using a Wald test at the 5% level. (Hint: a $\chi^2(1)$ random variable has probability .05 of being greater than 3.84.)

   (b) Carry out an LR test of the same hypothesis.

   (c) Construct a 95% confidence interval for θ.

4. Suppose that the vector (y_i, x_i) is IID from a distribution such that

$$E[y_i \mid x_i] = x_i'\beta,$$

for some $\beta \in \mathbb{R}^k$, and

$$V[y_i \mid x_i] = \exp(x_i'\gamma),$$

for some $\gamma \in \mathbb{R}^k$. In other words, the conditional variance of y_i given x_i is not a constant, but depends on x_i.

   (a) Set up the conditional log likelihood for this model (but do not solve for the conditional MLE).

   (b) Show that, for a sample of size n, the OLS estimator $\hat{\beta}$ is unbiased: $E[\hat{\beta}] = \beta$.

   (c) Derive the asymptotic variance of the OLS estimator, stating any additional assumptions you need.

5. Suppose that $x_i = (1, x_{i2}, x_{i3})'$, and we assume $E[y_i \mid x_i] = x_i'\beta$. However, we decide to estimate β in a two step procedure. First, we form the OLS estimate $\hat{\beta}$. Second, we test whether β_3, the coefficient on x_{i3}, is zero, using a standard t test at the 5% level. If the t-test rejects the null hypothesis, we report the original OLS estimate. If the t-test does not reject the null hypothesis, we re-run the regression, dropping the variable x_{i3}.

   We might worry that this procedure could produce some odd behavior if the true β_3 is close to, but not exactly, zero. Outline how you would construct a simulation (Monte-Carlo) study to examine this possibility. You do not have to provide specific Matlab code, but give enough detail so that a skilled programmer who knew no stastistics or econometrics could implement your proposed simulation study.