

**Economics 522A, Spring 2007**  
**Midterm, March 8, 2007**

Please write your answer to each question on a separate sheet of paper, and put your name on every sheet of paper. The exam has 3 questions, with a total of 70 possible points. Write clearly and legibly, and show all your work.

1. (20 points) Consider the following function file in Matlab:

```
function m = mystery(X);  
  
[n,k] = size(X);  
y = X*ones(k,1)+randn(n,1)*sqrt(3);  
P = X*inv(X'*X)*X';  
G = P*(P');  
m = (y-G*y)'*y;
```

What is the distribution of  $m$ ?

2. (30 points) Suppose that  $x_i$ , for  $i = 1, \dots, n$ , are IID with probability density function:

$$f(x|\gamma) = \frac{\gamma^{1/2}}{\sqrt{2\pi}} \exp\left(-\frac{\gamma}{2} \cdot x^2\right),$$

where  $\gamma$  is a strictly positive parameter.

- (a) Derive the MLE for  $\gamma$ , and provide its exact and large-sample distributions.
- (b) Suppose we wish to test  $H_0 : \gamma = 1$  against the alternative that  $\gamma = .5$ . Derive a most powerful test. (You do not need to give the exact cutoff for the test—just provide the general form of the test.) Will this test be most uniformly most powerful against the alternative that  $\gamma < 1$ ? What about the two-sided alternative that  $\gamma \neq 1$ ?
- (c) Explain how to construct a 95% confidence interval for  $\gamma$ , using asymptotic approximations.
3. (20 points) Suppose that  $(x_i, y_i), i = 1, \dots, n$ , are a random sample from a distribution, with

$$x_i \sim Unif[0, \theta], \quad y_i|x_i \sim Bernoulli(x_i \cdot \beta).$$

Both  $\theta$  and  $\beta$  are unknown.

- (a) Write down the conditional likelihood and conditional log likelihood for  $y_1, \dots, y_n$  given the  $x$ 's. You do not need to solve for the MLE explicitly.
- (b) Suppose we use the OLS estimator  $\hat{\beta}$  to estimate  $\beta$ . Derive the (unconditional) expected value of  $\hat{\beta}$ . Derive the probability limit of  $\hat{\beta}$ .