

## Economics 522A, Homework 9

Due Tuesday, April 10

1. Assume that  $y_1 = 0$ , and for  $t = 2, \dots, T$ , that

$$y_t = \alpha y_{t-1} + \epsilon_t,$$

where the  $\epsilon_t$  are IID  $N(0, 1)$ . Let  $\hat{\alpha}$  be the OLS coefficient in a regression of  $y_t$  on  $y_{t-1}$  (without a constant).

Simulate the distribution of  $\hat{\alpha}$  for  $T = 20$ , for different possible values of  $\alpha$ :  $\alpha = .5$ ,  $\alpha = .9$ , and  $\alpha = 1$ . For each value of  $\alpha$ , generate 1000 draws, plot the histograms of the draws for  $\hat{\alpha}$ , and calculate its bias.

2. Let  $y$  be an  $n \times 1$  vector, and let  $X$  be  $n \times k$ , and that  $E[y|X] = X\beta$ . We divide the variables in  $X$  into two groups:

$$E[y|X] = X_1\beta_1 + X_2\beta_2,$$

where  $X_1$  is  $n \times k_1$ ,  $X_2$  is  $n \times k_2$ , and  $k_1 + k_2 = k$ . Let  $\hat{\beta}_1$  be the OLS coefficients in a regression of  $y$  on  $X_1$ . Derive  $E[\hat{\beta}_1|X]$  (note: we are conditioning on all of  $X$ ), simplifying as much as possible. Compare your results to the findings in LN19, Section 2, and interpret your formula.