Economics 522A, Homework 8

Due Tuesday, April 3

1. Ruud, 18.1, part (a).

2. Suppose that $E[y|X] = X\beta$, and $V[y|X] = \Omega$, and the usual assumptions on X and Ω apply. Now suppose the researcher is mistaken and believes that $V[y|X] = \Sigma$, where $\Sigma \neq \Omega$.

   (a) Is the GLS estimator using the wrong variance matrix Σ unbiased?

   (b) What is the variance of the GLS estimator based on Σ?

3. Suppose that $y|X \sim N(X\beta, \Omega)$, where Ω is a known, symmetric, positive definite matrix, and X has full column rank. Consider a $m \times k$ matrix R with full row rank. Let $\hat{\beta}$ be the **GLS** estimator. Show that

$$\frac{(R\hat{\beta} - R\beta)' \left[R(X'\Omega^{-1}X)^{-1}R'\right]^{-1} (R\hat{\beta} - R\beta)/m}{(y - X\hat{\beta})'\Omega^{-1}(y - X\hat{\beta})/(n-k)}$$

   has an F distribution with m and $n - k$ degrees of freedom. Explain how you would use this result to test a set of linear restrictions on β.

4. Consider the data in `wage.dat`. Let y_i be the log of wage, and let x_i include a constant, the female dummy, the nonwhite dummy, the union indicator, years of schooling, and age.

   (a) Calculate the OLS regression of y_i on x_i, and report standard errors assuming homoskedasticity and also report the Eicker-White standard errors. Comment on the assumptions needed for the Eicker-White standard errors to be valid, and whether there is evidence of heteroskedasticity based on the two sets of standard errors.

   (b) Assume that $y_i|X \overset{\text{ind.}}{\sim} N(x_i'\beta, \sigma_i^2)$, where $\sigma_i^2 = \sigma_m^2$ for males, and $\sigma_i^2 = \sigma_f^2$ for females. Test the null hypothesis that $\sigma_m = \sigma_f$, against a two-sided alternative, at the 5% level.

   (c) Continue with the assumptions in part (b). Calculate the FGLS estimate of β. Provide asymptotically valid standard errors for the FGLS estimates.