

Economics 522A, Homework 6

Due Tuesday, February 27

1. Suppose we assume that $E[y_i|X] = x_i'\beta$, as in LN11.

(a) Show that this implies:

$$E[x_i(y_i - x_i'\beta)] = 0.$$

Here, “0” denotes a $k \times 1$ vector of zeros.

- (b) Now, suppose we replace the population expectation with a sample average, and define $\hat{\beta}$ as the solution to:

$$\frac{1}{n} \sum_{i=1}^n x_i(y_i - x_i'\hat{\beta}) = 0.$$

Assuming that $(X'X)$ is invertible, show that $\hat{\beta}$ defined above is equal to the OLS estimator.

- (c) Let $e_i = y_i - x_i'\hat{\beta}$. Show that, for any $j = 1, \dots, k$, the sample covariance between x_{ij} and e_i is zero.

2. Suppose we want to solve the OLS problem, but subject to a linear restriction on β :

$$\min_{\beta} (y - X\beta)'(y - X\beta),$$

subject to

$$R\beta = r,$$

where R is $(k - m) \times k$ and r is $(k - m) \times 1$. Assume X has full column rank. Show that the solution to the restricted least squares problem is:

$$\hat{\beta}_R = \hat{\beta} - (X'X)^{-1}R'[R(X'X)^{-1}R']^{-1}(R\hat{\beta} - r),$$

where $\hat{\beta}$ denotes the ordinary LS estimator.

3. The data set `hw6.dat` was drawn from an evaluation of a job training program for welfare recipients called the National Supported Work Demonstration Project. It contains a number of variables, including `re78`, earnings of the individual in 1978 (after the program), and `treat`, a dummy variable equal to 1 if the individual received the job training.

Assume that the normal linear regression model holds, where y_i is `re78`, and x_i contains a constant and `treat`. Calculate $\hat{\beta}$, the standard errors, s^2 , and R^2 . Test the hypothesis that $\beta_2 = 0$ at the 5% level, using both a t-test and an F-test. Construct a 95% confidence interval for β_2 .

Interpret your results. You should explain clearly what the meaning of β_1 and β_2 are, and explain what you conclude about these parameters based on your estimates, tests, and confidence intervals.