

Economics 522A, Homework 2

Due Tuesday, January 30

1. Suppose we want to test if a coin is fair (that is, the probability of heads is $p = .5$). We toss the coin 1000 times, and record the number of heads. Let T denote the number of heads divided by 1000. Consider a test that rejects the null hypothesis that $p = .5$ if $T > c$.
 - (a) Write down a formula for $P(T > c | p = .5)$. Write a short Matlab function to calculate this quantity for different values of c , and use it to calculate the probability for $c = 560/1000$.
 - (b) Give a large-sample approximate formula for $P(T > c | p = .5)$ based on the central limit theorem.
 - (c) Using the large-sample approximation, calculate the appropriate value c for a test at the 0.05 percent level. You can use the fact that if $Z \sim N(0, 1)$, then $P(Z > 1.645) = 0.05$. Would your test reject the null hypothesis if we observed 560 heads and 440 tails?

2. Suppose that X_1, \dots, X_n are IID with PDF

$$f(x; \theta) = \theta x^{\theta-1}, \quad \text{for } 0 \leq x \leq 1,$$

and 0 otherwise.

- (a) Obtain the Cramer-Rao bound for estimation of θ .
- (b) Calculate a general formula for the maximum likelihood estimator of θ .
- (c) Suppose that we have $n = 100$ and $\sum_{i=1}^n \log X_i = -25$. Test the hypothesis that $\theta = 3$ using a Wald test at the 5% level.
- (d) Carry out an LR and LM test of the same hypothesis.
- (e) Construct a 95% confidence interval for θ .