

Economics 522A, Homework 1

Due Tuesday, January 23 (at beginning of class)

Note: “CB” refers to exercises from Casella and Berger.

1. Let the random variable X have probability density function $f_X(x; \theta) = (1/\theta) \exp(-x/\theta)$ for $x > 0$. Consider the simple hypothesis $H_0 : \theta = 2$ and the alternative hypothesis $H_a : \theta = 4$. Let X_1, X_2 denote a random sample of size two from this distribution. Show that the best test of H_0 against H_a may be carried out by use of the statistic $X_1 + X_2$. Find the optimal critical region for $\alpha = 0.1$.
2. Let X_1, \dots, X_{10} be a random sample of size 10 from a normal distribution with mean zero and variance σ^2 . Find a best critical region for testing $H_0 : \sigma^2 = 1$ against $H_a : \sigma^2 = 2$ of size $\alpha = 0.05$. Is this also a best test of the same null hypothesis against the alternative hypothesis $H_a : \sigma^2 = 3$? And against the alternative hypothesis $H_a : \sigma^2 > 1$?
3. CB 8.20.
4. CB 8.22.