

Economics 522A, Final Review Questions

Note: the final exam will be cumulative and may cover any topic in the course, not just the topics in this set of review questions. The review questions mostly cover the second half of the course, but you should also go back and review material from the first half of the course.

1. Suppose that (y_i, x_i) are IID, with x_i and y_i scalar, and that y_i is a discrete random variable whose distribution conditional on x_i is

$$Pr(y_i = y | x_i = x) = \frac{e^{-\beta x} (\beta x)^y}{y!}.$$

Thus, y_i has a Poisson distribution with parameter βx_i conditional on x_i .

- (a) Write down the log likelihood function and derive the conditional maximum likelihood estimator for β .
 - (b) Find the asymptotic distribution of the conditional MLE for β .
 - (c) Find the exact distribution of the MLE conditional on the x_i s. (Hint: the sum of independent Poisson random variables with parameters λ_j is a Poisson random variable with parameter $\sum_i \lambda_j$.)
2. Suppose it is known that

$$\sqrt{n} \left(\begin{bmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{bmatrix} - \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix} \right) \xrightarrow{d} N(0, \Sigma).$$

(The “0” in the limiting distribution is a 2×1 vector of zeros.) Suppose that Σ is not known, but that we have a consistent estimator $\hat{\Sigma}$.

- (a) Construct an asymptotically valid 95% confidence interval for $\exp(\beta_1)$.
 - (b) Construct an asymptotically valid 95% confidence interval for $\beta_1 \cdot \beta_2$.
3. Show that if $y|X \sim N(X\beta, \sigma^2 I_n)$, where X has full column rank, and σ^2 is known, then the maximum likelihood estimator of β is equal to the OLS estimator.
 4. Suppose we run an OLS regression and calculate the R^2 . If we took the dependent variable (y_i) and multiplied it by a constant, and then ran OLS, would this change the R^2 ? What about if we add a constant to the dependent variable?
 5. Consider the following two-equation model relating schooling s_i and log-earnings y_i to an exogenous variable x_i , measuring distance from the nearest state university. We assume variables are recorded in deviations from sample means, so we model the system without intercept terms as:

$$\begin{aligned} s_i &= \beta x_i + \varepsilon_i \\ y_i &= \delta s_i + u_i \end{aligned}$$

We assume that (ε, u_i) are jointly normally distributed (with possibly nonzero covariance):

$$(\varepsilon, u_i)' \stackrel{\text{iid}}{\sim} N(0, \Sigma).$$

- (a) Will a least squares regression of y_i on s_i consistently estimate δ ? Explain your reasoning.
- (b) Suppose that a regression of s_i on x_i yields a slope coefficient of -2, and that a regression of y_i on x_i yields a slope coefficient of $-.18$. Provide a consistent estimate of δ .
- (c) Explain how your estimate in the previous part relates to the linear IV estimator.

6. Suppose that y_i, x_i are IID with

$$y_i = x_i\beta + u_i,$$

but that $E[x_i u_i] \neq 0$. A researcher proposes the following test of orthogonality: first, calculate the residuals $\hat{u}_i = y_i - x_i \hat{\beta}$, where $\hat{\beta}$ is the OLS estimator. Then calculate

$$\lambda = \frac{1}{n} \sum_i x_i \hat{u}_i.$$

If λ is close to 0, the researcher proposes to accept the assumption of orthogonality. What will happen if the researcher decides to use this test?

7. Suppose that (x_i, y_i) are jointly IID, and that

$$y_i | x_i \sim N(\alpha + \log(x_i)\beta, \sigma^2).$$

However, you estimate a linear regression of y_i on x_i (not on its log). Explain what the OLS estimator will converge to. Suppose you then run a test of heteroskedasticity. Would you expect to find evidence of heteroskedasticity?

8. Suppose (y_i, x_i) are IID, with $E[y_i | x_i] = x_i\beta$. (x_i is a scalar.) Let

$$y = (y_1, \dots, y_n)', \quad X = (x_1, \dots, x_n)', \quad W = (x_1^2, \dots, x_n^2)'$$

Let

$$\tilde{\beta} = (W'X)^{-1}W'y.$$

For each of the following parts, state any additional assumptions necessary for your conclusion.

- (a) Is $\tilde{\beta}$ a consistent estimator of β ?
- (b) Is $\tilde{\beta}$ an unbiased estimator of β ?