**Economics 520, Fall 2009**
**Homework 8**
**Due Tuesday, November 24 at beginning of class (9:30 AM)**

1. Show that the priors in the following cases are conjugate priors:

   (a) $X_1, \ldots, X_n$ is a random sample from the uniform distribution on $(0, \theta)$. The prior for $\theta$ is
   $$f(\theta) = ba^b \theta^{-(b+1)} \cdot 1(\theta > a).$$

   (b) $X_1, \ldots, X_n$ is a random sample from the exponential distribution with density $f(x; \lambda) = \lambda \exp(-\lambda x)$ for $x > 0$. The prior for $\lambda$ is a Gamma distribution with parameters $\alpha$ and $\gamma$.

2. Let $X_1, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution. Let $\tau = \sigma^{-2}$, so we can write the distribution as $N(\mu, \tau^{-1})$.

   Suppose the prior for $(\mu, \tau)$ has density
   $$f(\mu, \tau) \propto \frac{1}{\tau}, \quad -\infty < \mu < \infty, 0 < \tau < \infty.$$

   Note that this is an improper prior density.

   Show that the posterior density of $(\mu, \tau)$ is equal to
   $$f(\mu, \tau | x_1, \ldots, x_n) = f(\mu | \tau, x_1, \ldots, x_n) \cdot f(\tau | x_1, \ldots, x_n),$$

   where $f(\mu | \tau, x_1, \ldots, x_n)$ is a normal density with mean $\bar{x}$ and variance $1/(\tau n) = \sigma^2/n$, and $f(\tau | x_1, \ldots, x_n)$ is a gamma density with parameters
   $$\frac{n-1}{2}, \quad \text{and} \quad \left[ \sum_{i=1}^{n} (x_i - \bar{x})^2 / 2 \right]^{-1}.$$

3. $X_1$ and $X_2$ are independent normally distributed random variables with mean $\mu$ and variances $\sigma_1^2$ and $\sigma_2^2$ respectively. The variances are known and we are interested in estimating the means. Consider estimators of the form $W_{\lambda,\delta} = \lambda X_1 + \delta X_2$. Find the minimum variance unbiased estimator in this class of estimators.

4. $X$ has a binomial distribution with parameters $N = 1$ and $p = 1/2$. $Y$, which is independent of $X$, has a normal distribution with mean $\mu$ and variance 1. Consider the estimator for $\mu$ of the form $W_1 = Y + 2X - 1$.

   (a) Is $W_1$ unbiased?

   (b) What is the variance of $W_1$?

   (c) Consider the estimator $W_2 = E[W_1 | Y]$. Is $W_2$ unbiased? How does its variance compare to that of $W_1$?