**Economics 520, Fall 2009**
**Homework 7**
**Due THURSDAY, November 12 at beginning of class**

1. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with finite mean (not equal to zero) and finite variance, and let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^n X_i$. Let $Z_n = 1/(\bar{X}_n)$. Find the limiting distribution of $Z_n$.

2. Suppose $Y_i \overset{\text{iid}}{\sim} N(2,1)$. Let
$$\bar{Y}_n = \frac{1}{n}\sum_{i=1}^n Y_i.$$
Set $n = 100$, and simulate 1000 draws for $1/\bar{Y}_n$. Compare this to the approximate distribution you derived in the previous question. In particular, examine the mean, the variance, and the overall shape of the distribution of $1/\bar{Y}_n$.

3. Suppose $Z_i \overset{\text{iid}}{\sim} Poisson(\lambda)$, where $\lambda$ is the unknown parameter. Using a sample of size $n$, explain how to construct a method-of-moments estimator for $\lambda$. Is this estimator unbiased?

4. Suppose $Y_1, \ldots, Y_n$ are IID discrete random variables with
$$\begin{aligned} P(Y_i = 0) &= \theta_0, \\ P(Y_i = 1) &= \theta_1, \\ P(Y_i = 2) &= \theta_2, \end{aligned}$$
where the parameter vector $\theta = (\theta_0, \theta_1, \theta_2)$ satisfies: $\theta_j \geq 0$ and $\sum_{j=0}^2 \theta_j = 1$.
   (a) Calculate $E[Y_i]$ and $E[Y_i^2]$, and use the results to derive a method of moments estimator for the parameters $(\theta_1, \theta_2)$.
   (b) Show that the maximum likelihood estimator for $\theta = (\theta_0, \theta_1, \theta_2)$ is
$$\begin{aligned} \hat{\theta}_0 &= \frac{1}{n}\sum_i 1(Y_i = 0), \\ \hat{\theta}_1 &= \frac{1}{n}\sum_i 1(Y_i = 1), \\ \hat{\theta}_2 &= \frac{1}{n}\sum_i 1(Y_i = 2). \end{aligned}$$
   (Hint: be sure to impose the constraint that $\theta_0 + \theta_1 + \theta_2 = 1$.)

5. Suppose that $X_1, X_2, \ldots, X_n$ are IID with probability density function
$$f(x;\theta) = \theta x^{\theta-1}, \quad 0 \leq x \leq 1,$$

where the parameter $\theta > 0$.

(a) Derive the log likelihood function and show that it depends on the data $x_1, \ldots, x_n$ only through $\sum_{i=1}^{n} \log x_i$.

(b) Derive the maximum likelihood estimator for $\theta$.

(c) Now suppose that we observe data where $n = 30$, $\sum_{i=1}^{n} x_i = 20$, and $\sum_{i=1}^{n} \log x_i = -13.67$. Calculate the MLE.