Economics 520, Fall 2009
Homework 6
Due Tuesday, October 27 at beginning of class

1. Consider the following sequence of random variables $W_1, W_2, \ldots$ with

$$W_n = \begin{cases} \frac{1}{n} & \text{with probability } .5 \\ 0 & \text{with probability } .5 \end{cases}$$

   Using the definition of convergence in probability, show that $W_n \xrightarrow{p} 0$.

2. Suppose that ϵ_i (for $i = 0, 1, \ldots$) are IID with mean 0 and variance 1. For $i = 1, 2, \ldots$, let

$$Y_i = \alpha + \epsilon_i + \frac{1}{2}\epsilon_{i-1}.$$

   Let $\bar{Y}_n = \frac{1}{n}\sum_{i=1}^{n} Y_i$. Show that $\bar{Y}_n$ converges to α in probability as $n \to \infty$.

3. Continue the setup of the previous question, but now assume that the ϵ_i are IID standard normal. Derive the (marginal) distribution of

$$\sqrt{n}(\bar{Y}_n - \alpha).$$

   Does $\sqrt{n}(\bar{Y}_n - \alpha)$ converge in distribution?

4. Show that if the sequence $X_1, X_2, \ldots$ converges to a constant θ in probability, then it converges to θ in distribution (that is, it converges in distribution to a degenerate random variable equal to θ with probability 1).

5. R Exercise: Suppose that X_i are IID Uniform on the unit interval, for $i = 1, \ldots, 1000$. In R, generate a draw for the X_i and the running average:

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

   for $n = 1, \ldots, 1000$. (Hint: try using the `cumsum` function in R.) Plot X_n against n, using `plot(x, type="l")` to get a line plot.