Economics 520, Fall 2008
Midterm Solutions

1. $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and $P(\omega_1) = P(\omega_2) = P(\omega_3) = 1/3$. Define random variables X, Y, and Z by:

   $X(\omega_1) = 1, \quad X(\omega_2) = 2, \quad X(\omega_3) = 3$
   $Y(\omega_1) = 2, \quad Y(\omega_2) = 3, \quad Y(\omega_3) = 1$
   $Z(\omega_1) = 2, \quad Z(\omega_2) = 2, \quad Z(\omega_3) = 1$

   (a) $X + Y$:

   $$(X+Y)(\omega_1) = 3$$
   $$(X+Y)(\omega_2) = 5$$
   $$(X+Y)(\omega_3) = 4$$

   So
   $$Pr(X+Y=3) = Pr(X+Y=4) = Pr(X+Y=5) = \frac{1}{3}.$$

   By similar reasoning,
   $$Pr(XY=2) = Pr(XY=3) = Pr(XY=6) = \frac{1}{3}.$$

   (b)
   $$P(Y=2|Z=2) = \frac{1}{2}$$
   $$P(Y=3|Z=2) = \frac{1}{2}$$
   $$P(Y=1|Z=1) = 1.$$

   All other values are 0.

2. Markov's inequality: see lecture notes.

3.
   $$f_Y(y) = 1 - \frac{y}{2}, \quad \text{for } 0 < y < C,$$

   (a) Determine the value of C:
   $$1 = \int_0^C f_Y(y)dy$$
   Solution is $C = 2$.

   (b)
   $$E[Y] = \int_0^2 y f_Y(y)dy = \frac{2}{3}.$$

   (c) $X = 1(Y < 1)$. Note that X is binary, equal to either 0 or 1. It is equal to 1 with probability:
   $$Pr(X=1) = Pr(Y<1) = \int_0^1 f_Y(y)dy = \frac{3}{4}.$$
   So
   $$f_X(0) = \frac{1}{4}, \quad f_X(1) = \frac{3}{4}.$$

(d) First, note that

$$f(y|x=1) = \frac{f_Y(y)Pr(X=1|y)}{f_X(1)} = \frac{(1-y/2)1(y<1)}{3/4}.$$

Then

$$E[Y|X=1] = \int_0^1 y(1-y/2)(4/3)dy = \frac{4}{9}.$$