

Economics 520, Fall 2008
Midterm, October 14, 2008

The exam has 3 questions, with a total of 70 possible points. Write clearly and legibly, and show all your work.

1. (20 points) Let the sample space be $\Omega = \{\omega_1, \omega_2, \omega_3\}$, and suppose $P(\omega_1) = P(\omega_2) = P(\omega_3) = 1/3$. Define random variables X , Y , and Z by:

$$\begin{aligned} X(\omega_1) &= 1, & X(\omega_2) &= 2, & X(\omega_3) &= 3 \\ Y(\omega_1) &= 2, & Y(\omega_2) &= 3, & Y(\omega_3) &= 1 \\ Z(\omega_1) &= 2, & Z(\omega_2) &= 2, & Z(\omega_3) &= 1 \end{aligned}$$

- (a) Find the probability mass function of $X + Y$ and XY .
- (b) Find the conditional probability mass function of Y given Z .
2. (10 points) Prove Markov's inequality.
3. (40 points) Suppose that Y has a PDF function:

$$f_Y(y) = 1 - \frac{y}{2}, \quad \text{for } 0 < y < C,$$

where C is a positive constant.

- (a) Determine the value of C .
- (b) Calculate $E[Y]$.
- (c) Suppose $X = 1(Y < 1)$. Determine the distribution of X .
- (d) Continuing with the definition of X from the part c, determine $E[Y|X = 1]$.