Economics 520, Fall 2008
Homework 8
Due Thursday, November 13 (NOTE CHANGE)

1. Let X_1, X_2, …, X_N represent a random sample from each of the distributions having the following probability density functions. In each case find the maximum likelihood estimator $\hat{\theta}$ of θ.

   (a) $f(x;\theta) = \theta^x \exp(-\theta)/x!$, for $x = 0, 1, \ldots$ and zero elswhere, for $\theta \geq 0$.

   (b) $f(x;\theta) = \theta x^{\theta-1}$, for $0 < x < 1$, and zero elswhere, for $0 < \theta < \infty$.

   (c) $f(x;\theta) = (1/\theta)\exp(-x/\theta)$, for $0 < x < \infty$, and zero elswhere, for $0 < \theta < \infty$.

   (d) $f(x;\theta) = (1/2)\exp(-|x-\theta|$, for $-\infty < x < \infty$, and zero elswhere, for $-\infty < \theta < \infty$.

   (e) $f(x;\theta) = \exp(-(x-\theta))$, for $\theta < x < \infty$, and zero elswhere, for $-\infty < \theta < \infty$.

2. The following numbers were drawn independently from a Poisson distribution:

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   2 2 2 7 2 2 4 2 1 1 3 2 5 4 0 3 2 4 3 2
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   Calculate the maximum likelihood estimate of the Poisson parameter λ.

3. Suppose that X_i are IID $N(\theta, 1)$, where the variance is known, and the parameter θ is known to be ≥ 0. (In other words, $\Theta = [0, \infty)$.)

   (a) Derive the maximum likelihood estimator of θ.

   (b) Show that $\hat{\theta} \xrightarrow{p} \theta$ for all $\theta \in \Theta$. (Hint: consider the two cases $\theta = 0$ and $\theta > 0$.)

4. The pareto distribution is sometimes used to model incomes and has cumulative distribution function

   $$F_X(x;\theta_1,\theta_2) = 1 - (\theta_1/x)^{\theta_2}$$

   for $\theta_1 < x < \infty$ with $\theta_1, \theta_2 > 0$. If $X_1, X_2, \ldots, X_N$ is a random sample from this distribution, find the maximum likelihood estimator for θ_1 and θ_2.