Economics 520, Fall 2008
Homework 6
Due Tuesday, October 28 at beginning of class

1. Suppose that there is a classroom containing 3 white boards. Initially (at time $t = 1$) the boards are blank. In each time period, a professor either: (a) fills an extra board with equations (with probability .25); (b) erases *all* of the boards (with probability .25); or (c) talks and leaves the same number of boards filled with equations. If all of the boards are already filled with equations, then (a) is not an option, and the professor either erases all of the boards (with probability .5) or leaves the boards as they are (with probability .5). If all of the boards are currently empty, then (b) and (c) both result in the boards being empty in the next time period.

   Let X_t denote the number of boards filled with equations at time t. Write down the Markov chain transition matrix. Do you expect the chain to converge to a stationary distribution? If so what is the stationary distribution? (Give as precise a characterization as possible.)

2. Suppose that for each n, $X_n \sim N(1/n, 1/n)$.
   (a) Does $X_n \xrightarrow{p} 0$? Prove your answer.
   (b) Does $X_n \xrightarrow{qm} 0$? Prove your answer.

3. Show that if the sequence $X_1, X_2, \ldots$ converges to a constant θ in probability, then it converges to θ in distribution (that is, it converges in distribution to a degenerate random variable equal to θ with probability 1).

4. Let $X_1, X_2, \ldots$ be a sequence of i.i.d. random variables with finite mean and variance, and let $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Let $Z_n = 1/(\bar{X}_n)$. Find the limiting distribution of Z_n.

5. R Exercise: Suppose that X_i are IID standard normal, for $i = 1, \ldots, 1000$. Simulate the X_i and the running average:

   $$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i,$$

   for $n = 1, \ldots, 1000$. (Hint: try using the `cumsum` function in R.) Plot X_n against n, using `plot(x, type="l")` to get a line plot.