Economics 520, Fall 2008
Homework 3

Due Tuesday, September 23

1. Suppose that the random variable X has cdf

$$F_X(x) = \begin{cases} 0 & \text{if} \quad x < 0 \\ 1/2 & \text{if} \quad x = 0, \\ (x+1)/2 & \text{if} \quad 0 < x \leq 1, \\ 1 & \text{if} \quad x > 1. \end{cases}$$

   Is X a discrete random variable? Is X a continuous random variable? Calculate the mean and variance of X.

2. Let X be a random variable with mean μ and variance σ^2 such that $E((X-\mu)^3)$ exists. The value of the ratio $E((X-\mu)^3)/\sigma^3$ is often used as a measure of skewness. Calculate it for the following distributions.

   (a) $f(x) = (x+1)/2 \quad -1 < x < 1$, and 0 elsewhere.

   (b) $f(x) = 1/2 \quad -1 < x < 1$, and 0 elsewhere.

3. Let X be a random variable with moment generating function $M_X(t)$, $-h < t < h$. Prove that

$$P(X \geq a) \leq \exp(-at) \cdot M_X(t), \quad 0 < t < h,$$

   and

$$P(X \leq a) \leq \exp(-at) \cdot M_X(t), \quad -h < t < 0.$$

   Hint: Use Markov's inequality.

4. Let X be a discrete random variable with $P(X = -1) = 1/8$, $P(X = 0) = 6/8$ and $P(X = 1) = 1/8$, and $P(X = c) = 0$ for all other values of c. Calculate the bound on $P(|X - \mu_X| \geq k \cdot \sigma_X)$ for $k = 2$ using Chebyshev's inequality. Compare this to the actual probability $P(|X - \mu_X| \geq k \cdot \sigma_X)$. This shows that Chebyshev's inequality can be a sharp bound for some k for some random variables.

5. Consider the pdf defined by

$$f_X(x) = \frac{2}{x^3} \quad x > 1,$$

   and zero elsewhere. Show that

   (a) $\int_{-\infty}^{\infty} f_X(x)dx = 1$

   (b) $E(X) = 2$.

   (c) $Var(X)$ is infinite.